Chapterse-wise 15 Years Solved Papers

with Video Solutions

CLASS 10th

MATHEMATICS

Basic and Standard

With Reduced 30 % New Syllabus

CHAP 2 : Polynomials

How to See Vedio ?

Click on Youtube Icon attached with Question.

7. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1 2012]

We have $90 = 9 \times 10 = 9 \times 2 \times 5$

$= 2 \times 3^2 \times 5$

and $144 = 16 \times 9$

$= 2^4 \times 3^2$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

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CHAPTER 2

Polynomials

VERY SHORT ANSWER TYPE QUESTIONS

1. If α and β are the roots of $ax^2 - bx + c = 0\,(a \neq 0)$, then calculate $\alpha + \beta$.

Ans : [Board Term-1, 2014]

We know that

$$\text{Sum of the roots } = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

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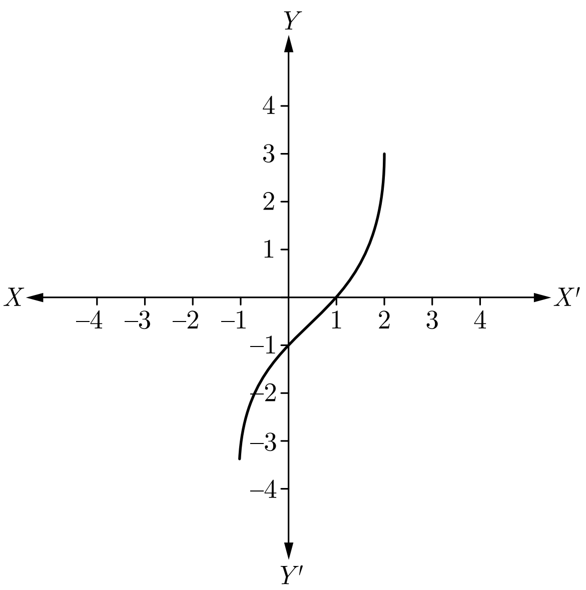
Thus $$\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$$

2. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



Ans :

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The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

3. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Ans :

We have
$$p(x) = 4x^2 - 12x + 9$$
$$= 4x^2 - 6x - 6x + 9$$

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$$= 2x(2x-3) - 3(2x-3)$$
$$= (2x-3)(2x-3)$$

Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$x = \frac{3}{2}, \frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$.

4. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans :

We have $$p(x) = 3x^2 - kx - 6$$

$$\text{Sum of the zeroes } = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

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Thus $$3 = -\frac{(-k)}{3} \Rightarrow k = 9$$

5. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $$f(x) = x^2 - 7x - 8$$

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Let other zero be k, then we have

Sum of zeroes, $$-1 + k = -\left(\frac{-7}{1}\right) = 7$$

or $$k = 8$$

SHORT ANSWER TYPE QUESTIONS - I

6. If zeroes of the polynomial $x^2 + 4x + 2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha} = 2$$

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or, $$2a = 2$$

Thus $$a = 1$$

7. Find all the zeroes of $f(x) = x^2 - 2x$.

Ans : [Board Term-1, 2013, LK-59]

We have $$f(x) = x^2 - 2x$$
$$= x(x-2)$$

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Substituting $f(x) = 0$, and solving we get $x = 0, 2$

Hence, zeroes are 0 and 2.

8. Find the zeroes of the quadratic polynomial $\sqrt{3}\,x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1, 2013, LK-59]

We have
$$p(x) = \sqrt{3}\,x^2 - 8x + 4\sqrt{3}$$
$$= \sqrt{3}\,x^2 - 6x - 2x + 4\sqrt{3}$$
$$= \sqrt{3}\,x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$$
$$= (\sqrt{3}\,x - 2)(x - 2\sqrt{3})$$

Substituting $p(x) = 0$, we have

$$(\sqrt{3}\,x - 2)(x - 2\sqrt{3})\ p(x) = 0$$

Solving we get $x = \frac{2}{\sqrt{3}},\ 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

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9. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1, 2016 Set- LGRKEGO]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x-3)(x-3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

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10. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1, 2012, Set-35]

Sum of zeroes, $\alpha + \beta = \frac{21}{8}$

Product of zeroes $\alpha\beta = \frac{5}{16}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \frac{21}{8}x + \frac{5}{16}$

or $p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

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11. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1, 2012, Set-64]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

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12. What should added to the polynomial $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $x - 3$.

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

We divide $x^3 - 3x^2 + 6x - 15$ by $x - 3$ as follows.

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$$\begin{array}{r|l} & x^2 + 6 \\ x-3 & x^3 - 3x^2 + 6x - 15 \\ & \underline{x^3 - 3x^2} \\ & \quad 6x - 15 \\ & \quad \underline{6x - 18} \\ & \qquad 3 \end{array}$$

Here remainder is 3, hence -3 must be added so that there is no remainder.

13. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1, 2012, Set-40]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

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Substituting in (1) we have

$$\frac{m}{n} + \frac{n}{m} = \frac{(m+n)^2 - 2mn}{mn}$$

$$= \frac{(-\frac{11}{3})^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}}$$

$$= \frac{121 + 4 \times 3 \times 2}{-4 \times 3}$$

or $\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$

14. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

Ans : [Board Term-1, 2012, Set-21]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$

$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$

Hence $p^2 + q^2 = \frac{37}{4}$.

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15. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Ans : [Board Term-1, 2012, Set-50]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

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16. Find the value of k if -1 is a zero of the polynomial

$p(x) = kx^2 - 4x + k$.

Ans : [Board Term-1, 2012, Set-62]

We have $p(x) = kx^2 - 4x + k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$
$$k(-1)^2 - 4(-1) + k = 0$$
$$k + 4 + k = 0$$
$$2k + 4 = 0$$
$$2k = -4$$

Hence, $k = -2$

17. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $p(x) = x^2 - 4\sqrt{3}x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha + \beta = 4\sqrt{3}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$.

18. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Ans : [Board Term-1, 2013, FFC],

We have $p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a \Rightarrow b = -2a = -2$

19. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

We have $f(x) = x^2 - 6x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$$(6)^2 - 2k = 40$$
$$36 - 2k = 40$$
$$-2k = 4$$

Thus $k = -2$

20. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1, 2012, Set-48]

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $0 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

21. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1, 2012, Set-71]

Let, the zero of $2x^2 + 3x + \lambda$ be $\frac{1}{2}$ and β.

Product of zeroes $\frac{c}{a}$, $\frac{1}{2}\beta = \frac{\lambda}{2}$

or, $\beta = \lambda$

and sum of zeroes $-\frac{b}{a}$, $\frac{1}{2} + \beta = -\frac{3}{2}$

or $\beta = -\frac{3}{2} - \frac{1}{2} = -2$

Hence $\lambda = \beta = -2$

Thus other zero is -2.

22. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1, 2013, Set FFC]

We have $f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

Sum of zeroes, $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$$\alpha + \beta = 1$$

...(1)

Given $\alpha - \beta = 9$ ...(2)

Solving (1) and (2) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $k = 20$

23. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1, 2012, Set-39]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\alpha+\beta = \frac{5}{2}$

Product of zeroes $\alpha\beta = -\frac{3}{2}$

According to the question, zeroes of x^2+px+q are 2α and 2β.

Sum of zeros, $2\alpha+2\beta = \frac{-p}{1}$

$$2(\alpha+\beta) = -p$$

Substituting $\alpha+\beta = \frac{5}{2}$ we have

$$2\times\frac{5}{2} = -p$$

or $p = -5$

Product of zeroes, $2\alpha 2\beta = \frac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4\times\frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

24. On dividing x^3-5x^2+6x+4 by a polynomial $g(x)$, the quotient and the remainder were $x-3$ and 4 respectively. Find $g(x)$.

Ans : [Board Term-1, 2012, Set-55]

We have $x^3-5x^2+6x+4 = g(x)(x-3)+4$

$$g(x) = \frac{x^3-5x^2+6x+4-4}{x-3}$$

or, $g(x) = \frac{x^3-5x^2+6x}{x-3}$

Now we divide x^3-5x^2+6x by $x-3$ as follows.

$$\begin{array}{r|l} & x^2-2x \\ x-3 & x^3-5x^2+6x \\ & \underline{x^3-3x^2} \\ & -2x^2+6x \\ & \underline{-2x^2+6x} \\ & 0 \end{array}$$

Hence $g(x) = x^2-2x$.

25. Find the quotient and remainder on dividing $p(x)$ by $g(x)$:
$p(x) = 4x^3+8x^2+8x+7$; $g(x) = 2x^2-x+1$

Ans : [Board Term-1, 2012, Set-55]

Dividing $4x^3+8x^2+8x+7$ by $2x^2-x+1$ we have

$$\begin{array}{r|l} & 2x+5 \\ 2x^2-x+1 & 4x^3+8x^2+8x+7 \\ & \underline{4x^3-2x^2+2x} \\ & 10x^2+6x+7 \\ & \underline{10x^2-5x+7} \\ & 11x+2 \end{array}$$

Thus, Quotient $= 2x+5$

and Remainder $= 11x+2$

26. Check whether the polynomial $g(x) = x^2+3x+1$ is a factor of the polynomial $f(x) = 3x^4+5x^3-7x^2+2x+4$.

Ans : [Board Term-1, 2012, Set-48]

Dividing $3x^4+5x^3-7x^2+2x+4$ by x^2+3x+1 we have

$$\begin{array}{r|l} & 3x^2-4x+2 \\ x^2+3x+1 & 3x^4+5x^3-7x^2+2x+4 \\ & \underline{3x^4+9x^3+3x^2} \\ & -4x^3-10x^2+2x \\ & \underline{-4x^3-12x^2-4x} \\ & 2x^2+6x+4 \\ & \underline{2x^2+6x+2} \\ & 2 \end{array}$$

Since remainder is not zero, polynomial $g(x) = x^2+3x+1$ is not a factor of the polynomial $f(x) = 3x^4+5x^3-7x^2+2x+4$.

27. What should be added in the polynomial $x^3-6x^2+11x+8$ so that is completely divisible by x^2-3x+2 ?

Ans : [Board Term-1, Set, 2015]

Dividing $x^3-6x^2+11x+8$ by x^2-3x+2 we have

$$\begin{array}{r|l} & x-3 \\ x^2-3x+2 & x^3-6x^2+11x+8 \\ & \underline{x^3-3x^2+2x} \\ & -3x^2+9x+8 \\ & \underline{-3x^2+9x-6} \\ & 14 \end{array}$$

Since remainder is 14 to make it zero, -14 should be added.

28. If the polynomial $6x^4+8x^3+17x^2+21x+7$ is divided by another polynomial $3x^2+4x+1$, the remainder comes out to be $(ax+b)$, find the values of a and b.

Ans : [Board Term-1, Set FHN8MGI, 2015]

Dividing $6x^4+8x^3+17x^2+21x+7$ by $3x^2+4x+1$ we have

$$\begin{array}{r|l} & 2x^2+5 \\ 3x^2+4x+1 & 6x^4+8x^3+17x^2+21x+7 \\ & \underline{6x^4+8x^3+2x^2} \\ & 15x^2+21x+7 \\ & \underline{15x^2+20x+5} \\ & x+2 \end{array}$$

Comparing both the sides we get $a = 1$ and $b = 2$

29. If $x^3 - 6x^2 + 6x + k$ is completely divisible by $x - 3$, then find the value of k.

Ans : [Board Term-1, Set-WJQZQBN]

Dividing $x^3 - 6x^2 + 6x + k$ by $x - 3$ we have

$$\begin{array}{r|l} & x^2 - 3x - 3 \\ x - 3 & x^3 - 6x^2 + 6x + k \\ & x^3 - 3x^2 \\ \hline & -3x^2 + 6x + k \\ & -3x^2 + 9x \\ \hline & -3x + k \\ & -3x + 9 \\ \hline & k - 9 \end{array}$$

Remainder should be zero, thus

$$k - 9 = 0$$

So, $k = 9$

30. Divide the polynomial $p(x) = x^3 - 4x + 6$ by the polynomial $g(x) = 2 - x^2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-1E]

Dividing $x^3 - 4x + 6$ by $2 - x^2$ we have

$$\begin{array}{r|l} & -x \\ -x^2 - 2 & x^3 - 4x + 6 \\ & x^3 - 2x \\ \hline & -2x + 6 \end{array}$$

Thus, Quotient $= -x$

and Remainder $= 6 - 2x$

31. Divide the polynomial $p(x) = x^2 - 5x + 16$ by the polynomial $g(x) = x - 2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

Dividing $x^2 - 5x + 16$ by $x - 2$ we have

$$\begin{array}{r|l} & x - 3 \\ x - 2 & x^2 - 5x + 16 \\ & x^2 - 2x \\ \hline & -3x + 16 \\ & -3x + 6 \\ \hline & 10 \end{array}$$

Quotient $= x - 3$, Remainder $= 10$

32. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

Ans : [KVS Practice Test 2017]

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha + \beta = -[-(k-6)] = k - 6$$

$$\alpha\beta = 2(2k-1)$$

Now $\alpha + \beta = \frac{1}{2}\alpha\beta$

Thus $k + 6 = \frac{2(2k-1)}{2}$

or, $k - 6 = 2k - 1$

$k = -5$

Hence the value of k is -5.

SHORT ANSWER TYPE QUESTIONS - II

33. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Ans : [Board Term-1, 2013, LK-59]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$
$$= 16 - 44 + 34 - 6$$
$$= 50 - 50$$

or $p(2) = 0$

(2) 3, $p(3)$

$$= 2(3)^3 - 11(3)^2 + 17(3) - 6$$
$$= 54 - 99 + 51 - 6$$
$$= 105 - 105$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

34. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1, 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

35. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1, 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

36. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2, 2015, Set-DDE-E]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$

$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$

$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

37. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1, 2011, Set-39]

We have $p(y) = 6y^2 - 7y + 2$

Sum of zeroes $\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

Product of zeroes $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$g(x) = y^2 - \frac{7}{2}y + 3$$

$$= \frac{1}{2}[2y^2 - 7y + 6]$$

38. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1, 2011, Set-21]

We have $p(x) = 4x^2 + 4x - 3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$

$$= 1 + 2 - 3 = 0$$

and $p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right) + 4\left(-\frac{3}{2}\right) - 3$

$$= 9 - 6 - 3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$ Verified

39. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1, 2015, Set-FHN8MG0]

We have $p(x)x^2 - 2\sqrt{2}\,x = 0$

$$x(x - 2\sqrt{2}) = 0$$

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $2\sqrt{2} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $0 = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

40. Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1, 2013, Set LK-59]

We have
$$\begin{aligned} p(x) &= 5x^2+8x-4=0 \\ &= 5x^2+10x-2x-4=0 \\ &= 5x(x+2)-2(x+2)=0 \\ &= (x+2)(5x-2) \end{aligned}$$

Substituting $p(x)=0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes } = -2+\frac{2}{5} = \frac{-8}{5}$$

$$\text{Product of zeroes } = (-2)\times\left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

$$\text{Sum of zeroes } \quad -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-8}{5}$$

$$\text{Product of zeroes } \quad \frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$$

Hence Verified.

41. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta=24$ and $\alpha-\beta=8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1, 2011, Set-44]

We have
$$\alpha+\beta = 24 \qquad ...(1)$$
$$\alpha-\beta = 8 \qquad ...(2)$$

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 24 \Rightarrow \beta = 12$$

Hence, the quadratic polynomial

$$\begin{aligned} p(x) &= x^2-(\alpha+\beta)x+\alpha\beta \\ &= x^2-(16+8)x+(16)(8) \\ &= x^2-24x+128 \end{aligned}$$

42. What should be added to x^3+5x^2+7x+3 so that it is completely divisible by x^2+2x.

Ans : [Board Term-1, 2016 Set-MV98HN3]

Dividing x^3+5x^2+7x+3 by x^2+2x we have

Quotient: $x+3$

$x^2+2x \,)\, x^3+5x^2+7x+3$

x^3+2x^2

$3x^2+7x+3$

$3x^2+6x$

$x+3$

Thus if we add $-(x+3)$ remainder will be zero and x^3+5x^2+7x+3 will be completely divisible by x^2+2x.

43. Divide $6x^3+2x^2-4x+3$ by $3x^2-2x+1$ and verify the division algorithm.

Ans : [Board Term-1, 2011, Set-74]

Dividing $6x^3+2x^2-4x+3$ by $3x^2-2x+1$ we have

Quotient: $2x+2$

$3x^2-2x+1 \,)\, 6x^3+2x^2-4x+3$

$6x^3-4x^2+2x$

$6x^2-6x+3$

$6x^2-4x+2$

$-2x+1$

Quotient $=2x+2$; Remainder $=-2x+1$

$$\begin{aligned} p(x) &= g(x)q(x)+r(x) \\ &= (3x^2-2x+1)(2x+2)+(-2x+1) \\ &= 6x^3-4x^2+2x+6x^2-4x+2-2x+1 \\ &= 6x^3+2x^2-4x+3 \end{aligned}$$

Verified

44. Find the value of a and b so that $8x^4+14x^3-2x^2+ax+b$ is exactly divisible by $4x^2+3x-2$.

Ans : [Board Term-1, 2011, Set-66]

Dividing $8x^4+14x^3-2x^2+ax+b$ by $4x^2+3x-2$ we have

Quotient: $2x^2+2x-1$

$4x^2+3x-2 \,)\, 8x^4+14x^3-2x^2+ax+b$

$8x^4+6x^3-4x^2$

$8x^3+2x^2+ax$

$8x^3+6x^2-4x$

$-4x^2+(a+4)x+b$

$-4x^2-3x+2$

$(a+7)x+b-2$

For exact division, remainder must be zero, so

$$\begin{aligned} (a+7)x+b-2 &= 0 \\ a+7 &= 0,\ b-2=0 \\ a &= -7, b=2 \end{aligned}$$

45. On dividing a polynomial $3x^3+4x^2+5x-13$ by a polynomial $g(x)$, the quotient and the remainder are $(3x+10)$ and $(16x-43)$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2011, Set-40]

Dividing $3x^3+4x^2+5x-13$ by $(3x+10)$ we have

Quotient: x^2-2x+3

$3x+10 \,)\, 3x^3+4x^2-11x+30$

3^3+10x^2

$-6x^2-11x$

$-6x^2-20x$

$9x+30$

$9x+30$

0

$$3x^3+4x^2+5x-13=(3x+10)g(x)+(16x-43)$$
$$g(x)(3x+10)=(3x^3+4x^2+5x-13)-(16x-43)$$

Hence, $\quad g(x) = x^2-2x+3$

46. When $p(x)=x^2+7x+9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1, 2011, Set-74]

We have
$$p(x) = x^2+7x+9$$
$$q(x) = x+2$$

$$r(x) = -1$$

Now $$p(x) = g(x)q(x) + r(x)$$

$$x^2 + 7x + 9 = g(x)(x+2) - 1$$

or, $$g(x) = \frac{x^2 + 7x + 10}{x+2}$$

$$= \frac{(x+2)(x+5)}{(x+2)} = x+5$$

Thus $g(x) = x+5$

47. Check by divisible, algorithm whether $x^2 - 2$ is a factor of $x^4 + x^3 + x^2 - 2x - 3$.

Ans : [Board Term-1, 2011, Set-39]

Dividing $x^4 + x^3 + x^2 - 2x - 3$ by $x^2 - 2$ we have

$$\begin{array}{r|l} & x^2 + x + 3 \\ x^2 - 2 & x^4 + x^3 + x^2 - 2x - 3 \\ & \underline{x^4 \qquad - 2x^2} \\ & x^3 + 3x^2 - 2x \\ & \underline{x^3 \qquad - 2x} \\ & 3x^2 \qquad - 3 \\ & \underline{3x^2 \qquad - 6} \\ & 3 \end{array}$$

Since Remainder $= 3 \neq 0$, hence $x^2 - 2$ is not a factor of the given polynomial.

48. On dividing $x^4 - x^3 - 3x^2 + 3x + 2$ by a polynomial $g(x)$, the quotient and the remainder are $x^2 - x - 2$ and $2x$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2015, Set-CJTOQ]

Dividing $x^4 - x^3 - 3x^2 + 3x + 2$ by $x^2 - x - 2$ we have

$$\begin{array}{r|l} & x^2 - 1 \\ x^2 - x - 2 & x^4 - x^3 - 3x^2 + x + 2 \\ & \underline{x^4 - x^3 - 2x^2} \\ & -x^2 + x + 2 \\ & \underline{-x^2 + x + 2} \\ & 0 \end{array}$$

Now

$$x^4 - x^3 - 3x^2 + 3x + 2 = (x^2 - x - 2)g(x) + 2x$$

$$g(x)(x^2 - x - 2) = (x^4 - x^3 - 3x^2 + 3x + 2) - 2x$$

$$g(x) = \frac{x^4 - x^3 - 3x^2 + x + 2}{x^2 - x - 2}$$

Hence, $$g(x) = x^2 - 1$$

49. What should be added in the polynomial $x^3 + 2x^2 - 9x + 1$ so that it is completely divisible by $x + 47$.

Ans : [Board Term-1, 2015, Set-DDE-M]

Dividing $x^3 + 2x^2 - 9x + 1$ by $x + 47$ we have

$$\begin{array}{r|l} & x^2 - 2x - 1 \\ x+4 & x^3 + 2x^2 - 9x + 1 \\ & \underline{x^3 + 4x^2} \\ & -2x^2 - 9x + 1 \\ & \underline{-2x^2 - 8x} \\ & -x + 1 \\ & \underline{-x - 4} \\ & 5 \end{array}$$

If we add -5 in $x^3 + 2x^2 - 9x + 1$, remainder will be zero.

50. If the polynomial $f(x) = 3x^4 + 3x^3 - 11x^2 - 5x + 10$ is completely divisible by $3x^2 - 5$, find all its zeroes.

Ans : [Board Term-1, 2013, FFC; 2011, Set-13]

Since $3x^2 - 5$ divides $f(x)$ completely, $(3x^2 - 5)$ is a factor of $f(x)$.

Thus $$3x^2 - 5 = 0$$

$$x^2 = \frac{5}{3}$$

$$x = \pm\sqrt{\frac{5}{3}}$$

$$\begin{array}{r|l} & x^2 + x - 2 \\ 3x^2 - 5 & 3x^4 + 3x^3 - 11x^2 - 5x + 10 \\ & \underline{3x^4 \qquad - 5x^2} \\ & 3x^3 - 6x^2 - 5x + 10 \\ & \underline{3x^3 \qquad - 5x} \\ & -6x^2 \qquad + 10 \\ & \underline{-6x^2 \qquad + 10} \\ & 0 \end{array}$$

Now $$x^2 + x - 2 = x^2 + 2x - x - 2$$

$$= x(x+2) - (x+2)$$

$$= (x+2)(x-1)$$

Since $(x^2 + x - 2) = (x+2)(x-1)$ is a factor of $p(x.)$, thus -2 and 1 are zeroes or $p(x)$.

All the zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -2$ and 1.

51. If α, β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

Ans : [KVS practice Test 2017, Board 2010]

We have $$p(x) = 6x^3 + 3x^2 - 5x + 1$$

Since α, β and γ are zeroes polynomial $p(x)$, we have

$$\alpha + \beta + \gamma = -\frac{b}{c} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

and $$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

Now $$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6} \times \frac{6}{-1} = 5$$

Hence $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

LONG ANSWER TYPE QUESTIONS

52. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q.

Ans : [Board Term-1, 2015, Set DDE-M]

We have $\quad f(x) = x^4 + 7x^3 + 7x^2 + px + q$

Now
$$x^2 + 7x + 12 = 0$$
$$x^2 + 4x + 3x + 12 = 0$$
$$x(x+4) + 3(x+4) = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4, -3$$

Since $f(x) = x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ must be its zeroes and these must satisfy $f(x) = 0$

So putting $x = -4$ and $x = -3$ in $f(x)$ and equating to zero we get

$$f(-4) : \ (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q = 0$$
$$256 - 448 + 112 - 4p + q = 0$$
$$-4p + q - 80 = 0$$
$$4p - q = -80 \qquad ...(1)$$
$$p(-3) : \ (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q = 0$$
$$81 - 189 + 63 - 3p + q = 0$$
$$-3p + q - 45 = 0$$
$$3p - q = -45 \qquad ...(2)$$

Subtracting eq, (2) from (1) we have
$$p = -35$$

Substituting the value of p in eq. (1) we have
$$4(-35) - q = -80$$
$$-140 - q = -80$$
$$-q = 140 - 80$$
or
$$-q = 60$$
$$q = -60$$

Hence, $p = -35$ and $q = -60$.

53. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

Ans : [Board Term-1, 2012, Set-50]

We have $\quad p(x) = 2x^2 + 5x + k$

Sum of zeroes, $\quad \alpha + \beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\quad \alpha\beta = \frac{c}{a} = \frac{k}{2}$

According to the question,
$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$
$$\alpha^2 + \beta^2 + 2\alpha\beta - \alpha\beta = \frac{21}{4}$$
$$(\alpha + b)^2 - \alpha\beta = \frac{21}{4}$$

Substituting values we have
$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$
$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$
$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

54. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1, 2012, Set-45, 62, 2010, Set-15]

We have $\quad p(x) = 3x^2 + 2x + 1$

Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$, we have
$$\alpha + \beta = -\frac{2}{3}$$

and
$$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,
$$\alpha_1 + \beta_1 = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$
$$= \frac{(1 - \alpha + \beta - \alpha\beta) + (1 + \alpha - \beta - \alpha\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{2 - 2\alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{2 - \frac{2}{3}}{1 - \frac{2}{3} + \frac{1}{3}}$$
$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,
$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$
$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{1 - \alpha - \beta + \alpha\beta}{1 + \alpha + \beta + \alpha\beta}$$
$$= \frac{1 - (\alpha + \beta) + \alpha\beta}{1 + (\alpha + \beta) + \alpha\beta}$$
$$= \frac{1 + \frac{2}{3} + \frac{1}{3}}{1 - \frac{2}{3} + \frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial
$$q(x) = x^2 - (\alpha_1 + \beta_1)2x + \alpha_1\beta_1$$
$$= x^2 - 2x + 3$$

55. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

Ans : [Board Term-1, 2013 LK-59]

We have $\quad p(x) = x^2 + 4x + 3$

Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$,

So, $\quad \alpha + \beta = -4$

and $\quad \alpha\beta = 3$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1 + \beta_1 = 1 + \frac{\alpha}{\beta} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, required polynomial

$$q(x) = x^2 - (\alpha_1 + \beta_1)2x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

56. If α and β are zeroes of the polynomial $p(x) = 6x - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Ans :

We have $p(x) = 6x - 5x + k$

Since α and β are zeroes of

$$p(x) = 6x - 5x + k,$$

Sum of zeroes, $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha - \beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

57. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2 + a)x^2 + 61x + 6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2 + a)x^2 + 61x + 6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2 + a}$

or, $1 = \frac{6}{a+1}$

$$a + 1 = 6$$

$$a = 5$$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \frac{-61 \pm \sqrt{(-61)^2 \times 4 \times 30 \times 30}}{2 \times 30}$

$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

58. If two zeroes of a polynomial $x^3 + 5x^2 + 7x + 3$ are -1 and -3, then find the third zero.

Ans : [Board Term-1, 2016 Set MV98HN3]

Since -1 and -3 are zeros of $x^3 + 5x^2 + 7x + 3$, $(x+1)$ and $(x+3)$ are factor of it and it divides it completely.

$$(x+1)(x+3) = x^2 + 4x + 3$$

$$\begin{array}{r|l} & x + 1 \\ x^2 + 4x + 3 & x^3 + 5x^2 + 7x + 3 \\ & x^3 + 4x^2 + 3x \\ \hline & x^2 + 4x + 3 \\ & x^2 + 4x + 3 \\ \hline & 0 \end{array}$$

Thus third zero is $x = -1$.

59. Given that $x - \sqrt{5}$ is a factor of the polynomial $x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$, find all the zeroes of the polynomial.

Ans : [Board Term-1, 2014] [Board Term-1, 2012, Set-39]

Dividing $x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$ by $x - \sqrt{5}$ we have

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x - 15 \\ x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5} \\ & x^3 - \sqrt{5}x^2 \\ \hline & -2\sqrt{5}x^2 - 5x + 15\sqrt{5} \\ & -2\sqrt{5}x^2 + 10x \\ \hline & -15x + 15\sqrt{5} \\ & -15x + 15\sqrt{5} \\ \hline & 0 \end{array}$$

Factorising the quotient we have

$$\begin{aligned} x^2 - 2\sqrt{5}\,x - 15 &= x^2 - 3\sqrt{5}\,x + \sqrt{5}\,x - 15 \\ &= x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5}) \\ &= (x + \sqrt{5})(x - 3\sqrt{5}) \end{aligned}$$

$$(x + \sqrt{5})(x - 3\sqrt{5}) = 0 \quad \Rightarrow \quad x = \sqrt{5},\ 3\sqrt{5}$$

Thus zeroes are $\sqrt{5}$, $-\sqrt{5}$ and $3\sqrt{5}$.

60. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $(x^2 - 2x + k)$, the remainder comes out to be $x + a$, find k and a.

Ans : [Board Term-1, 2012, Set-35]

Dividing $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $(x^2 - 2x + k)$ we have

$$\begin{array}{l} \text{Quotient: } x^2 - 4x + (8 - k) \\ x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 25x + 10 \\ \underline{x^4 - 2x^3 + kx^2} \\ -4x^3 + (16 - k)x^2 - 25x + 10 \\ \underline{-4x^3 + 8x^2 - 4kx} \\ (8 - k)x^2 - (25 - 4k)x + 10 \\ \underline{(8 - k)x - (16 - 2k)x + (8k - k^2)} \\ (2k - 9)x + (10 - 8k + k^2) \end{array}$$

Given, remainder $= x + a$

Comparing the multiples of x we have

$$\begin{aligned} (2k - 9)x &= 1 \times x \\ 2k - 9 &= 1 \\ k &= \frac{10}{2} = 5 \end{aligned}$$

Substituting this value of k into other portion of remainder, we have

and $\quad a = 10 - 8k + k^2 = 10 - 40 + 25 = -5$

61. Find the other zeroes of the polynomial $x^4 - 5x^3 + 2x^2 + 10x - 8$ if it is given that two zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Ans : [Board Term-1, 2012, Set-35]

We have two zeroes $\sqrt{2}$ and $-\sqrt{2}$. Thus two factors are $(x + \sqrt{2})$ and $(x - \sqrt{2})$.

Thus $g(x) = (x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$ is a factor of the given polynomial

Now dividing $x^4 - 5x^3 + 2x^2 + 10x - 8$ by $x^2 - 2$ we have

$$\begin{array}{l} \text{Quotient: } x^2 - 5x + 4 \\ x^2 - 2 \,\big)\, x^4 - 5x^3 + 2x^2 + 10x - 8 \\ \underline{x^4 \quad - 2x^2} \\ -5x^3 + 4x^2 + 10x - 8 \\ \underline{-5x^3 \quad - 10x} \\ 4x^2 \quad - 8 \\ \underline{4x^2 \quad - 8} \\ 0 \end{array}$$

Quotient $= x^2 - 5x + 4 = (x - 4)(x - 1)$

Hence other zeroes are 4 and 1.

62. Show that 3 is a zero of the polynomial $2x^2 - x^2 - 13x - 6$. Hence find all the zeroes of this polynomial.

Ans :

We have $\quad p(x) = 2x^3 - x^2 - 13x - 6$

Substituting $x = 3$ in $p(x) = 0$ we have

$$\begin{aligned} p(x) &= 2(3)^3 - (3)^2 - 13(3) - 6 \\ &= 2(27) - 9 - 39 - 6 \\ &= 54 - 54 = 0 \end{aligned}$$

So, $x - 3$ is a factor of $p(x)$. Now by long division

$$\begin{array}{l} \text{Quotient: } 2x^2 + 5x + 2 \\ x - 3 \,\big)\, 2x^3 - x^2 - 13x - 6 \\ \underline{2x^3 - 6x^2} \\ 5x^2 - 13x - 6 \\ \underline{5x^2 - 15x} \\ 2x - 6 \\ \underline{2x - 6} \\ 0 \end{array}$$

Factorising the quotient, we get

$$\begin{aligned} 2x^2 + 5x + 2 &= 2x^2 + 4x + x + 2 \\ &= 2x(x + 2) + (x + 2) \\ &= (2x + 1)(x + 2) \\ x &= -\frac{1}{2}, -2 \end{aligned}$$

Hence, all the zeroes of $p(x)$ are $-\frac{1}{2}, -2, 3$

63. Obtain all other zeroes of the polynomial $x^4 + 6x^3 + x^2 - 24x - 20$, if two of its zeroes are $+2$ and -5.

Ans : [Board Term-1, 2015, NCERT]

As $x = 2$ and $x = -5$ are the zeroes of $x^4 + 6x^3 + x^2 - 24x - 20$.

So $(x - 2)$ and $(x + 5)$ are two factors of $x^4 + 6x^3 + x^2 - 24x - 20$ and the product of factors is

$$(x - 2)(x + 5) = x^2 + 3x - 10 = 0$$

Dividing $x^4 + 6x^3 + x^2 - 24x - 20$ by $x^2 + 3x - 10$

$$\begin{array}{l} \text{Quotient: } x^2 + 3x + 2 \\ x^2 + 3x - 10 \,\big)\, x^4 + 6x^3 + x^2 - 24x - 20 \\ \underline{x^4 + 3x^3 - 10x^2} \\ 3x^3 + 11x^2 - 24x - 20 \\ \underline{3x^3 + 9x^2 - 30x} \\ 2x^2 + 6x - 20 \\ \underline{2x^2 + 6x - 20} \\ 0 \end{array}$$

Thus

$$\begin{aligned} x^4 &+ 6x^3 + x^2 - 24x - 20 \\ &= (x^2 + 3x - 10)(x^2 + 3x + 2) \\ &= (x - 2)(x + 5)(x + 2)(x + 1) \end{aligned}$$

Hence other two zeroes are -2 and 1.

64. Obtain all other zeroes of the polynomial $4x^4 + x^3 - 72x^2 - 18x$, if two of its zeroes are $3\sqrt{2}$ and

$-3\sqrt{2}$.

Ans : [Board Term-1, 2015, Set-C3TOQ]

As $3\sqrt{2}$ and $-3\sqrt{2}$ are the zeroes of $4x^4+x^3-72x^2-18x$, So $(x-3\sqrt{2})$ and $(x+3\sqrt{2})$ are its two factors.

Now, $(x-3\sqrt{2})(x+3\sqrt{2}) = 0$

or, $x^2-18 = 0$

Thus x^2-18 divides the polynomial $4x^4+x^3-72x^2-18x$ completely.

Now dividing $4x^4+x^3-72x^2-18x$ by x^2-18 we have

$$\begin{array}{r|l} & 4x^2+x \\ x^2-18 & 4x^4+x^3-72x^2-18x \\ & 4x^4 \quad -72x^2 \\ \hline & x^3 \quad -18x \\ & x^3 \quad -18x \\ \hline & 0 \end{array}$$

Factorising quotient $4x^2+x$ we have

$$4x^2+x = x(4x+1)$$

Now $x = 0$ and $-\frac{1}{4}$

Thus

$$4x^4+x^3-72x^2-18x = (x^2-18)x(4x+1)$$
$$= (x-3\sqrt{2})(x+3\sqrt{2})(x)(4x+1)$$

Hence, other two zeroes are 0 and $\frac{-1}{4}$.

65. Obtain all other zeroes of the polynomial $9x^4-6x^3-35x^2+24x-4$, if two of its zeroes are 2 and -2.

Ans : [Board Term-1, 2015, Set -DDE -M]

As 2 and -2 are the zeroes of $9x^4-6x^3-35x^2+24x-4$

So $(x-2)$ and $(x+2)$ are its two factors

Now $(x-2)(x+2) = x^2-4$

Dividing $9x^4-6x^3-35x^2+24x-4$ by x^2-4

$$\begin{array}{r|l} & 9x^2-6x+1 \\ x^2-4 & 9x^4-6x^3-35x^2+24x-4 \\ & 9x^4 \quad -36x^2 \\ \hline & -6x^3+x^2+24x-4 \\ & -6x^3 \quad +24x \\ \hline & x^2 \quad -4 \\ & x^2 \quad -4 \\ \hline & 0 \end{array}$$

Factorising this quotient

$$9x^2-6x+1 = 9x^2-3x-3x+1$$
$$= [3x(3x-1)-1(3x-1)]$$
$$= [(3x-1)(3x-1)]$$
$$= (3x-1)(3x-1)$$

Hence, other two zeroes are $\frac{1}{3}, \frac{1}{3}$.

66. Find all the zeros of the polynomial $3x^4+6x^3-2x^2-10x-5$ it two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans : [SQP 2017]

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the given polynomial, $\left(x-\sqrt{\frac{5}{3}}\right), \left(x+\sqrt{\frac{5}{3}}\right)$ will be its two factors

Now $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = \left(x^2-\frac{5}{3}\right) = \frac{1}{3}(3x^2-5)$

Since $\frac{1}{3}(3x^2-5)$ is a factor of given polynomial, dividing it by $3x^2-5$, we have

$$\begin{array}{r|l} & x^2+2x+1 \\ 3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\ & 3x^4 \quad -5x^2 \\ \hline & 6x^3+3x^2-10x-5 \\ & 6x^3 \quad -10x \\ \hline & 3x^2 \quad -5 \\ & 3x^2 \quad -5 \\ \hline & 0 \end{array}$$

$$x^2+2x+1 = (x+1)^2 = (x+1)(x+1)$$

Thus two other zeroes are -1 and -1.

Hence all the zeroes of given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, $-1$and -1.

HOTS QUESTIONS

67. Find the value for k for which $x^4+10x^3+25x^2+15x+k$ is exactly divisible by $x+7$.

Ans :

We have $f(x) = x^4+10x^3+25x^2+15x+k$

If $x+7$ is a factor then -7 is a zero of $f(x)$ and $x=-7$ satisfy $f(x)=0$.

Thus substituting $x=-7$ in $f(x)$ and equating to zero we have,

$$(-7)^4+10(-7)^3+25(-7)^2+15(-7)+k = 0$$
$$2401-3430+1225-105+k = 0$$
$$3626-3535+k = 0$$
$$91+k = 0$$
$$k = -91$$

68. If two zeroes of the polynomial $p(x) = x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$. Find the other zeroes.

Ans :

We have $p(x) = x^4-6x^3-26x^2+138x-35$

As $2\pm\sqrt{3}$ are the zeroes of $p(x)$, so $x-(2\pm\sqrt{3})$ are the factor of $p(x)$ and the product of zeros, is

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$$
$$= \{(x-2)-\sqrt{3}\}\{(x-2)+\sqrt{3}\}$$
$$= (x-2)^2-(\sqrt{3})^2$$
$$= x^2-4x+1$$

Dividing $p(x)$ by x^2-4x+1 we have

$$\begin{array}{r} x^2-2x-35 \\ x^2-4x+1 \overline{\big) x^4-6x^3-26x^2+138x-35} \\ \underline{x^4-4x^3+x^2} \\ -2x^3-27x^2+138x-35 \\ \underline{-2x^3+8x^2-2x} \\ -35x^2+140x-35 \\ \underline{-35x^2+140x-35} \\ 0 \end{array}$$

Factorising $(x^2-2x-35)$ we get

$$=(x+5)(x-7)$$

$$x=-5,7$$

Hence, other two zeroes of $p(x)$ are -5 and 7. 1

69. If α and β are the zeroes the polynomial $2x^2-4x+5$, find the values of

(i) $\alpha^2+\beta^2$ (ii) $\frac{1}{\alpha}+\frac{1}{\beta}$

(iii) $(\alpha-\beta)^2$ (iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

(v) $\alpha^2+\beta^2$

Ans :

We have $p(x)=2x^2-4x+5$

If α and β are then zeroes of $p(x)=2x^2-4x+5$, then

$$\alpha+\beta=-\frac{b}{a}=\frac{-(-4)}{2}=2$$

and $$\alpha\beta=\frac{c}{a}=\frac{5}{2}$$

(i) $$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=2^2-2\times\frac{5}{2}$$

$$=4-5=-1$$

(ii) $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{2}{\frac{5}{2}}=\frac{4}{5}$$

(iii) $$(\alpha-\beta)^2=(\alpha-\beta)^2-4\alpha\beta$$

$$=2^2-\frac{4\times5}{2}$$

$$4-10=-6$$

(iv) $$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=\frac{-1}{\left(\frac{5}{2}\right)^2}=\frac{-4}{25}$$

(v) $$(\alpha^3+\beta^3)=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$

$$=2^3-3\times\frac{5}{2}\times2=8-15=-7$$

70. On dividing the polynomial $4x^4-5x^3-39x^2-46x-2$ by the polynomial $g(x)$, the quotient is x^2-3x-5 and the remainder is $-5x+8$. Find the polynomial $g(x)$.

Ans :

Dividend = (Divisor × Quotient) + Remainder

$4x^4-5x^3-39x^3-46x-2$

$$=g(x)(x^2-3x-5)+(-5x+8)$$

or, $4x^2-5x^3-39x^2-46x-2+5x-8$

$$=g(x)(x^2-3x-5)$$

or, $$4x^4-5x^3-39x^2-41x-10=g(x)(x^2-3x-5)$$

$$g(x)=\frac{4x^4-5x^3-39x^2-41x-10}{(x^2-3x-5)}$$

Hence, $$g(x)=4x^2+7x+2$$

71. If the squared difference of the zeroes of the quadratic polynomial $f(x)=x^2+px+45$ is equal to 144, find the value of p.

Ans :

We have $f(x)=x^2+px+45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha+\beta=-p$

Product of zeroes $\alpha\beta=45$

Given, $$(\alpha-\beta)^2=144$$

$$(\alpha+\beta)^2-4\alpha\beta=144$$

Substituting value of $\alpha+\beta$ and $\alpha\beta$ we get

$$(-p)^2-4\times45=144$$

$$p^2-180=144$$

$$p^2=144+180=324$$

Thus $$p=\pm\sqrt{324}=\pm18$$

Hence, the value of p is ±18.