For JEE & Other Engineering Entrance Examinations

PHYSICS MCQ

MULTIPLE-CHOICE-QUESTION BANK

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Physics
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Preface

The entrance examinations for almost all the premier engineering colleges of the country are presently in the multiple-choice format. This includes the JEE for the IITs and the AIEEE for the other engineering colleges of excellence. This book is designed as the basic study material in physics for these two tests and various other competitive examinations.

Most of the problems presented in this book have actually evolved in my classrooms. While guiding students for competitive examinations for close to four decades, I have had to innovate problems to make them *think*. The sequence of their learning process has been first to grasp the basic concepts, then to correlate the problems with the concepts involved, and finally to get a lot of practice to achieve speed and accuracy. This book is a summary of my course material.

I have found it very useful to summarize most of the concepts into single sentences. One may almost call them the *sutras* of +2 physics. Many of these one-liners appear in the solutions. The questions have been devised to drill the students in all possible applications of these concepts. To limit the size of the book, only those questions have been used which have proved most effective in my classrooms.

This book is divided into nine parts. The first seven parts cover both theoretical and experimental physics, topicwise. One separate part is devoted to miscellaneous questions covering almost all types of objective questions that may be asked in the major competitive tests. Miscellaneous Questions—1 includes straight-objective questions of both single- and multiple-option-correct types. Miscellaneous Questions—2, 3 and 4 respectively contains assertion–reason-type, linked-comprehension-type and matrix-matching-type questions. The ninth part provides the students with several practice papers including model IIT-JEE test papers, which should help them take mock tests in a fixed time.

Both the IIT-JEE and the AIEEE are based on questions that have only one correct choice. This book, however, has a large number of questions in which more than one answer may be correct. These are the *teaching* questions and train the students towards the comprehension which IIT-JEE demands. Moreover, these more-than-one-option-correct objective-type questions are sometimes asked in various tests other than IIT-JEE and AIEEE. Several test papers in the single-choice format should help in the final run-up.

I hope that students and teachers will come forward with suggestions for improvements in the book. I thank the personnel in the editorial and production departments at Bharati Bhawan for their untiring efforts and enthusiasm in bringing out this book.

Author

To the Students

The purpose of this book is twofold—first, to teach you the concepts required to tackle the IIT-JEE and AIEEE, and second, to provide practice material for these and other engineering entrance tests. Therefore, although it is possible to frame a large number of multiple-choice questions, in this book only those questions have been given which teach you important concepts and show you the different ways in which they can be applied. If you master these concepts, you can be reasonably sure of working out any multiple-choice question that comes your way.

Some of the questions merit hints and some others, the full solutions. The hints and solutions are a very important part of the book. They will clarify your doubts and train you to think along the correct lines. Concepts and techniques which need special attention are highlighted with the TIP icon. Understand these well, for they have wide application.

In most parts of the book, the straight-objective questions are divided into two types, approximately in equal numbers. Type 1 questions have only one correct option, whereas type 2 questions can have more than one correct option. In this book, there is no difference in the level of difficulty between the two types. However, type 2 questions are more demanding as every option has to be examined carefully. Try to reason out why each option is either correct or incorrect. This will teach you to think logically. They will also help you in comprehension-type questions. Look at the solutions to *fine-tune* your thinking.

You will notice that some questions that have been asked in the previous IIT examinations have been included here. This is to give you an idea about the level of difficulty expected at the tests.

The IIT entrance examinations of 2007 and 2008 have introduced three new question patterns, viz., linked-comprehension type, assertion–reason type and matrix-matring type. Assuming that future examinations would also include such questions, separate sections and chapters have been added to help you prepare for them.

Contents

□

Part 1

Mechanics

1

Kinematics, Forces

• *Type 1* •

Choose the correct option (a, b, c or d).

1. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is

(a) 10 units (b) 7 units
(c) $7\sqrt{2}$ units (d) 8.5 units

2. A bird flies for 4 s with a velocity of $|t - 2|$ m/s in a straight line, where t = time in seconds. It covers a distance of

(a) 2 m (b) 4 m
(c) 6 m (d) 8 m

3. A particle has an initial velocity of 9 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in the fifth second of its motion is

(a) 0 (b) 0.5 m
(c) 2 m (d) none of these

4. A particle moves in a straight line with a retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x (b) x^2
(c) $\ln x$ (d) e^x

5. For a particle moving along a straight line, the displacement x depends on time t as $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$. The ratio of its initial acceleration to its initial velocity depends

(a) only on α and β
(b) only on β and γ
(c) only on α and γ
(d) only on α

6. From the top of a tower, a stone is thrown up and it reaches the ground in time t_1. A second stone is thrown down with the same speed and it reaches the ground in time t_2. A third stone is released from rest and it reaches the ground in time t_3.

(a) $t_3 = \frac{1}{2}(t_1 + t_2)$
(b) $t_3 = \sqrt{t_1 t_2}$
(c) $\frac{1}{t_3} = \frac{1}{t_2} - \frac{1}{t_1}$
(d) $t_3^2 = t_1^2 - t_2^2$

7. Water drops fall at regular intervals from a roof. At an instant when a drop is about to leave the roof, the separations between 3 successive drops below the roof are in the ratio

(a) 1 : 2 : 3
(b) 1 : 4 : 9
(c) 1 : 3 : 5
(d) 1 : 5 : 13

8. Three particles A, B and C are thrown from the top of a tower with the same speed. A is thrown straight up, B is thrown straight down and C is thrown horizontally. They hit the ground with speeds v_A, v_B and v_C respectively.

(a) $v_A = v_B = v_C$
(b) $v_B > v_C > v_A$
(c) $v_A = v_B > v_C$
(d) $v_A > v_B = v_C$

9. A balloon starts rising from the ground with an acceleration of 1.25 m/s^2. After 8 s, a stone is released from the balloon. The stone will

(a) cover a distance of 40 m
(b) have a displacement of 50 m
(c) reach the ground in 4 s
(d) begin to move down after being released

10. A particle thrown up vertically reaches its highest point in time t_1 and returns to the ground in a further time t_2. The air resistance exerts a constant force on the particle opposite to its direction of motion.

(a) $t_1 > t_2$

(b) $t_1 = t_2$

(c) $t_1 < t_2$

(d) May be (a) or (c) depending on the ratio of the force of air resistance to the weight of the particle.

11. An aeroplane flying at a constant velocity releases a bomb. As the bomb drops down from the aeroplane,

(a) it will always be vertically below the aeroplane

(b) it will always be vertically below the aeroplane only if the aeroplane is flying horizontally

(c) it will always be vertically below the aeroplane only if the aeroplane is flying at an angle of 45° to the horizontal

(d) it will gradually fall behind the aeroplane if the aeroplane is flying horizontally

12. A projectile is moving at 60 m/s at its highest point, where it breaks into two equal parts due to an internal explosion. One part moves vertically up at 50 m/s with respect to the ground. The other part will move at

(a) 110 m/s (b) 120 m/s

(c) 130 m/s (d) $10\sqrt{61}$ m/s

13. Two particles are projected simultaneously in the same vertical plane, from the same point, but with different speeds and at different angles to the horizontal. The path followed by one, as seen by the other, is

(a) a vertical straight line

(b) a straight line making a constant angle ($\neq 90°$) with the horizontal

(c) a parabola

(d) a hyperbola

14. Two particles are projected simultaneously in the same vertical plane from the same point, with different speeds u_1 and u_2, making angles θ_1 and θ_2 respectively with the horizontal, such that $u_1\cos\theta_1 = u_2\cos\theta_2$. The path followed by one, as seen by the other (as long as both are in flight), is

(a) a horizontal straight line

(b) a vertical straight line

(c) a parabola

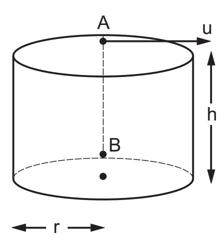
(d) a straight line making an angle $|\theta_1 - \theta_2|$ with the horizontal

15. A particle is thrown with a speed u at an angle θ to the horizontal. When the particle makes an angle ϕ with the horizontal, its speed changes to v.

(a) $v = u\cos\theta$ (b) $v = u\cos\theta \cdot \cos\phi$

(c) $v = u\cos\theta \cdot \sec\phi$ (d) $v = u\sec\theta \cdot \cos\phi$

16.

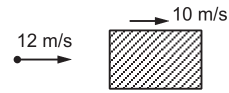


A hollow vertical cylinder of radius r and height h has a smooth internal surface. A small particle is placed in contact with the inner side of the upper rim, at point A, and given a horizontal speed u, tangential to the rim. It leaves the lower rim at point B, vertically below A. If n is an integer then

(a) $\frac{u}{2\pi r}\sqrt{2h/g} = n$ (b) $\frac{h}{2\pi r} = n$

(c) $\frac{2\pi r}{h} = n$ (d) $\frac{u}{\sqrt{2gh}} = n$

17. A light particle moving horizontally with a speed of 12 m/s strikes a very heavy block moving in the same direction at 10 m/s. The collision is one-dimensional and elastic. After the collision, the particle will



(a) move at 2 m/s in its original direction

(b) move at 8 m/s in its original direction

(c) move at 8 m/s opposite to its original direction

(d) move at 12 m/s opposite to its original direction

18. A ball falls vertically onto a floor, with momentum p, and then bounces repeatedly. The coefficient of restitution is e. The total momentum imparted by the ball to the floor is

(a) $p(1 + e)$ (b) $\frac{p}{1 - e}$

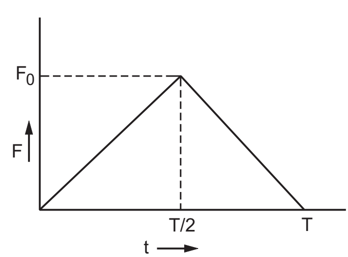
(c) $p\left(1 + \frac{1}{e}\right)$ (d) $p\left(\frac{1 + e}{1 - e}\right)$

19. A ball falls from rest from a height h onto a floor, and rebounds to a height $h/4$. The coefficient of restitution between the ball and the floor is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

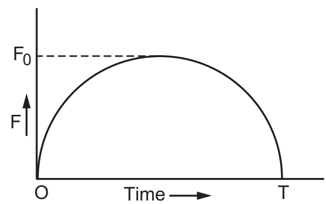
20.



A particle of mass m moving with velocity u makes an elastic one-dimensional collision with a stationary particle of mass m. They are in contact for a very brief time T. Their force of interaction increases from zero to F_0 linearly in time $T/2$, and decreases linearly to zero in a further time $T/2$. The magnitude of F_0 is

(a) mu/T (b) $2mu/T$
(c) $mu/2T$ (d) none of these

21.



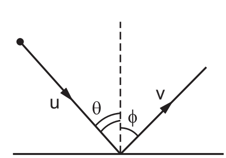
A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle.

(a) $u = \frac{\pi F_0^2}{2m}$ (b) $u = \frac{\pi T^2}{8m}$
(c) $u = \frac{\pi F_0 T}{4m}$ (d) $u = \frac{F_0 T}{2m}$

22. Two identical spheres move in opposite directions with speeds v_1 and v_2 and pass behind an opaque screen, where they may either cross without touching (Event 1) or make an elastic head-on collision (Event 2).

(a) We can never make out which event has occurred.
(b) We cannot make out which event has occurred only if $v_1 = v_2$.
(c) We can always make out which event has occurred.
(d) We can make out which event has occurred only if $v_1 = v_2$.

23. A particle strikes a horizontal frictionless floor with a speed u, at an angle θ to the vertical, and rebounds with a speed v, at an angle ϕ to the vertical. The coefficient of restitution between the particle and the floor is e. The magnitude of v is



(a) eu (b) $(1-e)u$

(c) $u\sqrt{\sin^2\theta + e^2\cos^2\theta}$ (d) $u\sqrt{e^2\sin^2\theta + \cos^2\theta}$

24. In the previous question the angle ϕ is equal to

(a) θ (b) $\tan^{-1}[e\tan\theta]$

(c) $\tan^{-1}\left[\frac{1}{e}\tan\theta\right]$ (d) $(1+e)\theta$

25. A particle of mass m_1 makes an elastic, one-dimensional collision with a stationary particle of mass m_2. What fraction of the kinetic energy of m_1 is carried away by m_2?

(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$

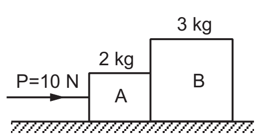
(c) $\frac{2m_1m_2}{(m_1+m_2)^2}$ (d) $\frac{4m_1m_2}{(m_1+m_2)^2}$

26. A block of mass M is attached to the lower end of a vertical rope of mass m. An upward force P acts on the upper end of the rope. The system is free to move. The force exerted by the rope on the block is $\frac{PM}{M+m}$

(a) in all cases (b) only if the rope is uniform

(c) in gravity-free space only (d) only if $P > (M+m)g$

27.



Blocks A and B have masses of 2 kg and 3 kg respectively. The ground is smooth. P is an external force of 10 N. The force exerted by B on A is

(a) 4 N (b) 6 N

(c) 8 N (d) 10 N

28. A man slides down a light rope whose breaking strength is η times his weight $(\eta < 1)$. What should be his maximum acceleration so that the rope just breaks?

(a) ηg
(b) $g(1 - \eta)$
(c) $\dfrac{g}{1 + \eta}$
(d) $\dfrac{g}{2 - \eta}$

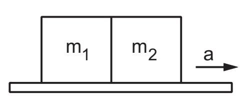
29. A particle of a small mass m is joined to a very heavy body by a light string passing over a light pulley. Both bodies are free to move. The total downward force on the pulley is

(a) mg
(b) $2\,mg$
(c) $4\,mg$
(d) $>> mg$

30. A uniform chain of mass m hangs from a light pulley, with unequal lengths of the chain hanging from the two sides of the pulley. The force exerted by the moving chain on the pulley is

(a) mg
(b) $> mg$
(c) $< mg$
(d) either (b) or (c) depending on the acceleration of the chain

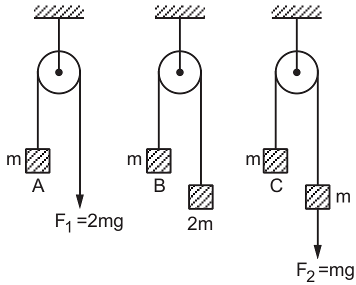
31.



Two blocks of masses m_1 and m_2 are placed in contact with each other on a horizontal platform. The coefficient of friction between the platform and the two blocks is the same. The platform moves with an acceleration. The force of interaction between the blocks is

(a) zero in all cases
(b) zero only if $m_1 = m_2$
(c) nonzero only if $m_1 > m_2$
(d) nonzero only if $m_1 < m_2$

32.

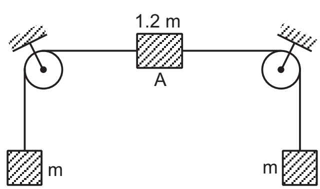


In the figure, the blocks A, B and C of mass m each have accelerations a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes $2mg$ and mg respectively.

(a) $a_1 = a_2 = a_3$ (b) $a_1 > a_3 > a_2$

(c) $a_1 = a_2, a_2 > a_3$ (d) $a_1 > a_2, a_2 = a_3$

33.



In the figure, the vertical sections of the string are long. A is released from rest from the position shown.

(a) The system will remain in equilibrium.
(b) The central block will move down continuously.
(c) The central block will undergo SHM.
(d) The central block will undergo periodic motion but not SHM.

34. A strip of wood of length l is placed on a smooth horizontal surface. An insect starts from one end of the strip, walks with constant velocity and reaches the other end in time t_1. It then flies off vertically. The strip moves a further distance l in time t_2.

(a) $t_2 = t_1$
(b) $t_2 < t_1$

(c) $t_2 > t_1$

(d) Either (b) or (c) depending on the masses of the insect and the strip

35. In gravity-free space, a man of mass M standing at a height h above the floor throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be

(a) $h\left(1+\frac{m}{M}\right)$ (b) $h\left(2-\frac{m}{M}\right)$

(c) $2h$ (d) a function of m, M, h and u

36. In a tug-of-war contest, two men pull on a horizontal rope from opposite sides. The winner will be the man who

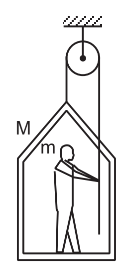
(a) exerts greater force on the rope

(b) exerts greater force on the ground

(c) exerts a force on the rope which is greater than the tension in the rope

(d) makes a smaller angle with the vertical

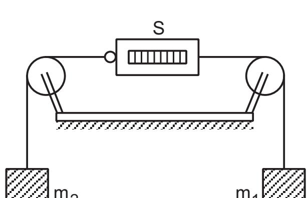
37. A man of mass m stands on a frame of mass M. He pulls on a light rope, which passes over a pulley. The other end of the rope is attached to the frame. For the system to be in equilibrium, what force must the man exert on the rope?



(a) $\frac{1}{2}(M+m)g$ (b) $(M+m)g$

(c) $(M-m)g$ (d) $(M+2m)g$

38.



In the arrangement shown, the pulleys are fixed and ideal, the strings are light, $m_1 > m_2$, and S is a spring balance which is itself massless. The reading of S (in units of mass) is

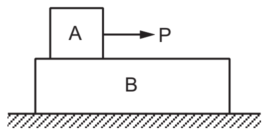
(a) $m_1 - m_2$

(b) $\frac{1}{2}(m_1 + m_2)$

(c) $\frac{m_1 m_2}{m_1 + m_2}$

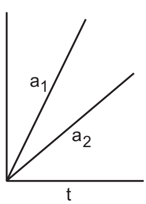
(d) $\frac{2m_1 m_2}{m_1 + m_2}$

39.

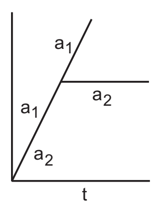


Block A is placed on block B, whose mass is greater than that of A. There is friction between the blocks, while the ground is smooth. A horizontal force P, increasing linearly with time, begins to act on A. The accelerations a_1 and a_2 of A and B respectively are plotted against time (t). Choose the correct graph.

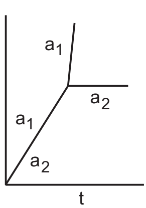
(a)



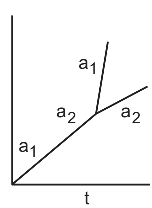
(b)



(c)



(d)



40. A bicycle moves on a horizontal road with some acceleration. The forces of friction between the road and the front and rear wheels are F_1 and F_2 respectively.

(a) Both F_1 and F_2 act in the forward direction.

(b) Both F_1 and F_2 act in the reverse direction.

(c) F_1 acts in the forward direction, F_2 acts in the reverse direction.

(d) F_2 acts in the forward direction, F_1 acts in the reverse direction.

41. A car starts from rest to cover a distance s. The coefficient of friction between the road and the tyres is μ. The minimum time in which the car can cover the distance is proportional to

(a) μ (b) $\sqrt{\mu}$

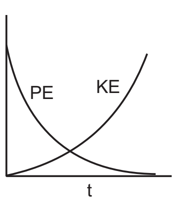
(c) $\frac{1}{\mu}$ (d) $\frac{1}{\sqrt{\mu}}$

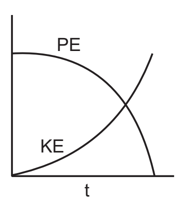
42. A car starts from rest and moves on a surface in which the coefficient of friction between the road and the tyres increases linearly with distance (x). The car moves with the maximum possible acceleration. The kinetic energy (E) of the car will depend on x as

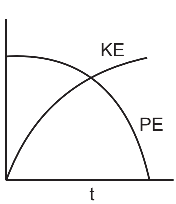
(a) $E \propto \frac{1}{x^2}$ (b) $E \propto \frac{1}{x}$

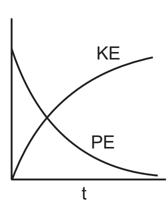
(c) $E \propto x$ (d) $E \propto x^2$

43. A particle falls from rest under gravity. Its potential energy with respect to the ground (PE) and its kinetic energy (KE) are plotted against time (t). Choose the correct graph.

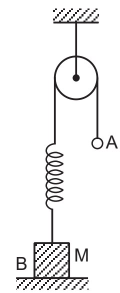
(a)


(b)


(c)


(d)


44. In the figure, the ball A is released from rest when the spring is at its natural (unstretched) length. For the block B of mass M to leave contact with the ground at some stage, the minimum mass of A must be



(a) $2M$

(b) M

(c) $\frac{M}{2}$

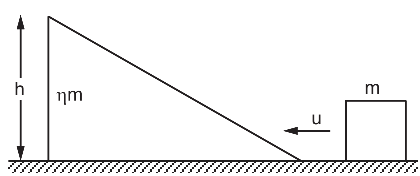
(d) a function of M and the force constant of the spring

45. A force $\vec{F} = -k(y\hat{i} + x\hat{j})$, where k is a positive constant, acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$, and then parallel to the y-axis to the point (a, a). The total work done by the force on the particle is

(a) $-2ka^2$ (b) $2ka^2$

(c) $-ka^2$ (d) ka^2

46.



A block of mass m is pushed towards a movable wedge of mass ηm and height h, with a velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge is

(a) $\sqrt{2gh}$ (b) $\eta\sqrt{2gh}$

(c) $\sqrt{2gh(1 + 1/\eta)}$ (d) $\sqrt{2gh(1 - 1/\eta)}$

47. A uniform chain of length l is placed on a rough table, with length l/n, where $n > 1$, hanging over the edge. If the chain just begins to slide off the table by itself from this position, the coefficient of friction between the chain and the table is

(a) $\frac{1}{n}$ (b) $\frac{1}{n-1}$

(c) $\frac{1}{n+1}$ (d) $\frac{n-1}{n+1}$

48. A uniform chain of length l and mass m is placed on a smooth table with one-fourth of its length hanging over the edge. The work that has to be done to pull the whole chain back onto the table is

(a) $\frac{1}{4}mgl$ (b) $\frac{1}{8}mgl$

(c) $\frac{1}{16}mgl$ (d) $\frac{1}{32}mgl$

49. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2.

(a) $W_2 = W_1$ (b) $W_2 = 2W_1$

(c) $W_2 = 3W_1$ (d) $W_2 = 4W_1$

• Type 2 •

Choose the correct options. One or more options may be correct.

50. An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects

(a) have the same speed (b) have the same velocity

(c) move in the same direction (d) move in opposite directions

51. Which of the following statements are true for a moving body?

(a) If its speed changes, its velocity must change and it must have some acceleration.

(b) If its velocity changes, its speed must change and it must have some acceleration.

(c) If its velocity changes, its speed may or may not change, and it must have some acceleration.

(d) If its speed changes but direction of motion does not change, its velocity may remain constant.

52. Let v and a denote the velocity and acceleration respectively of a body.

(a) a can be nonzero when $v = 0$.

(b) a must be zero when $v = 0$.

(c) a may be zero when $v \neq 0$.

(d) The direction of a must have some correlation with the direction of v.

53. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion.

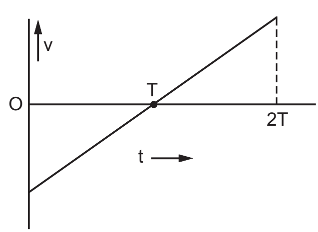
(a) $|\vec{v}|$ must decrease when $\vec{a} < 0$.

(b) Speed must increase when $\vec{a} > 0$.

(c) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0.

(d) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$.

54.



The figure shows the velocity (v) of a particle plotted against time (t).

(a) The particle changes its direction of motion at some point.

(b) The acceleration of the particle remains constant.

(c) The displacement of the particle is zero.

(d) The initial and final speeds of the particle are the same.

55. A particle moves along the x-axis as follows: it starts from rest at $t = 0$ from a point $x = 0$ and comes to rest at $t = 1$ at a point $x = 1$. No other information is available about its motion for the intermediate time $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle then

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$

(b) $|\alpha|$ cannot exceed 2 at any point in its path

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path

(d) α must change sign during the motion, but no other assertion can be made with the information given

56. The displacement (x) of a particle depends on time (t) as

$$x = \alpha t^2 - \beta t^3.$$

(a) The particle will return to its starting point after time α/β.

(b) The particle will come to rest after time $2\alpha/3\beta$.

(c) The initial velocity of the particle was zero but its initial acceleration was not zero.

(d) No net force will act on the particle at $t = \alpha/3\beta$.

57. A particle moves with an initial velocity v_0 and retardation αv, where v is its velocity at any time t.

(a) The particle will cover a total distance v_0/α.

(b) The particle will come to rest after a time $1/\alpha$.

(c) The particle will continue to move for a very long time.

(d) The velocity of the particle will become $v_0/2$ after a time $1/\alpha$.

58. A particle starts from the origin of coordinates at time $t = 0$ and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction is

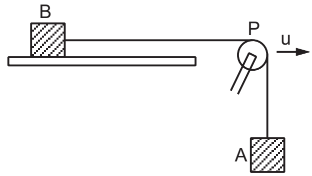
(a) variable

(b) $\sqrt{\dfrac{2\alpha}{\beta}}$

(c) $\dfrac{\alpha}{2\beta}$

(d) $\sqrt{\dfrac{\alpha}{2\beta}}$

59.



In the figure, the pulley P moves to the right with a constant speed u. The downward speed of A is v_A, and the speed of B to the right is v_B.

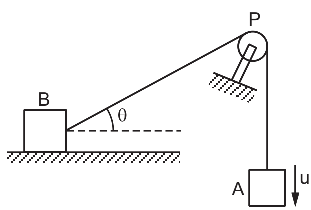
(a) $v_B = v_A$

(b) $v_B = u + v_A$

(c) $v_B + u = v_A$

(d) The two blocks have accelerations of the same magnitude.

60.



In the figure, the blocks are of equal mass. The pulley is fixed. In the position shown, A moves down with a speed u, and v_B = the speed of B.

(a) B will never lose contact with the ground.

(b) The downward acceleration of A is equal in magnitude to the horizontal acceleration of B.

(c) $v_B = u\cos\theta$

(d) $v_B = u/\cos\theta$

61. Two particles A and B start simultaneously from the same point and move in a horizontal plane. A has an initial velocity u_1 due east and acceleration a_1 due north. B has an initial velocity u_2 due north and acceleration a_2 due east.

(a) Their paths must intersect at some point.
(b) They must collide at some point.
(c) They will collide only if $a_1u_1 = a_2u_2$.
(d) If $u_1 > u_2$ and $a_1 < a_2$, the particles will have the same speed at some point of time.

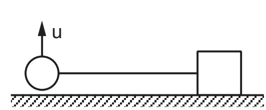
62. Two particles are projected from the same point with the same speed, at different angles θ_1 and θ_2 to the horizontal. They have the same horizontal range. Their times of flights are t_1 and t_2 respectively.

(a) $\theta_1 + \theta_2 = 90°$
(b) $\frac{t_1}{t_2} = \tan\theta_1$
(c) $\frac{t_1}{t_2} = \tan\theta_2$
(d) $\frac{t_1}{\sin\theta_1} = \frac{t_2}{\sin\theta_2}$

63. A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart.
(a) The particle will land somewhere on the circular path.
(b) The particle will land outside the circular path.
(c) The particle will follow an elliptical path.
(d) The particle will follow a parabolic path.

64. A man on a moving cart, facing the direction of motion, throws a ball straight up with respect to himself.
(a) The ball will always return to him.
(b) The ball will never return to him.
(c) The ball will return to him if the cart moves with a constant velocity.
(d) The ball will fall behind him if the cart moves with some acceleration.

65. A small ball is connected to a block by a light string of length l. Both are initially on the ground. There is sufficient friction on the ground to



prevent the block from slipping. The ball is projected vertically up with a velocity u, where $2gl < u^2 < 3gl$. The centre of mass of the 'block + ball' system is C.

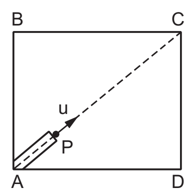
(a) C will move along a circle.

(b) C will move along a parabola.

(c) C will move along a straight line.

(d) The horizontal component of the velocity of the ball will first increase and then decrease.

66.



A large rectangular box ABCD falls vertically with an acceleration a. A toy gun fixed at A and aimed towards C fires a particle P.

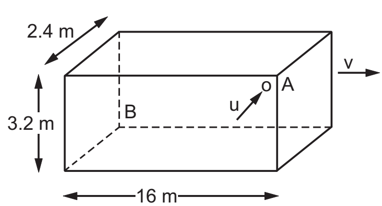
(a) P will hit C if $a = g$.

(b) P will hit the roof BC if $a > g$.

(c) P will hit the wall CD or the floor AD if $a < g$.

(d) May be either (a), (b) or (c), depending on the speed of projection of P.

67.



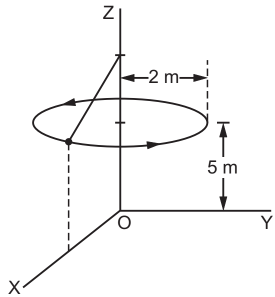
A railway compartment is 16 m long, 2.4 m wide and 3.2 m high. It is moving with a velocity v. A particle moving horizontally with a speed u, perpendicular to the direction of v, enters through a hole at an upper corner A and strikes the diagonally opposite corner B. Assume $g = 10\ \text{m/s}^2$.

(a) $v = 20$ m/s (b) $u = 3$ m/s

(c) To an observer inside the compartment, the path of the particle is a parabola.

(d) To a stationary observer outside the compartment, the path of the particle is a parabola.

68.



The upper end of the string of a simple pendulum is fixed to a vertical z-axis, and set in motion such that the bob moves along a horizontal circular path of radius 2 m, parallel to the xy plane, 5 m above the origin. The bob has a speed of 3 m/s. The string breaks when the bob is vertically above the x-axis, and it lands on the xy plane at a point (x, y).

(a) $x = 2$ m
(b) $x > 2$ m
(c) $y = 3$ m
(d) $y = 5$ m

69. Two shells are fired from a cannon with a speed u each, at angles of α and β respectively to the horizontal. The time interval between the shots is T. They collide in mid-air after time t from the first shot. Which of the following conditions must be satisfied?

(a) $\alpha > \beta$

(b) $t\cos\alpha = (t - T)\cos\beta$

(c) $(t - T)\cos\alpha = t\cos\beta$

(d) $(u\sin\alpha)t - \frac{1}{2}gt^2 = (u\sin\beta)(t - T) - \frac{1}{2}g(t - T)^2$

70. A man who can swim at a speed v relative to the water wants to cross a river of width d, flowing with a speed u. The point opposite him across the river is P.

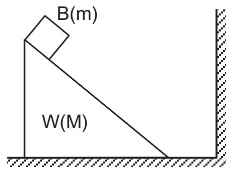
(a) The minimum time in which he can cross the river is $\frac{d}{v}$.

(b) He can reach the point P in time $\frac{d}{v}$.

(c) He can reach the point P in time $\frac{d}{\sqrt{v^2 - u^2}}$.

(d) He cannot reach P if $u > v$.

71.



In the figure, the block B of mass m starts from rest at the top of a wedge W of mass M. All surfaces are without friction. W can slide on the ground. B slides down onto the ground, moves along it with a speed v, has an elastic collision with the wall, and climbs back onto W.

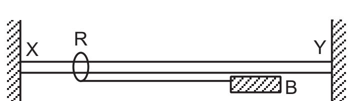
(a) B will reach the top of W again.

(b) From the beginning, till the collision with the wall, the centre of mass of 'B plus W' does not move horizontally.

(c) After the collision, the centre of mass of 'B plus W' moves with the velocity $\frac{2mv}{m+M}$.

(d) When B reaches its highest position on W, the speed of W is $\frac{2mv}{m+M}$.

72.



The ring R in the arrangement shown can slide along a smooth, fixed, horizontal rod XY. It is attached to the block B by a light string. The block is released from rest, with the string horizontal.

(a) One point in the string will have only vertical motion.

(b) R and B will always have momenta of the same magnitude.

(c) When the string becomes vertical, the speeds of R and B will be inversely proportional to their masses.

(d) R will lose contact with the rod at some point.

73. A strip of wood of mass M and length l is placed on a smooth horizontal surface. An insect of mass m starts at one end of the strip and walks to the other end in time t, moving with a constant speed.

(a) The speed of the insect as seen from the ground is $< \frac{l}{t}$.

(b) The speed of the strip as seen from the ground is
$$\frac{l}{t}\left(\frac{M}{M+m}\right).$$

(c) The speed of the strip as seen from the ground is
$$\frac{l}{t}\left(\frac{m}{M+m}\right).$$

(d) The total kinetic energy of the system is $\frac{1}{2}(m+M)\left(\frac{l}{t}\right)^2$.

74. A charged particle X moves directly towards another charged particle Y. For the 'X plus Y' system, the total momentum is p and the total energy is E.

(a) p and E are conserved if both X and Y are free to move.

(b) (a) is true only if X and Y have similar charges.

(c) If Y is fixed, E is conserved but not p.

(d) If Y is fixed, neither E nor p is conserved.

75. In a one-dimensional collision between two particles, their relative velocity is $\vec{v}_1$ before the collision and $\vec{v}_2$ after the collision.

(a) $\vec{v}_1 = \vec{v}_2$ if the collision is elastic.

(b) $\vec{v}_1 = -\vec{v}_2$ if the collision is elastic.

(c) $|\vec{v}_2| = |\vec{v}_1|$ in all cases.

(d) $\vec{v}_1 = -k\,\vec{v}_2$ in all cases, where $k \geq 1$.

76. A sphere A moving with a speed u and rotating with an angular velocity ω, makes a head-on elastic collision with an identical stationary sphere B. There is no friction between the surfaces of A and B. Disregard gravity.

(a) A will stop moving but continue to rotate with an angular velocity ω.

(b) A will come to rest and stop rotating.

(c) B will move with a speed u without rotating.

(d) B will move with a speed u and rotate with an angular velocity ω.

77. In an elastic collision between spheres A and B of equal mass but unequal radii, A moves along the x-axis and B is stationary before impact. Which of the following is possible after impact?

(a) A comes to rest.

(b) The velocity of B relative to A remains the same in magnitude but reverses in direction.

(c) A and B move with equal speeds, making an angle of 45° each with the x-axis.

(d) A and B move with unequal speeds, making angles of 30° and 60° with the x-axis respectively.

78. In a one-dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

(a) The total momentum of the 'A plus B' system is p before and after the impact, and $(p - J)$ during the impact.

(b) During the impact, A gives impulse J to B.

(c) The coefficient of restitution is $\frac{2J}{p} - 1$.

(d) The coefficient of restitution is $\frac{J}{p} + 1$.

79. When a cannon shell explodes in mid-air,

(a) the momentum of the system is conserved in all cases

(b) the momentum of the system is conserved only if the shell was moving horizontally

(c) the kinetic energy of the system either remains constant or decreases

(d) the kinetic energy of the system always increases

80. A cannon shell is fired to hit a target at a horizontal distance R. However, it breaks into two equal parts at its highest point. One part (A) returns to the cannon. The other part

(a) will fall at a distance of R beyond the target

(b) will fall at a distance of $3R$ beyond the target

(c) will hit the target

(d) have nine times the kinetic energy of A

81. A particle moving with a speed v changes direction by an angle θ, without change in speed.

(a) The change in the magnitude of its velocity is zero.

(b) The change in the magnitude of its velocity is $2v\sin(\theta/2)$.

(c) The magnitude of the change in its velocity is $2v\sin(\theta/2)$.

(d) The magnitude of the change in its velocity is $v(1 - \cos\theta)$.

82. A block of weight 9.8 N is placed on a table. The table surface exerts an upward force of 10 N on the block. Assume $g = 9.8\ \text{m/s}^2$.

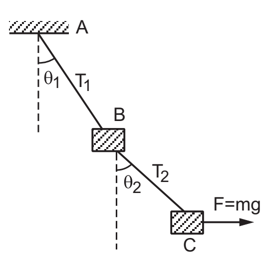
(a) The block exerts a force of 10 N on the table.

(b) The block exerts a force of 19.8 N on the table.

(c) The block exerts a force of 9.8 N on the table.

(d) The block has an upward acceleration.

83.

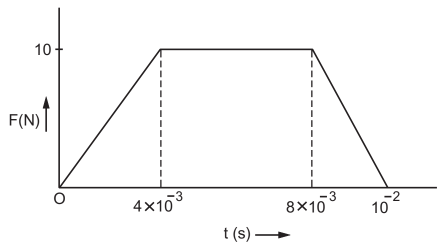


The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.

(a) $\tan\theta_1 = 1/2$ (b) $\tan\theta_2 = 1$

(c) $T_1 = \sqrt{5}mg$ (d) $T_2 = \sqrt{2}mg$

84. A particle of mass 70 g, moving at 50 cm/s, is acted upon by a variable force opposite to its direction of motion. The force F is shown as a function of time t.



(a) Its speed will be 50 cm/s after the force stops acting.

(b) Its direction of motion will reverse.

(c) Its average acceleration will be 1 m/s^2 during the interval in which the force acts.

(d) Its average acceleration will be 10 m/s^2 during the interval in which the force acts.

85. A monkey of mass m kg slides down a light rope attached to a fixed spring balance, with an acceleration a. The reading of the spring balance is W kg. [g = acceleration due to gravity]

(a) The force of friction exerted by the rope on the monkey is $m(g-a)$ N.

(b) $m = \dfrac{Wg}{g-a}$

(c) $m = W\left(1 + \dfrac{a}{g}\right)$

(d) The tension in the rope is Wg N.

86. A block of weight W is suspended from a spring balance. The lower surface of the block rests on a weighing machine. The spring balance reads W_1 and the weighing machine reads W_2. (W, W_1, W_2 are in the same unit.)

(a) $W = W_1 + W_2$ if the system is at rest.

(b) $W > W_1 + W_2$ if the system moves down with some acceleration.

(c) $W_1 > W_2$ if the system moves up with some acceleration.

(d) No relation between W_1 and W_2 can be obtained with the given description of the system.

87. A simple pendulum with a bob of mass m is suspended from the roof of a car moving with a horizontal acceleration a.

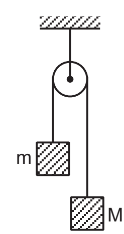
(a) The string makes an angle of $\tan^{-1}(a/g)$ with the vertical.

(b) The string makes an angle of $\tan^{-1}\left(1 - \frac{a}{g}\right)$ with the vertical.

(c) The tension in the string is $m\sqrt{a^2 + g^2}$.

(d) The tension in the string is $m\sqrt{g^2 - a^2}$.

88. Two masses M and m $(M > m)$ are joined by a light string passing over a smooth light pulley.



(a) The acceleration of each block is $\left(\frac{M - m}{M + m}\right)g$.

(b) The tension in the string is $\frac{2Mmg}{M + m}$.

(c) The centre of mass of the 'M plus m' system moves down with an acceleration of $g\left(\frac{M - m}{M + m}\right)^2$.

(d) The tension in the string by which the pulley is attached to the roof is $(M + m)g$.

89. In the previous question, the blocks are allowed to move for some time, after which M is stopped momentarily (brought to rest and released at once). After this,

(a) both blocks will move with the same acceleration

(b) the string will become taut (under tension) again when the blocks acquire the same speed

(c) the string will become taut again when the blocks cover equal distances

(d) at the instant when the string becomes taut again, there may be some exchange of impulse between the string and blocks

90. A block of mass 1 kg moves under the influence of external forces on a rough horizontal surface. At some instant, it has a speed of 1 m/s due east and an acceleration of 1 m/s^2 due north. The force of friction acting on it is F.

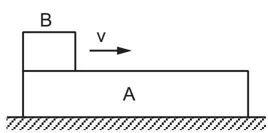
(a) F acts due west.

(b) F acts due south.

(c) F acts in the south-west direction.

(d) The magnitude of F cannot be found from the given data.

91.



A long block A is at rest on a smooth horizontal surface. A small block B, whose mass is half of A, is placed on A at one end and projected along A with some velocity u. The coefficient of friction between the blocks is μ.

(a) The blocks will reach a final common velocity $\frac{u}{3}$.

(b) The work done against friction is two-thirds of the initial kinetic energy of B.

(c) Before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$.

(d) Before the blocks reach a common velocity the acceleration of A relative to B is $\frac{3}{2}\mu g$.

92. A 10-kg block is placed on a horizontal surface. The coefficient of friction between them is 0.2. A horizontal force $P = 15$ N first acts on it in the eastward direction. Later, in addition to P a second horizontal force $Q = 20$ N acts on it in the northward direction.

(a) The block will not move when only P acts, but will move when both P and Q act.

(b) If the block moves, its acceleration will be 0.5 m/s^2.

(c) When the block moves, its direction of motion will be $\tan^{-1}(4/3)$ east of north.

(d) When both P and Q act, the direction of the force of friction acting on the block will be $\tan^{-1}(3/4)$ west of south.

93. A block of mass m is placed on a rough horizontal surface. The coefficient of friction between them is μ. An external horizontal force is applied to the block and its magnitude is gradually increased. The force exerted by the block on the surface is R.

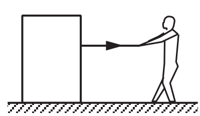
(a) The magnitude of R will gradually increase.

(b) $R \le mg\sqrt{\mu^2 + 1}$.

(c) The angle made by R with the vertical will gradually increase.

(d) The angle made by R with the vertical $\le \tan^{-1}\mu$.

94.



A man pulls a block heavier than himself with a light rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground.

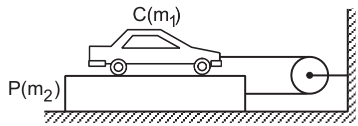
(a) The block will not move unless the man also moves.

(b) The man can move even when the block is stationary.

(c) If both move, the acceleration of the man is greater than the acceleration of the block.

(d) None of the above assertions is correct.

95. A car C of mass m_1 rests on a plank P of mass m_2. The plank rests on a smooth floor. The string and pulley are ideal. The car starts and moves towards the pulley with acceleration.



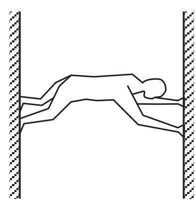
(a) If $m_1 > m_2$, the string will remain under tension.

(c) If $m_1 < m_2$, the string will become slack.

(c) If $m_1 = m_2$, the string will have no tension, and C and P will have accelerations of equal magnitude.

(d) C and P will have accelerations of equal magnitude if $m_1 \geq m_2$.

96.



A man tries to remain in equilibrium by pushing with his hands and feet against two parallel walls. For equilibrium,

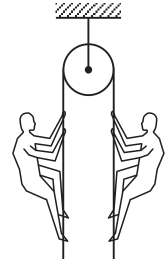
(a) he must exert equal forces on the two walls

(b) the forces of friction at the two walls must be equal

(c) friction must be present on both walls

(d) the coefficients of friction must be the same between both walls and the man

97. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley. Which of the following are possible?



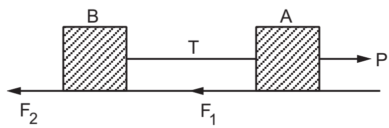
(a) The lighter man is stationary while the heavier man slides with some acceleration.

(b) The heavier man is stationary while the lighter man climbs with some acceleration.

(c) The two men slide with the same acceleration in the same direction.

(d) The two men slide with accelerations of the same magnitude in opposite directions.

98.



Two blocks A and B of the same mass are joined by a light string and placed on a horizontal surface. An external horizontal force P acts on A. The tension in the string is T. The forces of friction acting on A and B are F_1 and F_2 respectively. The limiting value of F_1 and F_2 is F_0. As P is gradually increased,

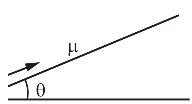
(a) for $P < F_0$, $T = 0$

(b) for $F_0 < P < 2F_0$, $T = P - F_0$

(c) for $P > 2F_0$, $T = P/2$

(d) none of the above

99.



A block is placed at the bottom of an inclined plane and projected upwards with some initial speed. It slides up the incline, stops after time t_1, and slides back in a further time t_2. The angle of inclination of the plane with the horizontal is θ and the coefficient of friction is μ.

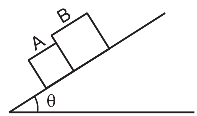
(a) $t_1 > t_2$

(b) $t_1 < t_2$

(c) The retardation of the block while moving up is $g(\sin\theta + \mu\cos\theta)$.

(d) The acceleration of the block while moving down is $g(\sin\theta - \mu\cos\theta)$.

100.



The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively.

(a) If $\mu_1 > \mu_2$, the blocks will always remain in contact.

(b) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations.

(c) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1 + \mu_2)g\sin\theta$.

(d) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2 g}{\mu_1 + \mu_2}\sin\theta$.

101. A ball of mass m is attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, and comes to rest again after descending through a distance x.

(a) $x = mg/k$

(b) $x = 2mg/k$

(c) The ball will have no acceleration at the position where it has descended through $x/2$.

(d) The ball will have an upward acceleration equal to g at its lowermost position.

Answers

1. c	**2.** b	**3.** b	**4.** b	**5.** b
6. b	**7.** c	**8.** a	**9.** c	**10.** c
11. a	**12.** c	**13.** b	**14.** b	**15.** c
16. a	**17.** b	**18.** d	**19.** b	**20.** b
21. c	**22.** a	**23.** c	**24.** c	**25.** d
26. a	**27.** b	**28.** b	**29.** c	**30.** c
31. a	**32.** b	**33.** d	**34.** c	**35.** a
36. b	**37.** a	**38.** d	**39.** c	**40.** d
41. d	**42.** d	**43.** b	**44.** c	**45.** c
46. c	**47.** b	**48.** d	**49.** c	**50.** a, b, c
51. a, c	**52.** a, c	**53.** c, d	**54.** a, b, c, d	**55.** a, c
56. a, b, c, d	**57.** a, c	**58.** d	**59.** b, d	**60.** a, d
61. a, c, d	**62.** a, b, d	**63.** b, d	**64.** c, d	**65.** a, d
66. a, b, c	**67.** a, b, d	**68.** a, c	**69.** a, b, d	**70.** a, c, d
71. b, c, d	**72.** a, c	**73.** a, c	**74.** a, c	**75.** b, c, d
76. a, c	**77.** a, b, c, d	**78.** b, c	**79.** a, d	**80.** a, d
81. a, c	**82.** a, d	**83.** a, b, c, d	**84.** a, b	**85.** a, b, d
86. a, b, d	**87.** a, c	**88.** a, b, c	**89.** a, c, d	**90.** a, d
91. a, b, d	**92.** a, b, d	**93.** a, b, c, d	**94.** a, b, c	**95.** a, b, c, d
96. a, c	**97.** a, b, d	**98.** a, b, c	**99.** b, c, d	**100.** a, b
101. b, c, d				

Hints and Solutions to Selected Questions

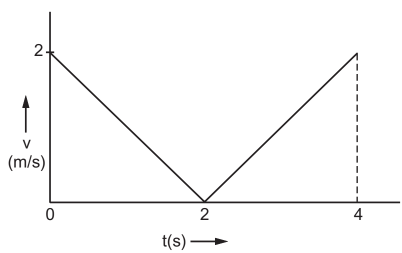
1. $\vec{u} = 3\hat{i} + 4\hat{j}$ $\quad\therefore\quad u_x = 3, u_y = 4$

$\vec{a} = 0.4\hat{i} + 0.3\hat{j}$ $\quad\therefore\quad a_x = 0.4, a_y = 0.3$

$v_x = u_x + a_x t = 3 + 0.4 \times 10 = 7.$

$v_y = u_y + a_y t = 7.$ $\qquad v = \sqrt{v_x^2 + v_y^2}.$

2. Plot its velocity v against time t. The area under the curve gives the distance.



3. TIP To find the distance, first find the turning point, i.e., where $v = 0$. If the interval required in the question includes this point, calculate the displacements separately for the periods before and after this point, and add the magnitudes of these displacements.

Here, $u = 9$ m/s, $a = -2$ m/s^2. For $v = 0, t = 4.5$ s. The required interval is from $t = 4$ s to $t = 5$ s. This includes the turning point. Hence, find displacements for $t = 4$ s, $t = 4.5$ s, $t = 5$ s.

4. $v\dfrac{dv}{dx} = -\alpha x$ (α = constant) $\qquad \displaystyle\int_{v_i}^{v_f} v\,dv = \int_0^x -\alpha x\,dx$

$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}\alpha m x^2.$

5. $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$ $\qquad \dot{x} = v = 3\alpha t^2 + 2\beta t + \gamma$

For $t = 0, v_i = \gamma$ $\qquad \ddot{x} = a = 6\alpha t + 2\beta$

For $t = 0, a_i = 2\beta$ $\qquad\therefore\quad$ for $t = 0, a_i/v_i = 2\beta/\gamma.$

8. The increase in kinetic energy = the loss in potential energy. This is the same for all three.

9. TIP When a particle separates from a moving body, it retains the velocity of the body but not its acceleration.

At the instant of release, the balloon is 40 m above the ground and has an upward velocity of 10 m/s. For the motion of the stone from the balloon to the ground, $u = 10$ m/s, $s = -40$ m, $a = -10\ \text{m/s}^2 (g)$.

10. Let F be the force of air resistance. For the upward motion, F acts downwards. Let a_1 = retardation (downward acceleration).

$$mg + F = ma_1 \quad \text{or} \quad a_1 = g + \frac{F}{m}\cdot$$

For the downward motion, F acts upwards. Let a_2 = downward acceleration.

$$mg - F = ma_2 \quad \text{or} \quad a_2 = g - \frac{F}{m}\cdot \text{ Thus } a_1 > a_2.$$

Let h = maximum height reached.

$$h = \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2.$$

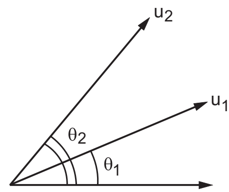
11. The horizontal component of the velocity of the bomb is the same as the horizontal component of the velocity of the aeroplane in all cases. Hence, in all cases, the two have the same horizontal displacement in the same time.

12. For conservation of vertical momentum, the second part must have a vertical downward velocity of 50 m/s. For conservation of horizontal momentum, the second part must have a horizontal velocity of 120 m/s.

13. The velocities of the two particles after time t are

$\vec{v_1} = (u_1\cos\theta_1)\,\hat{i} + (u_1\sin\theta_1 - gt)\,\hat{j}$, and

$\vec{v_2} = (u_2\cos\theta_2)\,\hat{i} + (u_2\sin\theta_2 - gt)\,\hat{j}$.

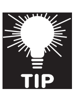


Their relative velocity is $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$, which is constant, having both horizontal and vertical components.

14. See the hint to Q. No. 13. $\vec{v}_{12}$ has only a vertical component.

15. The horizontal component of the velocity remains constant. Hence, $u\cos\theta = v\cos\phi$.

16. Considering the vertical motion, the time of travel from A to B is $t = \sqrt{2h/g}$. In this time, the particle must make an integral number of rotations, say n. Considering the horizontal motion, distance covered $= 2\pi rn = ut$.

17.  In an elastic, one-dimensional collision between a very heavy and a very light particle, to find the velocity of the light particle after collision, reverse the velocity of the light particle before collision, and add to it twice the velocity of the heavy particle (which remains unchanged).

18. When a particle undergoes normal collision with a floor or a wall, with coefficient of restitution e, the speed after collision is e times the speed before collision. Thus, in this case, the change in momentum for the first impact is $ep - (-p) = p(1 + e)$, for the second impact it is $e(ep) - (-ep) = ep(1 + e)$ and so on. The total change in momentum is $p(1 + e)[1 + e + e^2 + \ldots]$.

20. This collision will cause an exchange of velocities. The change in momentum of any one particle is mu, which is equal to the impulse given by one to the other.

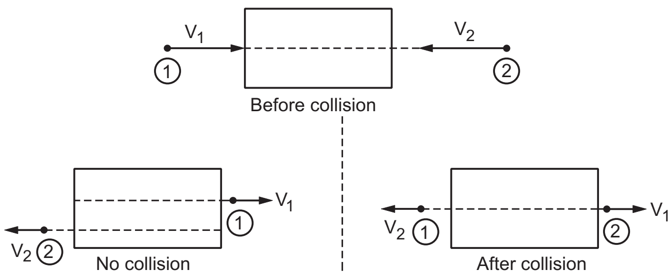
TIP

In a force–time graph, the impulse is equal to the area under the curve.

Thus, $mu = \text{area} = \frac{1}{2} F_0 T$.

21. See the hint to Q. No. 20. To find the area, treat the semicircle as half of an ellipse with semimajor and semiminor axes $T/2$ and F_0.

22.



If a collision does occur, the velocities of the two spheres will interchange. If a collision does not occur, they retain their original velocities. As the spheres are identical, an observer cannot tell from the velocities of the two spheres whether a collision occurred or not.

23. In the horizontal direction, momentum must be conserved, as the floor is frictionless and there is no horizontal force. $mu\sin\theta = mv\sin\phi$. In the vertical direction, $v\cos\phi = eu\cos\theta$.

24. See the hint to Q. No. 23.

25.

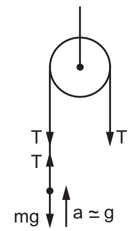
Let v_1 and v_2 be the velocities of m_1 and m_2 after collision. Then,

$$m_1 u = m_1 v_1 + m_2 v_2 \text{ and } v_2 - v_1 = -1(0 - u).$$

Find v_2 in terms of u. The required fraction is $\dfrac{\frac{1}{2} m_2 v_2^2}{\frac{1}{2} m_1 u^2}$.

26, 27. Find the acceleration of the two-body system. Then make a free-body diagram of any one unit. Apply $F = ma$ to that unit.

29. The downward acceleration of the heavy body $\cong g$ = upward acceleration of the small mass.



If T = tension in the string,

$T - mg = ma \cong mg$ or $T \cong 2mg$.

The downward force on the pulley is $2T$.

30. See the hint to Q. No. 88. Let the masses on the two sides be $\left(\frac{m}{2} + M\right)$ and $\left(\frac{m}{2} - M\right)$, with $M < \frac{m}{2}\cdot$ The acceleration of the system is,

$$a = \frac{(m/2 + M)g - (m/2 - M)g}{(m/2 + M) + (m/2 - M)} = \frac{2Mg}{m}.$$

The tension in the chain is $T = \frac{2g}{m}\left(\frac{m^2}{4} - M^2\right)$.

The force on the pulley is $2T = \frac{4g}{m}\left(\frac{m^2}{4} - M^2\right) < mg$.

31. The two blocks will have the same acceleration and identical free-body diagrams.

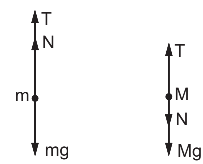
32. $a_1 = g$, $a_2 = g/3$, $a_3 = g/2$.

34. The strip and insect will move in opposite directions such that their centre of mass is stationary. Their relative velocity is l/t_1 and each will have speed $< l/t_1$. When the insect flies off, the strip will continue to move at its previous speed and hence will cover a further distance l in time $t_2 > t_1$.

35. Due to conservation of momentum in the vertical direction, the man will begin to move up with speed $\frac{mu}{M}\cdot$ The ball will reach the floor in time h/u. In this time, the man will move up by $\left(\frac{mu}{M}\right)\left(\frac{h}{u}\right)$. Total distance of the man from the floor $= h + \frac{mh}{M}$.

36. For the 'two men plus rope' system, the external forces are those exerted by the ground on the men, which, by the third law of motion, are equal to the forces they exert on the ground. The system will move in the direction of the greater external force.

37. Let $T =$ tension in the rope = force exerted by the man on the rope = force exerted by the rope on the man. $N =$ normal reaction between man and frame. For equilibrium, $T + N = mg$, $T = N + Mg$.



38. The acceleration of the system $= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$ and hence the tension T in the string is $\left(\frac{2m_1m_2}{m_1 + m_2}\right)g$. The reading of the spring balance is T (in units of force) and T/g (in units of mass).

39. The two blocks will move together with the same acceleration as long as the force of friction between them is less than the limiting friction, as the only force on the lower block B is the force of friction. Once limiting friction is reached, the acceleration of B becomes constant $\left(= \frac{F_{\text{lim}}}{\text{mass of B}}\right)$, and the acceleration of A continues to increase at a faster rate.

41. The maximum possible acceleration is μg. Hence $s = \frac{1}{2}(\mu g)t_{\text{min}}^2$.

42. $a = v\frac{dv}{dx} = \mu g = (\alpha x)g$, where $\alpha =$ constant.

or $\int v\,dv = \int \alpha g\,x\,dx$ or $\frac{1}{2}mv^2 = \frac{1}{2}m\alpha g\,x^2$.

44. The spring will exert the maximum force when the ball is at its lowest position. If the ball has descended through a distance x to reach this position, $mgx = \frac{1}{2}kx^2$, where k is the force constant of the spring. For the block B to leave contact, spring force $= kx = Mg$.

45. For the first displacement, $y = 0$. Hence $F_x = 0$ and no work is done. For the second displacement, $F_y = -ka$ and $\Delta y = a$.

Work $= F_y \Delta_y = -ka^2$.

46. The centre of mass of the 'block plus wedge' must move with speed $\dfrac{mu}{m + \eta m} = \dfrac{u}{1 + \eta} = v_{CM}$.

$$\therefore \quad \frac{1}{2}mu^2 - mgh = \frac{1}{2}(m + \eta m)v_{CM}^2.$$

47. The mass of the hanging part

$$= m_1 = m\frac{l}{n}.$$

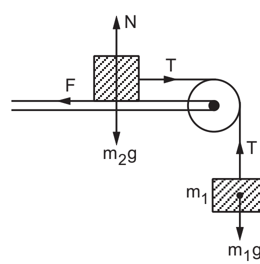
[m = mass per unit length of chain]

The mass of section on table

$$= m_2 = m\left(l - \frac{l}{n}\right).$$

For the chain to just move,

$$m_1 g = T = F = \mu m_2 g.$$



49. Let k = force constant of the spring.

Potential energy of the spring after the first stretching

$$= E_1 = \frac{1}{2}kx^2.$$

Potential energy of the spring after the second stretching

$$= E_2 = \frac{1}{2}k(2x)^2.$$

$$W_1 = E_1, \qquad W_2 = E_2 - E_1.$$

50. TIP Superimpose on the observer and the two objects a velocity equal and opposite to the velocity of the observer. The system now reduces to what is seen by the observer.

57. $v\dfrac{dv}{dx} = -\alpha v$ or $\displaystyle\int_{v_0}^{0} dv = -\alpha \int_0^{x_0} dx$ or $x_0 = v_0/\alpha$.

$\dfrac{dv}{dt} = -\alpha v$ or $\displaystyle\int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_0^{t} dt$

or $v = v_0 e^{-\alpha t}$. $\therefore$ $v = 0$ for $t \to \infty$.

58. $y = \beta x^2$ or $\dot{y} = 2\beta x \dot{x}$

or $\ddot{y} = 2\beta(\dot{x})^2 = \alpha$

or $\dot{x} = \sqrt{\alpha/2\beta}$.

[$\ddot{x} = 0$ as it has acceleration only in the y-direction.]

59. At any instant of time, let the length of the string BP = l_1 and the length PA = l_2. In a further time t, let B move to the right by x and A move down by y, while P moves to the right by ut. As the length of the string must remain constant,

$$l_1 + l_2 = (l_1 - x + ut) + (l_2 + y)$$

or $x = ut + y$

or $\dot{x} = u + \dot{y}$.

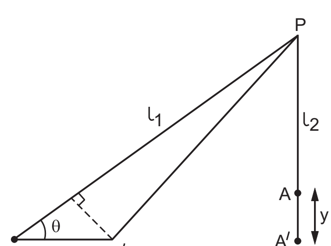
$\dot{x}$ = speed of B to the right = v_B, $\dot{y}$ = downward speed of $A = v_A$

$\therefore$ $v_B = u + v_A$. Also, $\dot{v}_B = \dot{v}_A$

or $a_B = a_A$.

60. Let the lengths of the sections of the string be BP = l_1 and PA = l_2 in the position shown. Let B move through a small horizontal distance x to B′ and A move down through a distance y to A′. Length of the string

= BP + PA = B′P + PA′.



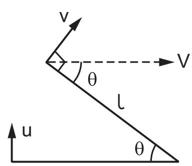
$$l_1 + l_2 = (l_1 - x\cos\theta) + l_2 + y$$

or $\quad \dot{y} = \dot{x}\cos\theta$

or $\quad v_A = v_B \cos\theta = u.$

62. TIP Projectiles with the same initial speed have the same horizontal range for complimentary angles of projection. Here, $\theta_1 = 90 - \theta_2$.

65.



As the block does not move, the ball moves along a circular path of radius l. The centre of mass of the system always lies somewhere on the string.

Let v = speed of ball when the string makes an angle θ with the horizontal.

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgl\sin\theta$$

The horizontal component of $v = V = v\sin\theta = \sin\theta\sqrt{u^2 - 2gl\sin\theta}$.

For V to be maximum, $\frac{dV}{d\theta} = 0$, which gives $\sin\theta = \frac{u^2}{3gl}\cdot$

66. Superimpose an upward acceleration a on the system. The box becomes stationary. The particle has an upward acceleration a and a downward acceleration g. If $a = g$, the particle has no acceleration and will hit C. If $a > g$, the particle has a net upward acceleration, and if $a < g$, the particle has a net downward acceleration.

67. Consider the vertical motion of the particle after entering the compartment. Let it reach the floor in time t.

$$3.2 = \frac{1}{2}(10)t^2 \quad \text{or} \quad t = 0.8 \text{ s}.$$

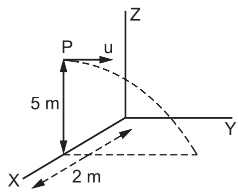
Due to the velocity component u, which remains constant, it covers a distance of 2.4 m in 0.8 s.

$$\therefore \quad u = \frac{2.4 \text{ m}}{0.8 \text{ s}} = 3 \text{ m/s}.$$

Due to the velocity v, point B must move forward by 16 m in 0.8 s.

$$\therefore \quad v = \frac{16\text{ m}}{0.8\text{ s}} = 20\text{ m/s}.$$

68. When the string breaks, the particle starts from the point (2, 0, 5) with speed $u_y = 3$ m/s, and moves as a projectile. It will reach the xy plane in 1 second.



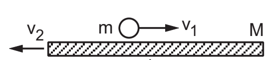
69. For two shells to collide in mid-air, they must reach the same point at the same time. Here, the first shell is fired at an angle α and has a longer time of flight, t. The second shell is fired at an angle $\beta\,(<\alpha)$ and has a shorter time of flight, $(t - T)$. Option (b) satisfies the condition for equal horizontal displacement, and (d) for equal vertical displacement.

71. The horizontal momentum of the system is conserved (= 0) till the collision, as there are no horizontal forces acting on the system. At the collision, an additional impulse $2mv$ is given by the wall to the system.

72. There are no horizontal forces acting on the 'R plus B' system. Hence, its centre of mass will move down vertically, and horizontal momentum will be conserved.

73. See the hint to Q. No. 34. $mv_1 = Mv_2$, where v_1 and v_2 are speeds of m and M, as seen from ground.

The velocity of m relative to M is



$$v_{12} = v_1 - (-v_2).$$

$$\text{Hence, } t = \frac{l}{v_{12}} = \frac{l}{v_1 + v_2}$$

or $\quad v_1 + v_2 = l/t.$

74. If the system is isolated (no external forces), p and E are conserved. Electrostatic forces are internal forces. To fix Y, external forces must act on the system, viz., on Y. In that case, p is not conserved. However, these external forces do no work, as there is no displacement of Y. E is conserved.

76. There is an exchange of linear velocities. However, the two spheres cannot exert torques on each other, as their surfaces are frictionless, and the angular velocities of the spheres do not change.

78. Let u = speed of A before impact. Thus, $p = mu$.

Let v_1, v_2 = speeds of A and B after impact.

$u = v_1 + v_2$ and $v_1 - v_2 = -eu$

$$\therefore \quad v_1 = \frac{1}{2}u(1-e) \quad \text{and} \quad v_2 = \frac{1}{2}u(1+e)$$

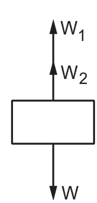
$$J = mv_2 = m\left[\frac{1}{2}u(1+e)\right] = \frac{1}{2}p(1+e).$$

83. Draw the free-body diagrams for B and C. Balance horizontal and vertical forces separately for both.

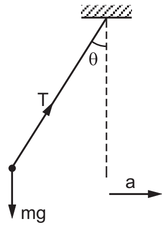
84. The impulse given to the particle is equal to the area under the F–t graph = 0.07 kg m/s = 2 × initial momentum of the particle. Hence, the particle will reverse in direction and move with its initial speed.

85. TIP The force exerted by the rope and the monkey on each other = the force of friction between the rope and the monkey = tension in the rope = reading of the spring balance.

86. The readings of the spring balance and the weighing machine are equal to the forces exerted by them on the body.



87.



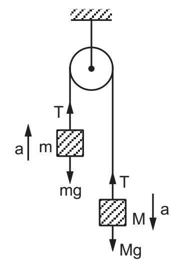
$$T\cos\theta = mg$$

$$T\sin\theta = ma$$

$$\therefore\quad \tan\theta = a/g.$$

$$T = m\sqrt{g^2 + a^2}.$$

88. $Mg - T = Ma \qquad T - mg = ma$



Solving, $a = \left(\dfrac{M-m}{M+m}\right)g$ and $T = \dfrac{2Mmg}{M+m}$

$$a_{CM} = \frac{Ma - ma}{M+m} = \left(\frac{M-m}{M+m}\right)\left(\frac{M-m}{M+m}\right)g.$$

90. Force of friction acts opposite to the direction of motion or tendency of motion.

91. As there are no external forces acting on the 'A + B' system, its total momentum is conserved. If the masses of A and B are $2m$ and m respectively, and v is the final common velocity, $mu = (m + 2m)v$ or $v = u/3$.

Work done against friction = loss in KE $= \dfrac{1}{2}mu^2 - \dfrac{1}{2}(3m)v^2$

$$= \frac{1}{2}mu^2 - \frac{1}{2}(3m)\frac{u^2}{9} = \frac{1}{2}mu^2\left[1 - \frac{1}{3}\right] = \frac{2}{3}\times\frac{1}{2}mu^2.$$

The force of friction between the blocks is μmg.

Acceleration of A (to the right) $= a_1 = \dfrac{\mu mg}{2m} = \dfrac{\mu g}{2}$.

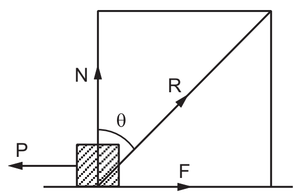
Acceleration of B (to the left) $= a_2 = \dfrac{\mu mg}{m} = \mu g$.

Acceleration of A relative to B $= a_1 - (-a_2) = \dfrac{3}{2}\mu g$.

92. Force of limiting friction $= F_{lim} = \mu mg = 0.2 \times 10 \times 10 = 20$ N.

The block will move when the resultant external force on it exceeds F_{lim}. When it moves, the direction of the force of friction will be opposite to the block's direction of motion.

93. R is the resultant of the normal reaction, $N = mg$, and the force of friction, $F \leq \mu mg$. As P is increased, F (= P) increases, while N is constant.

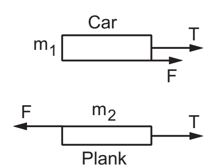


94. Let T = tension in the rope. Force T acts on both. The limiting force of friction is larger for the block than for the man. Each body will move when T exceeds the force of friction.

95. Let T = tension in the string,

F = force of friction between C and P.

If the string is under tension, the acceleration of C to the right = acceleration of P to the left = a.



$T + F = m_1 a \qquad F - T = m_2 a$

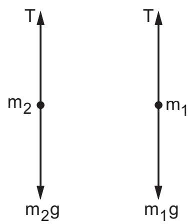
$\therefore \quad T = \frac{1}{2}(m_1 - m_2)\, a \quad$ or $\quad T > 0$ if $m_1 > m_2$.

If $m_1 < m_2$, T becomes < 0, i.e., it becomes slack.

If $m_1 = m_2$, $T = 0$.

96. The horizontal forces on the man must balance, i.e., the forces exerted by the two walls on him must be equal.

The vertical forces can balance even if the forces of friction on the two walls are unequal. The torques due to the forces of friction about his centre of mass must balance. This requires friction on both walls.

97. 

Let $m_1 > m_2$, T = tension in rope.

If m_2 is stationary, $T = m_2 g$.

$\therefore$ m_1 must slide down with some acceleration.

If m_1 is stationary, $T = m_1 g$.

$\therefore$ m_2 must slide up with some acceleration.

If $m_2 g < T < m_1 g$, m_1 can move down and m_2 can move up.

98. TIP When the external force P acts on A, it is balanced by the force of friction. The block does not move and there is no tension in the string as long as $F_1 < F_0$, i.e., $P < F_0$. Where P exceeds F_0, A moves slightly and $T > 0$.

100. The block with lower value of μ will tend to have greater acceleration down the slope.

101. Loss in the potential energy of the ball = gain in the potential energy of the spring.

$$mgx = \frac{1}{2}kx^2 \quad \text{or} \quad x = \frac{2mg}{k}.$$

When the ball is at its lowest position, spring force $= kx = 2mg$.

□

• *Revision Exercise 1* •

Choose the correct option in each of the following questions. Only one option is correct in each question.

R1. Take the z-axis as vertical and the xy plane as horizontal. A particle A is projected at $4\sqrt{2}$ m/s at an angle of 45° to the horizontal, in the xz plane. Particle B is projected at 5 m/s at an angle $\theta = \tan^{-1}(4/3)$ to the y-axis, in the yz plane. Which of the following is not correct for the velocity of B with respect to A?

(a) Its initial magnitude is 5 m/s.

(b) Its magnitude will change with time.

(c) It lies in the xy plane.

(d) It will initially make an angle $(\theta + \pi/2)$ with the positive x-axis.

R2. Two particles moving initially in the same direction undergo a one-dimensional, elastic collision. Their relative velocities before and after the collision are $\vec{v}_1$ and $\vec{v}_2$. Which of the following is not correct?

(a) $|\vec{v}_1| = |\vec{v}_2|$

(b) $\vec{v}_1 = -\vec{v}_2$ only if the two are of equal mass.

(c) $\vec{v}_1 \cdot \vec{v}_2 = -|\vec{v}_1|^2$

(d) $|\vec{v}_2 \cdot \vec{v}_1| = |\vec{v}_2|^2$

R3. A ball A, moving with kinetic energy E, makes a head-on, elastic collision with a stationary ball with mass n times that of A. The maximum potential energy stored in the system during the collision is

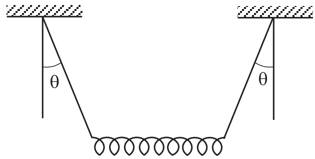
(a) $nE/(n+1)$

(b) $(n+1)E/n$

(c) $(n-1)E/n$

(d) E/n

R4.



A spring of weight W and force constant k is suspended in a horizontal position by two light strings attached to its two ends. Each string makes an angle θ with the vertical. The extension of the spring is

(a) $(W/4k)\tan\theta$ (b) $(W/2k)\tan\theta$

(c) $(W/4k)\sin\theta$ (d) 0

R5. A man balances himself in a horizontal position by pushing his hands and feet against two parallel walls. (See figure of Q. No. 96). His centre of mass lies midway between the walls. The coefficients of friction at the walls are equal. Which of the following is not correct?

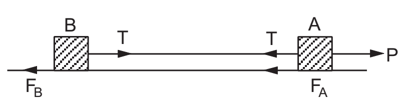
(a) He exerts equal forces on the walls.

(b) He exerts only horizontal forces on the walls.

(c) The forces of friction at the walls are equal.

(d) The forces exerted by the walls on him are not horizontal.

R6.



Two blocks A and B are placed on a table and joined by a string. The limiting friction for both blocks is F. The tension in the string is T. The forces of friction acting on the blocks are F_A and F_B. An external horizontal force $P = 3F/2$ acts on A, directed away from B.

(a) $F_A = F_B = T = 3F/4$ (b) $F_A = F/2, F_B = F, T = F$

(c) $F_A = F_B = 3F/4, T = 0$ (d) $F_A = F, F_B = T = F/2$

R7. A train of length 200 m switches on its headlight when it starts moving with acceleration 0.5 m/s^2. Some time later, its tail light is switched on. An observer on the ground notices that the two events occur at the same place. The time interval between the two events is

(a) $10\sqrt{2}$ s (b) 20 s
(c) $20\sqrt{2}$ s (d) 40 s

R8. A stick is thrown in the air and lands at some distance from the thrower. The centre of mass of the stick will move along a parabolic path

(a) in all cases
(b) only if the stick is uniform
(c) only if the stick does not have any rotational motion
(d) only if the centre of mass of the stick lies at some point on it and not outside it

R9. A car starts from rest and moves on a surface on which the coefficient of friction between the road and the tyres increases linearly with distance x. The car moves with the maximum possible acceleration. The kinetic energy E of the car will be proportional to

(a) x^{-2} (b) x^{-1}
(c) x (d) x^{2}

R10. A uniform chain of mass m hangs from a light pulley, with unequal lengths hanging from the two sides of the pulley. The force exerted by the moving chain on the pulley is

(a) mg
(b) $> mg$
(c) $< mg$
(d) may be any of these depending on the time elapsed

R11. A ball of mass m falls from rest onto a floor, from a height h. It makes elastic collisions with the floor repeatedly. The force exerted by it on the floor, averaged over a long time, is

(a) mg (b) $2mg$
(c) $3mg$ (d) proportional to m and h

R12. An insect of mass m is initially at one end of a stick of length L and mass M, which rests on a smooth floor. The coefficient of friction between the insect and the stick is k. The minimum time in which the insect can reach the other end of the stick is t. Then t^2 is equal to

(a) $2L/kg$
(b) $2Lm/kg(M+m)$
(c) $2LM/kg(M+m)$
(d) $2Lm/kgM$

R13. Three ships A, B and C are in motion. The motion of A as seen by B is with speed V towards the north-east. The motion of B as seen by C is with speed V towards the north-west. Then, as seen by A, C will be moving towards the

(a) north
(b) south
(c) east
(d) west

R14. A point moving along the x-direction starts from rest at $x=0$ and comes to rest at $x=1$ after 1 s. Its acceleration at any point is denoted by α. Which of the following is not correct?

(a) α must change sign during the motion.
(b) $|\alpha| \geq 4$ units at some or all points during the motion.
(c) It is not possible to specify an upper limit for $|\alpha|$ from the given data.
(d) $|\alpha|$ cannot be less that 1/2 during the motion.

R15. A man of mass m stands on a long flat car of mass M, moving with velocity V. If he now begins to run with velocity u, with respect to the car, in the same direction as V, the velocity of the car will be

(a) $V - mu/M$
(b) $V - mu/(m+M)$
(c) $V + mu/(m+M)$
(d) $V - u(M-m)/(M+m)$

R16. A small body B starts from rest at the highest point A of a large fixed sphere, with centre C, and slides down with a small but constant speed. Then, the coefficient of friction between B and the sphere, at any point P on the surface of the sphere such that $\angle ACP = \theta$, must be equal to

(a) $\sin\theta$
(b) $\cos\theta$
(c) $\tan\theta$
(d) $|\cos\theta - \sin\theta|$

R17. A spring of force constant k rests on a smooth floor, with one end fixed to a wall. A block of mass m hits the free end of the spring with velocity v. The maximum force exerted by the spring on the wall is

(a) $v\sqrt{(mk)}$ (b) $mv\sqrt{k}$

(c) $m\sqrt{(vk)}$ (d) $k\sqrt{(mv)}$

R18. A variable force F acts on a body which is free to move. The displacement of the body is proportional to t^3, where t = time. The power delivered by F to the body will be proportional to

(a) t (b) t^2 (c) t^3 (d) t^4

R19. A cannon shell lands 2 km away from the cannon. A second shell, fired identically, breaks into two equal parts at the highest point. One part falls vertically. How far from the cannon will the other land?

(a) 2 km (b) 3 km

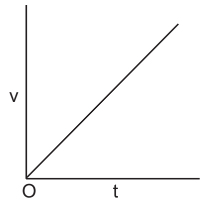
(c) 4 km (d) 5 km

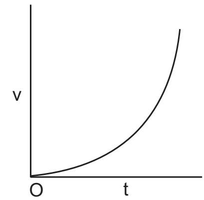
R20. A uniform heavy chain is placed on a table with a part of it hanging over the edge. It just begins to slide when this part is one-third of its length. The coefficient of friction between the table and the chain is

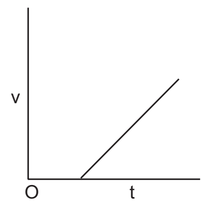
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

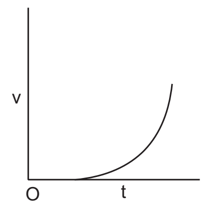
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

R21. A block rests on a rough floor. A horizontal force which increases linearly with time (t), begins to act on the block at $t = 0$. Its velocity (v) is plotted against t. Which of the given graphs is correct?

(a) 

(b) 

(c) 

(d) 

R22. A particle moving with velocity V changes its direction of motion by an angle θ without change in speed. Which of the following statements is not correct?

(a) The magnitude of the change in its velocity is $2V\sin\left(\frac{\theta}{2}\right)$.

(b) The change in the magnitude of its velocity is zero.

(c) The change in its velocity makes an angle $\frac{\pi}{2}+\frac{\theta}{2}$ with its initial direction of motion.

(d) The change in velocity is equal to the negative of the resultant of the initial and final velocities.

R23. A frame of reference F_2 moves with velocity $\vec{v}$ with respect to another frame F_1. When an object is observed from both frames, its velocity is found to be $\vec{v}_1$ in F_1 and $\vec{v}_2$ in F_2. Then, $\vec{v}_2$ is equal to

(a) $\vec{v}_1+\vec{v}$ (b) $\vec{v}_1-\vec{v}$

(c) $\vec{v}-\vec{v}_1$ (d) $|\vec{v}_1-\vec{v}|\dfrac{\vec{v}_1}{|\vec{v}_1|}$

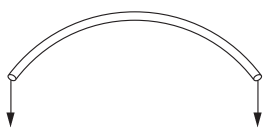
R24. Two bodies of masses m and M are attached to the two ends of a light string passing over a fixed ideal pulley ($M >> m$). When the bodies are in motion, the tension in the string is approximately

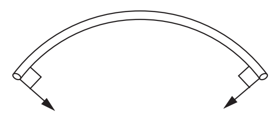
(a) $(M-m)g$ (b) mg (c) $2mg$ (d) $(m/M)mg$

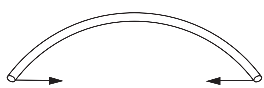
R25. A particle starts from the origin at $t = 0$ and moves in the xy plane with constant acceleration a in the y-direction. Its equation of motion is $y = bx^2$. The x-component of its velocity is

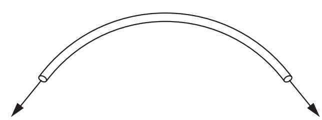
(a) variable
(b) $\sqrt{\left(\frac{2a}{b}\right)}$
(c) $\frac{a}{2b}$
(d) $\sqrt{\left(\frac{a}{2b}\right)}$

R26. A man holds a thin stick at its two ends and bends it in an arc, like a bow without a string. Which of the following figures correctly show the directions of the forces exerted by him on the stick? (Neglect gravity)

(a)


(b)


(c)


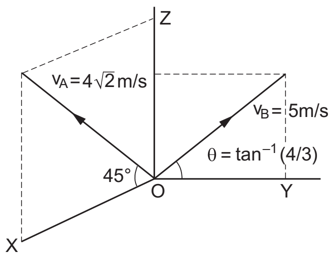
(d)


Answers to Revision Exercise 1

R1. b	**R2.** b	**R3.** a	**R4.** b	**R5.** b
R6. d	**R7.** c	**R8.** a	**R9.** d	**R10.** c
R11. a	**R12.** c	**R13.** b	**R14.** d	**R15.** b
R16. c	**R17.** a	**R18.** c	**R19.** b	**R20.** a
R21. d	**R22.** d	**R23.** b	**R24.** c	**R25.** d
R26. c				

Hints and Solutions to Selected Questions

R1.



$$\vec{v}_A = 4\hat{i} + 4\hat{k}$$
$$\vec{v}_B = 3\hat{j} + 4\hat{k}$$
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -4\hat{i} + 3\hat{j}$$

R2. For a head-on elastic collision,
relative velocity after collision
= – relative velocity before collision.
This is independent of the masses of the particles.

R3. Let A have mass m and initial velocity u. $E_i = \frac{1}{2}mu^2 = E$ and momentum $= p = mu =$ constant. The potential energy becomes maximum when the kinetic energy is minimum. This occurs when the bodies move with the same velocity.

Minimum kinetic energy $= E_f = \dfrac{p^2}{2(m + nm)}$.

Maximum PE $= E_i - E_f = \dfrac{1}{2}mu^2 - \dfrac{m^2u^2}{2m(1+n)}$

$$= E\left[1 - \frac{1}{1+n}\right] = \frac{nE}{n+1}.$$

R4. Let T = tension in each string.

$2T\cos\theta = W$ and $T\sin\theta = kx$, x = extension of spring

or $\left(\dfrac{W}{2\cos\theta}\right)\sin\theta = kx$ or $x = \dfrac{W\tan\theta}{2k}$.

R5. The forces acting on the man at the walls are
(a) horizontal normal reactions of the walls, and
(b) forces of friction acting upwards.
The man exerts equal and opposite forces on the walls.

R6. Tension will appear in the string only when its length tends to change, i.e., when A tends to move. This can occur only when F_A reaches its limiting value.

R7. The train has covered a distance of 200 m in the time interval.

$$200\text{ m} = 0 + \frac{1}{2}(0.5\text{ m/s}^2)t^2.$$

$$t = 20\sqrt{2}\text{ s}.$$

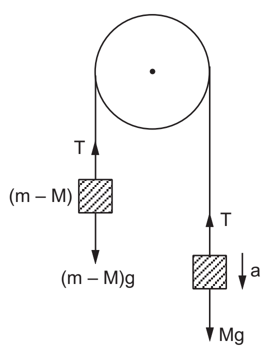
R9. Coefficient of friction $= \mu = cx$, $c =$ constant.

Maximum acceleration $= \mu g = (cx)g$

$$v^2 = 0 + 2(cgx)x$$

or $\quad v^2 \propto x^2 \quad$ or $\quad \text{KE} \propto x^2$

R10.



Let the masses of the section of the chain on the two sides be M and $(m - M)$, with $M > m/2$. Let a be the acceleration of the chain, and T be the tension in the chain at the points where it meets the pulley. Then, $Mg - T = ma$ and $T - (m - M)g = (m - M)a$. Eliminating a and rearranging, $T = 2\left(1 - \frac{M}{m}\right)Mg$. The force exerted by the chain on the pulley is $2T$. This will be less than mg as $M > m/2$.

R11. Time of fall $= t = \sqrt{2h/g}$.

Velocity of impact $= v = \sqrt{2gh}$.

Change of momentum at each impact, $\Delta p = 2mv$.

Time interval between successive impacts, $\Delta t = 2t$.

Average force $\dfrac{\Delta p}{\Delta t} = mg$.

R12. Maximum force of friction $= kmg$.

$\therefore$ maximum acceleration of insect $= a_1 = \dfrac{kmg}{m} = kg$

and maximum acceleration of stick $= a_2 = \dfrac{kmg}{M}$.

$\therefore$ acceleration of insect with respect to stick

$$= a = a_1 - (-a_2) = kg\left(1 + \frac{m}{M}\right).$$

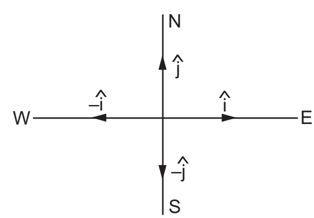
$$\therefore \quad L = \frac{1}{2}at^2 \qquad \text{or} \qquad t^2 = \frac{2L}{a} = \frac{2\,ML}{kg(M+m)}.$$

R13. $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v\,\hat{i} + v\,\hat{j}, \;\; v = \dfrac{V}{\sqrt{2}}$.

$\vec{v}_{BC} = \vec{v}_B - \vec{v}_C = -v\,\hat{i} + v\,\hat{j}$.

$\vec{v}_{CA} = \vec{v}_C - \vec{v}_A$

$= -[\,\vec{v}_{AB} + \vec{v}_{BC}\,] = -2v\,\hat{j}$.

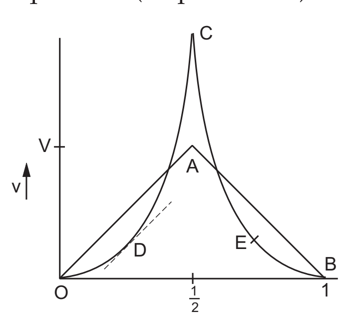


R14. The simplest solution to the motion is shown by the plot OAB. The area under the v~t plot must be equal to 1 (displacement).

For this, $\dfrac{V}{2} = 1$ or $V = 2$.

The acceleration $\dfrac{V}{1/2} = 4$ units.

$\therefore$ slope of OA $= 4$.



Taking OAB as reference, the condition of the motion can be satisfied by any other curve, e.g., OCB, as long as the area under it is equal to 1. Also, the slope of the curve at any point gives the acceleration α. Points D and E have $\alpha = 4$. Points below D have $\alpha < 4$ and above D have $\alpha > 4$.

As the shape of the curve OCB is arbitrary, it is not possible to set upper or lower limits on the acceleration.

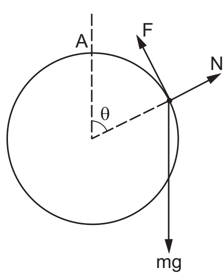
R15. Momentum of the 'man + car' system $= (m + M)V =$ constant. When the man begins to run, let $v_M =$ velocity of man and $v_C =$ velocity of car.

$\therefore$ velocity of man with respect to the car $= u = v_M - v_C$.

By conservation of momentum, $(m + M)V = mv_M + Mv_C$.

Rearranging, $v_C = V - \dfrac{mu}{m + M}$.

R16.



$mg\cos\theta = N$

$mg\sin\theta = F = \mu N = \mu\, mg\cos\theta$

or $\mu = \tan\theta$.

R17. Let $x =$ maximum compression of spring.

By conservation of energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ or $x = v\sqrt{m/k}$.

Maximum spring force $= kx = v\sqrt{mk}$.

R18. Displacement $= s = kt^3$ where $k =$ constant.

$v = \dot{s} = 3kt^2 \qquad a = \dot{v} = 6kt \qquad F = ma = 6mkt$

Power $= Fv = 18mk^2t^3 \propto t^3$.

R19. Let $v =$ horizontal component of velocity of shell,

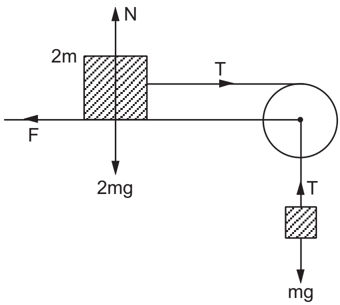
and $T =$ total time of flight.

$\therefore \quad vT = 2$ km.

When the shell breaks, the part which falls vertically does not have any horizontal velocity, and hence the other part acquires horizontal velocity $2v$, by conservation of momentum. Also, its remaining time of flight will be $T/2$. It will travel a further distance $(2v)(T/2) = vT = 2$ km.

$\therefore$ it will land 3 km away from the cannon.

R20.



$$mg = T = F = \mu N = \mu(2mg)$$

$$\mu = \frac{1}{2}$$

R21. The block will begin to move only when the external force becomes equal to the limiting friction at the floor, i.e., at $t > 0$. Also, as the external force is variable, its acceleration will be variable. The plot of v against t will be a curve.

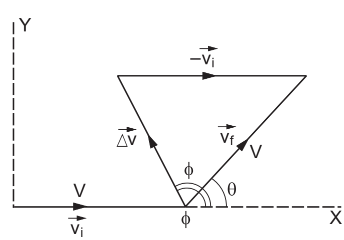
R22. Initial velocity $\vec{v_i} = V\hat{i}$.

Final velocity $\vec{v_f} = (V\cos\theta)\hat{i} + (V\sin\theta)\hat{j}$.

Change in velocity, $\vec{\Delta v} = \vec{v_f} - \vec{v_i}$

$$= (V\cos\theta)\hat{i} + (V\sin\theta)\hat{j} - V\hat{i}$$

$$= V(\cos\theta - 1)\hat{i} + (V\sin\theta)\hat{j}.$$



$$\Delta v = \sqrt{\{V(\cos\theta - 1)\}^2 + (V\sin\theta)^2}$$

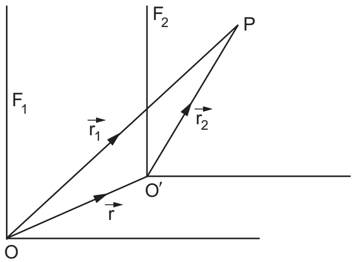
$$= 2V\sin\frac{\theta}{2}.$$

$$\tan\phi = \frac{V\sin\theta}{V(\cos\theta - 1)}$$

$$= -\cot\frac{\theta}{2} = \tan\left(\frac{\pi}{2} + \frac{\theta}{2}\right)$$

or $\quad \phi = \dfrac{\pi}{2} + \dfrac{\theta}{2}.$

R23.



$\vec{v} = \dot{\vec{r}}$, $\vec{v}_1 = \dot{\vec{r}}_1$, $\vec{v}_2 = \dot{\vec{r}}_2$,

$\vec{r}_1 = \vec{r} + \vec{r}_2$

or $\dot{\vec{r}}_1 = \dot{\vec{r}} + \dot{\vec{r}}_2$

or $\vec{v}_1 = \vec{v} + \vec{v}_2$.

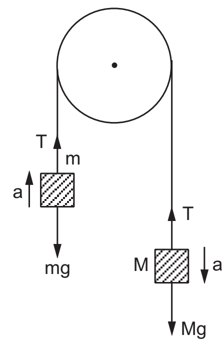
R24. $Mg - T = Ma$, $T - mg = ma$.

$$\therefore \quad a = \left(\frac{M-m}{M+m}\right)g.$$

$$T = mg + ma$$

$$= mg\left[1 + \frac{M-m}{M+m}\right] = \frac{2mMg}{M+m}$$

$$= \frac{2mg}{1 + m/M} \simeq 2\,mg \text{ as } M >> m.$$



R25. $y = bx^2$, $\dot{y} = 2bx\,\dot{x}$, $\ddot{y} = a = 2b\dot{x}^2 + 2bx\ddot{x}$,

Here, $\ddot{x} = 0$, $\therefore$ $\dot{x} = \sqrt{a/2b}$.

R26. The net force on the stick must be zero for it to be in equilibrium.

□

2

Circular, Rotational and Simple Harmonic Motions

• *Type 1* •

Choose the correct option (a, b, c or d).

1. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are

(a) mg and T only

(b) mg, T and an additional force of mv^2/r directed inwards

(c) mg, T and an additional force of mv^2/r directed outwards

(d) only a force mv^2/r directed outwards

2. A particle of mass m is tied to a light string of length l and rotated along a vertical circular path. What should be the minimum speed at the highest point of its path so that the string does not become slack at any position?

(a) $\sqrt{2gl}$ (b) $\sqrt{gl}$

(c) zero (d) $\sqrt{gl/2}$

3. A simple pendulum has a string of length l and bob of mass m. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a circular path about the point of suspension. The tension in the string at the lowest position of the bob is

(a) $3mg$ (b) $4mg$
(c) $5mg$ (d) $6mg$

4. In the previous question, when the string is horizontal, the net force on the bob is

(a) mg (b) $3mg$
(c) $\sqrt{10}mg$ (d) $4mg$

5. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical.

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60°$
(c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 0$

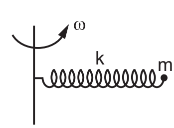
6. A particle of mass m is fixed to one end of a light rigid rod of length l and rotated in a vertical circular path about its other end. The minimum speed of the particle at its highest point must be

(a) zero (b) $\sqrt{gl}$
(c) $\sqrt{1.5gl}$ (d) $\sqrt{2gl}$

7. In the previous question, if the particle is released from rest at its highest position then the tension in the rod

(a) is zero when it is vertical
(b) is zero when it is horizontal
(c) is zero when it is making an angle of $\cos^{-1}(2/3)$ with the vertical
(d) cannot be zero in any position

8.



A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about the other end of the spring with an angular velocity ω, in gravity-free space. The increase in length of the spring will be

(a) $\frac{m\omega^2 l}{k}$ (b) $\frac{m\omega^2 l}{k - m\omega^2}$

(c) $\frac{m\omega^2 l}{k + m\omega^2}$ (d) none of these

9. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2 r t^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi m k^2 r^2 t$ (b) $m k^2 r^2 t$

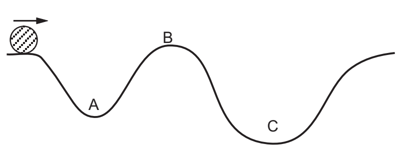
(c) $\frac{1}{3} m k^4 r^2 t^5$ (d) 0

10. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with the vertical is

(a) zero (b) 30°

(c) 45° (d) 60°

11.



A body moves along an uneven horizontal road surface with constant speed at all points. The normal reaction of the road on the body is

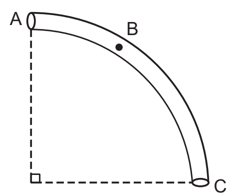
(a) maximum at A (b) maximum at B

(c) minimum at C (d) the same at A, B and C

12. In gravity-free space, a particle is in contact with the inner surface of a hollow vertical cylinder and moves in a horizontal circular path along the surface. There is some friction between the particle and the surface. The retardation of the particle is

(a) zero
(b) independent of its velocity
(c) proportional to its velocity
(d) proportional to the square of its velocity

13.



The tube AC forms a quarter circle in a vertical plane. The ball B has an area of cross-section slightly smaller than that of the tube, and can move without friction through it. B is placed at A and displaced slightly. It will

(a) always be in contact with the inner wall of the tube
(b) always be in contact with the outer wall of the tube
(c) initially be in contact with the inner wall and later with the outer wall
(d) initially be in contact with the outer wall and later with the inner wall

14. A car moves along a horizontal circular road of radius r with velocity v. The coefficient of friction between the wheels and the road is μ. Which of the following statements is not true?

(a) The car will slip if $v > \sqrt{\mu r g}$.
(b) The car will slip if $\mu < \frac{v^2}{rg}$.
(c) The car will slip if $r > \frac{v^2}{\mu g}$.
(d) The car will slip at a lower speed, if it moves with some tangential acceleration, than if it moves at constant speed.

15. For a car taking a turn on a horizontal surface, let N_1 and N_2 be the normal reactions of the road on the inner and outer wheels respectively.

(a) N_1 is always greater than N_2.

(b) N_2 is always greater than N_1.

(c) N_1 is always equal to N_2.

(d) Either (a) or (b) depending on the speed of the car and the radius of curvature of the road.

16. A curved section of a road is banked for a speed v. If there is no friction between the road and the tyres then

(a) a car moving with speed v will not slip on the road

(b) a car is more likely to slip on the road at speeds higher than v, than at speeds lower than v

(c) a car is more likely to slip on the road at speeds lower than v, than at speeds higher than v

(d) a car can remain stationary on the road without slipping

17. A railway track is banked for a speed v, by making the height of the outer rail h higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r.

(a) $\frac{h}{d} = \frac{v^2}{rg}$

(b) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$

(c) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$

(d) $\frac{h}{r} = \frac{v^2}{dg}$

18. A cyclist moves along a curved road with a velocity v. The road is banked for speed v. The angle of banking is θ. Which of the following statements is not true?

(a) The cyclist will lean away from the vertical at an angle θ.

(b) The normal reaction of the road will pass through the centre of gravity of the 'cycle plus cyclist' system.

(c) There will be no force of friction between the tyres and the road.

(d) The cyclist is in equilibrium with respect to the ground.

19. Two point masses m and M are separated by a distance L. The distance of the centre of mass of the system from m is

(a) $L(m/M)$

(b) $L(M/m)$

(c) $L\left(\frac{M}{m+M}\right)$ (d) $L\left(\frac{m}{m+M}\right)$

20. A man stands at one end of a boat which is stationary in water. Neglect water resistance. The man now moves to the other end of the boat and again becomes stationary. The centre of mass of the 'man plus boat' system will remain stationary with respect to water

(a) in all cases
(b) only when the man is stationary initially and finally
(c) only if the man moves without acceleration on the boat
(d) only if the man and the boat have equal masses

21. A stick is thrown in the air and lands on the ground at some distance from the thrower. The centre of mass of the stick will move along a parabolic path

(a) in all cases
(b) only if the stick is uniform
(c) only if the stick has linear motion but no rotational motion
(d) only if the stick has a shape such that its centre of mass is located at some point on it and not outside it

22. A man hangs from a rope attached to a hot-air balloon. The mass of the man is greater than the mass of the balloon and its contents. The system is stationary in air. If the man now climbs up to the balloon using the rope, the centre of mass of the 'man plus balloon' system will

(a) remain stationary
(b) move up
(c) move down
(d) first move up and then return to its initial position

23. There are some passengers inside a stationary railway compartment. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment,

(a) both C_1 and C_2 will move with respect to the ground
(b) neither C_1 nor C_2 will move with respect to the ground
(c) C_1 will move but C_2 will be stationary with respect to the ground
(d) C_2 will move but C_1 will be stationary with respect to the ground

24. For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about

(a) the centre of the system
(b) the centre of mass of the system
(c) any point on the system
(d) any point on the system or outside it

25. A uniform horizontal metre scale of mass *m* is suspended by two vertical strings attached to its two ends. A body of mass 2*m* is placed on the 75-cm mark. The tensions in the two strings are in the ratio

(a) 1 : 2 (b) 1 : 3
(c) 2 : 3 (d) 3 : 4

26. The line of action of the resultant of two like parallel forces shifts by one-fourth of the distance between the forces when the two forces are interchanged. The ratio of the two forces is

(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) 3 : 5

27. A uniform metre scale balances at the 40-cm mark when weights of 10 g and 20 g are suspended from the 10-cm and 20-cm marks respectively. The weight of the metre scale is

(a) 50 g (b) 60 g
(c) 70 g (d) 80 g

28. Weights of 1 g, 2 g, …, 100 g are suspended from the 1-cm, 2-cm, …, 100-cm marks respectively of a light metre scale. Where should it be supported for the system to be in equilibrium?

(a) 55-cm mark (b) 60-cm mark
(c) 66-cm mark (d) 72-cm mark

29. A rectangular block has a square base measuring $a \times a$, and its height is h. It moves with a speed v on a smooth horizontal surface.

(a) It will topple if $v > \sqrt{2gh}$.
(b) It will topple if $v > \sqrt{2ga}$.
(c) It will topple if $v > \sqrt{2ga^2/h}$.
(d) It will not topple for any value of v.

30. A rectangular block has a square base measuring $a \times a$, and its height is h. It moves on a horizontal surface in a direction perpendicular to one of the edges. The coefficient of friction is μ. It will topple if

(a) $\mu > \frac{h}{a}$ (b) $\mu > \frac{a}{h}$
(c) $\mu > \frac{2a}{h}$ (d) $\mu > \frac{a}{2h}$

31. A flywheel rotates with a uniform angular acceleration. Its angular velocity increases from 20π rad/s to 40π rad/s in 10 seconds. How many rotations did it make in this period?

(a) 80 (b) 100
(c) 120 (d) 150

32. When a ceiling fan is switched on, it makes 10 rotations in the first 3 seconds. How many rotations will it make in the next 3 seconds? (Assume uniform angular acceleration.)

(a) 10 (b) 20
(c) 30 (d) 40

33. When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest? (Assume uniform angular retardation.)

(a) 36 (b) 24 (c) 18 (d) 12

34. A flywheel rotates about an axis. Due to friction at the axis, it experiences an angular retardation proportional to its angular velocity. If its angular velocity falls to half while it makes n rotations, how many more rotations will it make before coming to rest?

(a) $2n$ (b) n
(c) $n/2$ (d) $n/3$

35. A flywheel rotating about an axis experiences an angular retardation proportional to the angle through which it rotates. If its rotational kinetic energy gets reduced by ΔE while it rotates through an angle θ then

(a) $\Delta E \propto \theta^2$ (b) $\Delta E \propto \sqrt{\theta}$
(c) $\Delta E \propto \theta$ (d) $\Delta E \propto \theta^{3/2}$

36. An external device, e.g., an electric motor, supplies constant power to a rotating system, e.g., a flywheel, through a torque τ. The angular velocity of the system is ω. Both τ and ω are variable.

(a) $\omega \propto \tau$ (b) $\omega \propto \frac{1}{\tau}$
(c) $\omega \propto \sqrt{\tau}$ (d) $\omega \propto \frac{1}{\sqrt{\tau}}$

37. Two identical rods are joined to form an 'X'. The smaller angle between the rods is θ. The moment of inertia of the system about an axis passing through the point of intersection of the rods and perpendicular to their plane is

(a) $\propto \theta$ (b) $\propto \sin^2\theta$
(c) $\propto \cos^2\theta$ (d) independent of θ

38. A uniform rod of mass m and length l makes a constant angle θ with an axis of rotation which passes through one end of the rod. Its moment of inertia about this axis is

(a) $\frac{ml^2}{3}$ (b) $\frac{ml^2}{3}\sin\theta$
(c) $\frac{ml^2}{3}\sin^2\theta$ (d) $\frac{ml^2}{3}\cos^2\theta$

39. A disc of mass m and radius R has a concentric hole of radius r. Its moment of inertia about an axis through its centre and perpendicular to its plane is

(a) $\frac{1}{2}m(R-r)^2$ (b) $\frac{1}{2}m(R^2-r^2)$

(c) $\frac{1}{2}m(R+r)^2$ (d) $\frac{1}{2}m(R^2+r^2)$

40. The radius of gyration of a thin disc of radius 4 cm about a diameter is

(a) 4 cm (b) $2\sqrt{2}$ cm

(c) 2 cm (d) $\sqrt{2}$ cm

41. The radius of gyration of a solid sphere of radius r about a certain axis is r. The distance of this axis from the centre of the sphere is

(a) r (b) $0.5r$

(c) $\sqrt{0.6}\,r$ (d) $\sqrt{0.4}\,r$

42. Three identical rods, each of length l, are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is

(a) $l/2$ (b) $\sqrt{\frac{3}{2}}\,l$

(c) $l/\sqrt{2}$ (d) $l/\sqrt{3}$

43. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate and it passes through the centre of the plate, making an angle θ with AB. The moment of inertia of the plate about the axis CD is equal to

(a) I (b) $I\sin^2\theta$

(c) $I\cos^2\theta$ (d) $I\cos^2\left(\frac{\theta}{2}\right)$

44. If the radius of the earth becomes half of its present value, with its mass remaining the same, the duration of one day will become

(a) 6 h (b) 12 h

(c) 48 h (d) 96 h

45. A man spinning in free space can change the moment of inertia of his body (I) by changing its shape. In this process,

(a) he will have to expend some energy to increase I

(b) he will have to expend some energy to decrease I

(c) he does not have to expend any energy to change I

(d) either (a) or (b) depending on the initial value of I

46. A small ball strikes a stationary uniform rod, which is free to rotate, in gravity-free space. The ball does not stick to the rod. The rod will rotate about

(a) its centre of mass

(b) the centre of mass of 'rod plus ball'

(c) the point of impact of the ball on the rod

(d) the point about which the moment of inertia of the 'rod plus ball' is minimum

47. Three identical solid spheres move down three inclines A, B and C—all of the same dimensions. A is without friction, the friction between B and a sphere is sufficient to cause rolling without slipping, the friction between C and a sphere causes rolling with slipping. The kinetic energies of A, B, C at the bottom of the inclines are E_A, E_B, E_C.

(a) $E_A = E_B = E_C$ (b) $E_A = E_B > E_C$

(c) $E_A > E_B > E_C$ (d) $E_A > E_B = E_C$

48. A body of mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing

49. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity.) The length of the string is reduced gradually, keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle, and n is

(a) 1 (b) –1
(c) –2 (d) –3

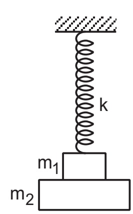
50. For a particle undergoing SHM, the velocity is plotted against displacement. The curve will be

(a) a straight line (b) a parabola
(c) a circle (d) an ellipse

51. A particle undergoes SHM with a time period of 2 seconds. In how much time will it travel from its mean position to a displacement equal to half of its amplitude?

(a) $\frac{1}{2}$ s (b) $\frac{1}{3}$ s
(c) $\frac{1}{4}$ s (d) $\frac{1}{6}$ s

52.



Two blocks of masses m_1 and m_2 are attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. When the system is in equilibrium, the lower block (m_2) drops off. The other block (m_1) will

(a) remain undisturbed
(b) move up through a distance m_2g/k and come to rest
(c) undergo vertical SHM with a time period of $2\pi\sqrt{m_1/k}$
(d) undergo vertical SHM with a time period of $2\pi\sqrt{(m_1 + m_2)/k}$

53. The displacement y of a particle executing a certain periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$. This expression may be considered to be the superposition of n independent harmonic motions. Then, n is equal to

(a) 2 (b) 3
(c) 4 (d) 5

• *Type 2* •

Choose the correct options. One or more options may be correct.

54. A simple pendulum has a bob of mass m and swings with an angular amplitude ϕ. The tension in the thread is T. At a certain time, the string makes an angle θ with the vertical ($\theta \leq \phi$).

(a) $T = mg\cos\theta$, for all values of θ.
(b) $T = mg\cos\theta$, only for $\theta = \phi$.
(c) $T = mg$, for $\theta = \cos^{-1}\left[\frac{1}{3}(2\cos\phi + 1)\right]$.
(d) T will be larger for smaller values of θ.

55. A simple pendulum rotates in a horizontal plane with an angular velocity of ω about a fixed point P in gravity-free space. There is a negative charge at P. The bob gradually emits photoelectrons (disregard the energy and momentum of the incident photons and emitted electrons). The total force acting on the bob is T.

(a) T will decrease, ω will decrease.
(b) T will decrease, ω will remain constant.
(c) T and ω will remain unchanged.
(d) The elastic strain in the string will decrease.

56. A simple pendulum of length l is set in motion such that the bob, of mass m, moves along a horizontal circular path, and the string makes a constant angle θ with the vertical. The time period of rotation of the bob is t and the tension in the thread is T.

(a) $t = 2\pi\sqrt{l/g}$ (b) $t = 2\pi\sqrt{l\cos\theta/g}$

(c) $T = \frac{4\pi^2 ml}{t^2}$

(d) The bob is in equilibrium.

57. A stone tied to a string of length L is whirled in a vertical circle, with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

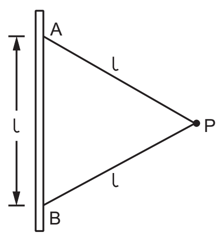
(a) $\sqrt{u^2 - 2gL}$

(b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$

(d) $\sqrt{2(u^2 - gL)}$

58.



A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation $AB = l$. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2.

(a) $T_1 = T_2$

(b) $T_1 + T_2 = m\omega^2 l$

(c) $T_1 - T_2 = 2mg$

(d) BP will remain taut only if $\omega \geq \sqrt{2g/l}$.

59. A uniform rod of mass m and length l rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is

(a) $\frac{1}{2} m\omega^2 x$

(b) $\frac{1}{2} m\omega^2 \frac{x^2}{l}$

(c) $\frac{1}{2} m\omega^2 l \left(1 - \frac{x}{l}\right)$

(d) $\frac{1}{2} \cdot \frac{m\omega^2}{l} [l^2 - x^2]$

60. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{1}{2}M\omega^2L$ (b) $M\omega^2L$

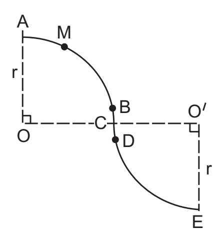
(c) $\frac{1}{4}M\omega^2L$ (d) $\frac{1}{2}M\omega^2L^2$

61. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is

(a) zero (b) $\frac{1}{2}m\omega^2r^2$

(c) $m\omega^2r^2$ (d) $mr\omega^2$

62.



ABCDE is a smooth iron track in the vertical plane. The sections ABC and CDE are quarter circles. Points B and D are very close to C. M is a small magnet of mass m. The force of attraction between M and the track is F, which is constant and always normal to the track. M starts from rest at A.

(a) If M is not leave the track at C then $F \geq 2mg$.

(b) At B, the normal reaction of the track is $F - 2mg$.

(c) At D, the normal reaction of the track is $F + 2mg$.

(d) The normal reaction of the track is equal to F at some point between A and B.

63. The earth rotates from west to east. A wind mass begins moving due north from the equator, along the earth's surface.

Neglect all effects other than the rotation of the earth. The wind mass will

(a) always move due north
(b) shift a little to the east as it moves to higher latitudes
(c) shift a little to the west as it moves to higher latitudes
(d) move along a loop and return to its starting point on the equator

64. A geostationary satellite S is stationed above a point P on the equator. A particle is fired from S directly towards P.

(a) With respect to the axis of rotation of the earth, P and S have the same angular velocity but different linear velocities.
(b) The particle will hit P.
(c) The particle will hit the equator east of P.
(d) The particle will hit the equator west of P.

65. A body moves on a horizontal circular road of radius r, with a tangential acceleration a_t. The coefficient of friction between the body and the road surface is μ. It begins to slip when its speed is v.

(a) $v^2 = \mu rg$

(b) $\mu g = \frac{v^2}{r} + a_t$

(c) $\mu^2 g^2 = \frac{v^4}{r^2} + a_t^2$

(d) The force of friction makes on angle $\tan^{-1}(v^2/a_t r)$ with the direction of motion at the point of slipping.

66. The density of a rod AB increases linearly from A to B. Its midpoint is O and its centre of mass is at C. Four axes pass through A, B, O and C, all perpendicular to the length of the rod. The moments of inertia of the rod about these axes are I_A, I_B, I_O and I_C respectively.

(a) $I_A > I_B$ (b) $I_A < I_B$
(c) $I_O > I_C$ (d) $I_O < I_C$

67. A square plate lies in the xy plane with its centre at the origin and its edges parallel to the x and y axes. Its moments of inertia about the x, y and z axes are I_x, I_y and I_z respectively, and about a diagonal it is I_D.

(a) $I_x = I_y = \frac{1}{2} I_z$ (b) $I_x = I_y = 2I_z$

(c) $I_D = I_x$ (d) $I_D = I_z$

68. Four identical rods, each of mass m and length l, are joined to form a rigid square frame. The frame lies in the xy plane, with its centre at the origin and the sides parallel to the x and y axes. Its moment of inertia about

(a) the x-axis is $\frac{2}{3} ml^2$

(b) the z-axis is $\frac{4}{3} ml^2$

(c) an axis parallel to the z-axis and passing through a corner is $\frac{10}{3} ml^2$

(d) one side is $\frac{5}{2} ml^2$

69. P is the centre of mass of four point masses A, B, C and D, which are coplanar but not collinear.

(a) P may or may not coincide with one of the point masses.
(b) P must lie within the quadrilateral ABCD.
(c) P must lie within or on the edge of at least one of the triangles formed by taking A, B, C and D three at a time.
(d) P must lie on a line joining two of the points A, B, C, D.

70. When slightly different weights are placed on the two pans of a beam balance, the beam comes to rest at an angle with the horizontal. The beam is supported at a single point P by a pivot.

(a) The net torque about P due to the two weights is nonzero at the equilibrium position.
(b) The whole system does not continue to rotate about P because it has a large moment of inertia.

(c) The centre of mass of the system lies below P.

(d) The centre of mass of the system lies above P.

71. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

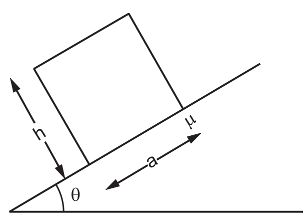
(a) 2, if their lines of action pass through the centre of mass of the body

(b) 3, if their lines of action are not parallel

(c) 3, if their lines of action are parallel

(d) 4, if their lines of action are parallel and all the forces have the same magnitude

72.



A block with a square base measuring $a \times a$, and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

(a) topple before sliding if $\mu > \frac{a}{h}$

(b) topple before sliding if $\mu < \frac{a}{h}$

(c) slide before toppling if $\mu > \frac{a}{h}$

(d) slide before toppling if $\mu < \frac{a}{h}$

73. Two men support a uniform horizontal beam at its two ends. If one of them suddenly lets go, the force exerted by the beam on the other man will

(a) remain unaffected (b) increase

(c) decrease
(d) become unequal to the force exerted by him on the beam

74. A uniform rod kept vertically on the ground falls from rest. Its foot does not slip on the ground.

(a) No part of the rod can have acceleration greater than g in any position.
(b) At any one position of the rod, different points on it have different accelerations.
(c) Any one particular point on the rod has different accelerations at different positions of the rod.
(d) The maximum acceleration of any point on the rod, at any position, is $1.5g$.

75. A man spinning in free space changes the shape of his body, e.g., by spreading his arms or curling up. By doing this, he can change his

(a) moment of inertia
(b) angular momentum
(c) angular velocity
(d) rotational kinetic energy

76. A man standing on a platform holds weights in his outstretched arms. The system rotates freely about a central vertical axis. If he now draws the weights inwards close to his body,

(a) the angular velocity of the system will increase
(b) the angular momentum of the system will decrease
(c) the kinetic energy of the system will increase
(d) he will have to expend some energy to draw the weights in

77. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity.

(a) Some friction exists between the disc and the ring.
(b) The angular momentum of the 'disc plus ring' is conserved.

(c) The final common angular velocity is $\frac{2}{3}$rd of the initial angular velocity of the disc

(d) $\frac{2}{3}$rd of the initial kinetic energy changes to heat.

78. Two horizontal discs of different radii are free to rotate about their central vertical axes. One is given some angular velocity, the other is stationary. Their rims are now brought in contact. There is friction between the rims.

(a) The force of friction between the rims will disappear when the discs rotate with equal angular speeds.

(b) The force of friction between the rims will disappear when they have equal linear velocities.

(c) The angular momentum of the system will be conserved.

(d) The rotational kinetic energy of the system will not be conserved.

79. A constant external torque τ acts for a very brief period Δt on a rotating system having moment of inertia I.

(a) The angular momentum of the system will change by $\tau\,\Delta t$.

(b) The angular velocity of the system will change by $\frac{\tau\,\Delta t}{I}$.

(c) If the system was initially at rest, it will acquire rotational kinetic energy $\frac{(\tau\,\Delta t)^2}{2I}$.

(d) The kinetic energy of the system will change by $\frac{(\tau\,\Delta t)^2}{I}$.

80. Two identical spheres A and B are free to move and to rotate about their centres. They are given the same impulse J. The lines of action of the impulses pass through the centre of A, and away from the centre of B.

(a) A and B will have the same speed.

(b) B will have greater kinetic energy than A.

(c) They will have the same kinetic energy, but the linear kinetic energy of B will be less than that of A.

(d) The kinetic energy of B will depend on the point of impact of the impulse on B.

81. The motion of a sphere moving on a rough horizontal surface changes from pure sliding (without rolling) to pure rolling (without slipping). In this process, the force of friction

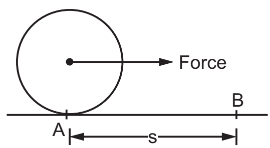
(a) initially acts opposite to the direction of motion and later in the direction of motion

(b) causes linear retardation

(c) causes angular acceleration

(d) stops acting when pure rolling begins

82.



A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB $= s$. The disc moves from A to B in time T. To the right of B,

(a) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged

(b) linear acceleration of the disc will increase

(c) the disc will make one rotation in time $T/2$

(d) the disc will cover a distance greater than s in a further time T

83. A solid sphere starts from rest at the top of an incline of height h and length l, and moves down. The force of friction between the sphere and the incline is F. This is insufficient to prevent slipping. The kinetic energy of the sphere at the bottom of the incline is W.

(a) The work done against the force of friction is Fl.

(b) The heat produced is Fl.

(c) $W = mgh - Fl$

(d) $W > (mgh - Fl)$

84. A ring (R), a disc (D), a solid sphere (S) and a hollow sphere with thin walls (H), all having the same mass but different radii, start together from rest at the top of an inclined plane and roll down without slipping.

(a) All of them will reach the bottom of the incline together.

(b) The body with the maximum radius will reach the bottom first.

(c) They will reach the bottom in the order S, D, H, R.

(d) All of them will have the same kinetic energy at the bottom of the incline.

85. A solid sphere rolls without slipping on a rough horizontal floor, moving with a speed v. It makes an elastic collision with a smooth vertical wall. After impact,

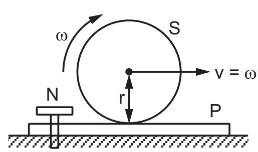
(a) it will move with a speed v initially

(b) its motion will be rolling without slipping

(c) its motion will be rolling with slipping initially and its rotational motion will stop momentarily at some instant

(d) its motion will be rolling without slipping only after some time

86.



A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin N. If N is suddenly removed,

(a) S will begin to slip on P

(b) P will begin to move backwards

(c) the speed of S will decrease and its angular velocity will increase

(d) there will be no change in the motion of S and P will still be at rest

87. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.

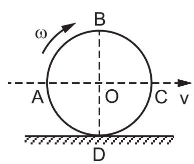
(a) $0 \leq v \leq 2u$

(b) $v = u$, if CP is horizontal.

(c) $v = u$, if CP makes an angle of 60° with the horizontal and P is below the horizontal level of C.

(d) $v = \sqrt{2}u$, if CP is horizontal.

88.



A ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure.

(a) Section ABC has greater kinetic energy than section ADC.

(b) Section BC has greater kinetic energy than section CD.

(c) Section BC has the same kinetic energy as section DA.

(d) The sections AB, BC, CD and DA have the same kinetic energy.

89. A wheel of radius r rolls without slipping with a speed v on a horizontal road. When it is at a point A on the road, a small blob of mud separates from the wheel at its highest point and lands at point B on the road.

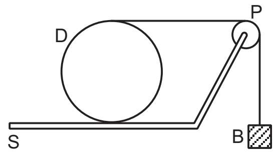
(a) $AB = v\sqrt{r/g}$

(b) $AB = 2v\sqrt{r/g}$

(c) $AB = 4v\sqrt{r/g}$

(d) If $v > \sqrt{4rg}$, the blob of mud will land on the wheel and not on the road.

90.



In the figure, the disc D does not slip on the surface S. The pulley P has mass, and the string does not slip on it. The string is wound around the disc.

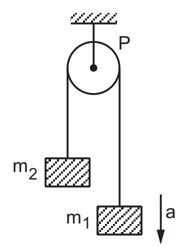
(a) The acceleration of the block B is double the acceleration of the centre of D.

(b) The force of friction exerted by D on S acts to the left.

(c) The horizontal and the vertical sections of the string have the same tension.

(d) The sum of the kinetic energies of D and B is less than the loss in the potential energy of B as it moves down.

91. In the figure, the blocks have unequal masses m_1 and m_2 $(m_1 > m_2)$. m_1 has a downward acceleration a. The pulley P has a radius r, and some mass. The string does not slip on the pulley.



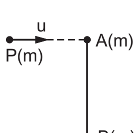
(a) The two sections of the string have unequal tensions.

(b) The two blocks have accelerations of equal magnitude.

(c) The angular acceleration of P is a/r.

(d) $a < \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

92.



Two particles A and B, of mass m each, are joined by a rigid massless rod of length l. A particle P of mass m, moving with a speed u normal to AB, strikes A and sticks to it. The centre of mass of the 'A + B + P' system is C.

(a) The velocity of C before impact is $u/3$.

(b) The velocity of C after impact is $u/3$.

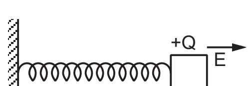
(c) The velocity of 'A + P' immediately after impact is $u/2$.
(d) The velocity of B immediately after impact is zero.

93. In the previous question, immediately after the impact,
(a) $AC = l/3$
(b) the angular momentum of the 'A + B + P' system about C is $\frac{1}{3}mul$
(c) the moment of inertia of the 'A + B + P' system about C is $\frac{2}{3}ml^2$
(d) the angular velocity of the 'A + B + P' system is $u/2l$

94. In Q. No. 92, immediately after impact,
(a) the velocity of 'A + P' with respect to C is $u/6$, to the right
(b) the angular velocity of 'A + P' with respect to C is $u/2l$, clockwise
(c) the velocity of B with respect to C is $u/3$, to the left
(d) the angular velocity of B with respect to C is $u/2l$, clockwise

95. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact,
(a) the angular momentum of the rod is Jl
(b) the angular velocity of the rod is $3J/ml$
(c) the kinetic energy of the rod is $3J^2/2m$
(d) the linear velocity of the midpoint of the rod is $3J/2m$

96.



A spring–block system undergoes simple harmonic motion on a smooth horizontal surface. The block is now given some positive charge, and a uniform horizontal electric field to the right is switched on. As a result,

(a) the time period of oscillation will increase
(b) the time period of oscillation will decrease
(c) the time period of oscillation will remain unaffected
(d) the mean position of simple harmonic motion will shift to the right

97. A simple pendulum has a time period T. The bob is now given some positive charge.

(a) If some positive charge is placed at the point of suspension, T will increase.
(b) If some positive charge is placed at the point of suspension, T will not change.
(c) If a uniform downward electric field is switched on, T will increase.
(d) If a uniform downward electric field is switched on, T will decrease.

98. A coin is placed on a horizontal platform, which undergoes horizontal simple harmonic motion about a mean position O. The coin does not slip on the platform. The force of friction acting on the coin is F.

(a) F is always directed towards O.
(b) F is directed towards O when the coin is moving away from O, and away from O when the coin moves towards O.
(c) $F = 0$ when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.
(d) F is maximum when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.

99. In the previous question, the angular frequency of the simple harmonic motion is ω. The coefficient of friction between the coin and the platform is μ. The amplitude of oscillation is gradually increased. The coin will begin to slip on the platform for the first time

(a) at the extreme positions of oscillations
(b) at the mean position
(c) for an amplitude of $\mu g/\omega^2$
(d) for an amplitude of $g/\mu\omega^2$

100. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

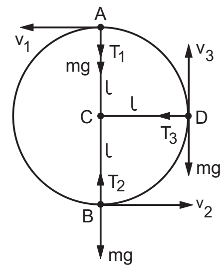
(a) at the highest position of the platform
(b) at the mean position of the platform
(c) for an amplitude of g/ω^2
(d) for an amplitude of $\sqrt{g}/\omega$

Answers

1. a	**2.** b	**3.** d	**4.** c	**5.** c
6. a	**7.** c	**8.** b	**9.** b	**10.** c
11. a	**12.** d	**13.** c	**14.** c	**15.** b
16. a	**17.** b	**18.** d	**19.** c	**20.** a
21. a	**22.** a	**23.** c	**24.** d	**25.** a
26. d	**27.** c	**28.** c	**29.** d	**30.** b
31. d	**32.** c	**33.** d	**34.** b	**35.** a
36. b	**37.** d	**38.** c	**39.** d	**40.** c
41. c	**42.** c	**43.** a	**44.** a	**45.** b
46. a	**47.** b	**48.** b	**49.** d	**50.** d
51. d	**52.** c	**53.** b	**54.** b, c, d	**55.** c, d
56. b, c	**57.** d	**58.** b, c, d	**59.** d	**60.** a
61. c	**62.** a, b, c, d	**63.** b	**64.** a, c	**65.** c, d
66. a, c	**67.** a, c	**68.** a, b, c, d	**69.** a, c	**70.** a, c
71. b, c, d	**72.** a, d	**73.** c	**74.** b, c, d	**75.** a, c, d
76. a, c, d	**77.** a, b, d	**78.** b, d	**79.** a, b, c	**80.** a, b, d
81. b, c, d	**82.** b, c, d	**83.** a, d	**84.** c, d	**85.** a, c, d
86. d	**87.** a, c, d	**88.** a, b	**89.** c	**90.** a, b, d
91. a, b, c, d	**92.** a, b, c, d	**93.** a, b, c, d	**94.** a, b, c, d	**95.** a, b, c, d
96. c, d	**97.** b, d	**98.** a, d	**99.** a, c	**100**. a, c

Hints and Solutions to Selected Questions

1. The force mv^2/r directed outwards, called centrifugal force, is not a real force but a pseudoforce.



2, 3, 4. At A, $mv_1^2/l = T_1 + mg$.

For v_1 to be minimum, $T_1 = 0$.

or $v_1 = \sqrt{gl}$.

Applying the principle of conservation of energy between A and B.

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg \times 2l$$

or $$\frac{mv_2^2}{l} = \frac{mv_1^2}{l} + 4mg$$

At B, $$\frac{mv_2^2}{l} = T_2 - mg$$

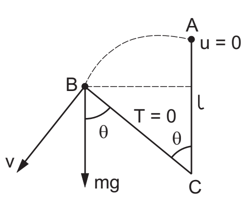
or $mg + 4mg = T_2 - mg$ or $T_2 = 6mg$.

Applying the principle of conservation of energy between A and D,

$$\frac{1}{2}mv_3^2 - \frac{1}{2}mv_1^2 = mgl \qquad \text{or} \qquad \frac{mv_3^2}{l} = \frac{mv_1^2}{l} + 2mg = 3mg = T_3.$$

The net force on the bob at D $= \sqrt{(3mg)^2 + (mg)^2} = \sqrt{10}\,mg$.

7.



Applying the principle of conservation of energy between A and B,

$$mg(l - l\cos\theta) = \frac{1}{2}mv^2$$

or $$\frac{mv^2}{l} = 2mg(1 - \cos\theta) = mg\cos\theta.$$

[The only force towards C is $mg\cos\theta$ as $T = 0$.]

$$\text{or} \quad \cos\theta = 2/3.$$

8. Let x = the increase in the length of spring. Then the particle moves along a circular path of radius $(l + x)$, and the spring force $= kx$ = centripetal force.

$$\therefore \quad m\omega^2(l + x) = kx \qquad \text{or} \quad x = \frac{m\omega^2 l}{k - m\omega^2}.$$

9. $a_c = k^2rt^2 = v^2/r$ or $v = krt$.

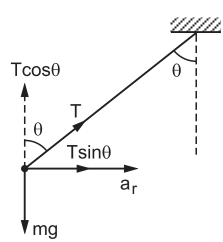
The tangential acceleration is $a_t = \dfrac{dv}{dt} = kr$.

$\therefore$ the net tangential force on the particle $= ma_t = mkr = F_t$.

Work is done on the particle only by tangential forces, as the radial forces are perpendicular to v.

$\therefore$ the power delivered to the particle $= F_t v = (mkr)(krt) = mk^2r^2t$.

10.



The radial acceleration is $a_r = v^2/r = \dfrac{(10\ \text{m/s})^2}{10\ \text{m}} = 10\ \text{m/s}^2$.

If T = tension in the rod then $T\cos\theta = mg$, $T\sin\theta = ma_r$.

or $\tan\theta = a_r/g = 1$ or $\theta = 45°$.

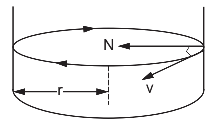
11. Let r_A, r_B and r_C be the radii of curvature of the road surface at A, B and C respectively. Let N_A, N_B and N_C be the normal reactions of the road on the body at A, B and C respectively. The centres of curvature of the road surface lie above the road at A and C, and below the road at B.

$$\therefore \quad N_A - mg = mv^2/r_A, \qquad mg - N_B = mv^2/r_B, \qquad N_C - mg = mv^2/r_C.$$

Clearly, N_A and N_C are $> N_B$. Also, as $r_A < r_C$, from the figure,

$$N_A = mg + \frac{mv^2}{r_A} > N_C = mg + \frac{mv^2}{r_C}.$$

12. The horizontal normal reaction of the wall on the particle is $N = mv^2/r$.

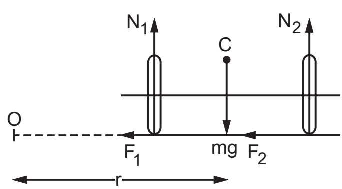


The force of friction, $F = \mu N = \mu mv^2/r$.

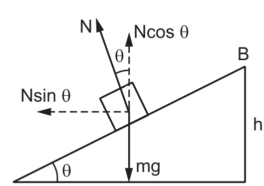
The retardation, $a = \frac{F}{m} = \frac{\mu v^2}{r} \propto v^2$.

14. The required centripetal force (mv^2/r) is supplied by the force of friction, which has a maximum value of μmg. Hence, the car will slip when $mv^2/r > \mu mg$. This yields (a) and (b) but not (c). To understand why (d) is a correct statement and, therefore, an incorrect choice, see the hint to Q. No. 65.

15.



Let O be the centre of curvature of the road and C be the centre of mass on the car. F_1 and F_2 are the forces of friction on the inner and outer wheels respectively. The torques due to N_1, F_1 and F_2 about C are clockwise; they must balance the anticlockwise torque due to N_2 about C. As N_1 and N_2 are equidistant from C, $N_2 > N_1$.



16, 17. The horizontal component of N supplies the necessary centripetal force.

$$N\sin\theta = mv^2/r$$

Hence a car moving with a speed v does not slip even if there is no friction.

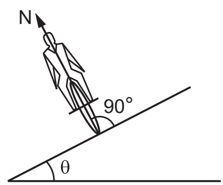
For any speed other than v, either higher or lower than v, the above condition is not satisfied, and the car will slip. This is true for $v = 0$ also, i.e., for a stationary car.

Also, $N\cos\theta = mg$ $\quad\therefore\quad \tan\theta = v^2/rg$

For the railway track, $AB = d$, $BC = h$, $\sin\theta = h/d$

or $\theta = \sin^{-1}\left(\frac{h}{d}\right)$ or $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$.

18.



TIP A body moving along a circular path is in accelerated motion, and is therefore not in equilibrium.

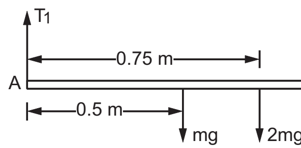
20. There are no external horizontal forces acting on the 'man plus boat' system. (The forces exerted by the man and the boat on each other are internal forces for the system.) Therefore, the centre of mass of the system, which is initially at rest, will always be at rest.

21. We may consider the entire mass of the stick to be concentrated as a point mass at the centre of mass of the stick. As the centre of mass moves as a projectile, it will move along a parabolic path.

22. As the system is stationary in air, the net external vertical forces on it is zero. Apply the arguments of Q. No. 20.

23. Choosing the compartment as the system, the passengers are external to the compartment, and can apply horizontal forces on it. Thus, C_1 may move. For the 'compartment plus passengers' system, there are no external horizontal forces. Thus, C_2 will not move.

25.



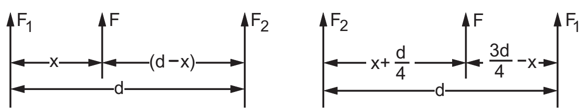
$T_1 + T_2 = 3mg.$

Taking torques about A,

$$0.5mg + 0.75 \times 2mg = 1 \times T_2$$

$\therefore \quad T_2 = 2mg, \qquad T_1 = mg.$

26.



Taking torque about F, which is the resultant of F_1 and F_2,

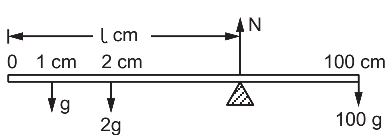
$$F_1 x = F_2(d - x)$$

and $F_1\left(\frac{3d}{4} - x\right) = F_2\left(x + \frac{d}{4}\right)$.

Dividing, $\dfrac{x}{\frac{3d}{4} - x} = \dfrac{d - x}{x + \frac{d}{4}}$.

Solve for x and use $\dfrac{F_1}{F_2} = \dfrac{d - x}{x}$.

28.



Let the scale be supported at the l-cm mark.

$$N = g + 2\,g + \ldots + 100\,g = 5050\,g.$$

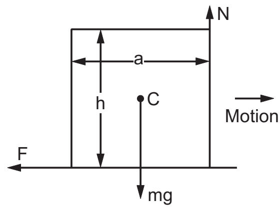
Taking torque about the zero mark,

$$Nl = 1 \times g + 2 \times 2\,g + \ldots + 100 \times 100\,g.$$

Solve for l.

29. A body topples due to an unbalanced torque about its centre of mass. Here, the two forces acting on the block, viz., mg and N, both pass through the centre of mass and hence produce no torque.

30.



Let C be the centre of mass of the block. When the force of friction acts on the block, the normal reaction (N) of the ground shifts to the right so as to balance the torque due to F about C. The condition of toppling arises when N reaches the edge of the cube. Taking torque about C, toppling occurs when

$$F \times \frac{h}{2} > N \times \frac{a}{2}$$

or $\mu mgh > mga$ or $\mu > \frac{a}{h}$.

31 to 35. (Common concepts)

Let ω_i = initial angular velocity,

ω_f = final angular velocity,

α = angular acceleration,

θ = angle through which the body rotates,

n = number of rotations, t = time.

Then, for constant α,

$$\theta = 2\pi n = \left(\frac{\omega_i + \omega_f}{2}\right) t = \omega_i t + \frac{1}{2}\alpha t^2$$

$\omega_f = \omega_i + \alpha t$ or $\omega_f^2 = \omega_i^2 + 2\alpha\,\theta$.

For variable α, $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$.

31. $\theta = 2\pi n = \left(\frac{\omega_i + \omega_f}{2}\right) t = \left(\frac{20\pi + 40\pi}{2}\right) \cdot 10 = 300\pi$

$\therefore \quad n = 150$

32. $\theta = 2\pi n = \omega_i t + \frac{1}{2}\alpha t^2$

or $\quad 2\pi \times 10 = \frac{1}{2}\alpha(3)^2 \qquad$ or $\quad \alpha = \frac{40\pi}{9}$.

Let it make N rotations in the first 6 seconds.

$$2\pi N = \frac{1}{2}\alpha(6)^2 \qquad \text{or} \quad N = \frac{1}{2\pi} \cdot \frac{36}{2} \cdot \frac{40\pi}{9} = 40.$$

$\therefore \quad$ the required number of rotations = 40 – 10 = 30.

33. Let Ω = angular velocity of the fan when it is switched off.

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$\therefore \quad (\Omega/2)^2 = \Omega^2 + 2\alpha(2\pi \times 36)$

and $\; 0 = (\Omega/2)^2 + 2\alpha(2\pi N)$.

Solve for N.

34. $\alpha = \omega \frac{d\omega}{d\theta} = -c\omega$, where c is a constant.

or $\quad d\omega = -c\,d\theta$

or $\quad \omega = \omega_i - c\theta$

For the first part,

$$\omega_i/2 = \omega_i - c\theta \qquad \text{or} \quad c(2\pi n) = \omega_i/2.$$

For the second part,

$$0 = \omega_i/2 - c(2\pi n') \qquad \text{or} \quad n' = n.$$

35. $\alpha = \omega \cdot \frac{d\omega}{d\theta} = -c\theta \qquad$ or $\quad \int_{\omega_i}^{\omega_f} \omega\, d\omega = -\int_0^{\theta} c\theta\, d\theta$

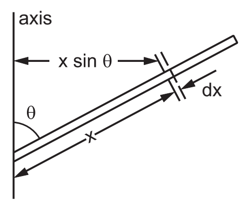
$$\frac{1}{2}\omega_i^2 - \frac{1}{2}\omega_f^2 = \frac{1}{2}c\theta^2$$

or $\frac{1}{2} I\omega_i^2 - \frac{1}{2} I\omega_f^2 = \Delta E = \frac{1}{2} cI\theta^2 \propto \theta^2.$

36. In rotational motion, power delivered, $P = \tau\omega$.

[Compare this with linear motion, where P = force × velocity.]

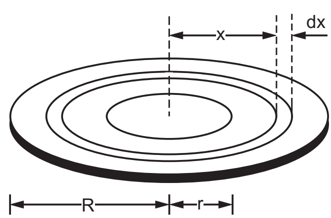
38. Mass of the element $= \left(\frac{m}{l}\right) dx$.



Moment of inertia of the element about the axis $= \left(\frac{m}{l} dx\right) (x\sin\theta)^2$.

$$I = \frac{m}{l} \sin^2\theta \cdot \int_0^l x^2 dx = \frac{ml^2}{3} \sin^2\theta.$$

39.

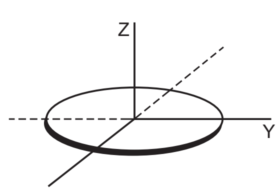


Mass of the elemental ring $= (2\pi x\, dx)\left(\frac{m}{\pi R^2 - \pi r^2}\right) = dm$.

Moment of inertia of the ring $= (dm)\, x^2$.

$$I = \frac{2m}{R^2 - r^2} \int_r^R x^3 dx = \frac{1}{2} m(R^2 + r^2).$$

40.



Moment of inertia about a diameter $= I_D = I_X = I_Y$.

$$I_Z = \frac{1}{2}mr^2 = I_X + I_Y = 2I_D$$

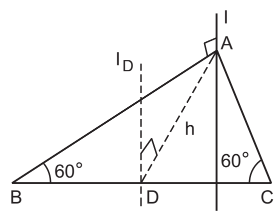
or $\quad I_D = \frac{1}{4}mr^2 = mk^2$

or $\quad k = \frac{r}{2} = \frac{4\text{ cm}}{2} = 2\text{ cm}.$

41. $I = mk^2 = mr^2 = I_{CM} + mh^2 = \frac{2}{5}mr^2 + mh^2$

or $\quad h^2 = \frac{3}{5}r^2.$

42.

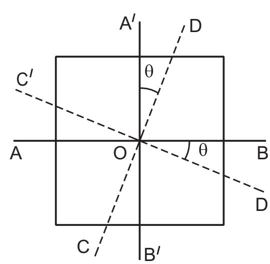


$$I = I_{AB} + I_{AC} + I_{BC} = \frac{ml^2}{3} + \frac{ml^2}{3} + (I_D + mh^2)$$

$$= \frac{2}{3}ml^2 + \frac{ml^2}{12} + m(l\sin 60)^2 = ml^2\left[\frac{2}{3} + \frac{1}{12} + \frac{3}{4}\right]$$

$$= \frac{3}{2}ml^2 = (3m)\,k^2 \qquad \text{or} \quad k = \frac{l}{\sqrt{2}}\cdot$$

43.



$I_{AB} = I_{A'B'}$.

$I_{AB} + I_{A'B'} = I_O =$ moment of inertia about O, perpendicular to plane of the plate.

$I_{CD} = I_{C'D'}$ $\quad I_{CD} + I_{C'D'} = I_O$

$\therefore \quad I_{AB} = I_{CD}$.

This is independent of θ.

45. Let L = angular momentum $\quad \omega$ = angular velocity

I = moment of inertia.

Here, L is conserved. $\quad \therefore \quad L = I\omega$ = constant.

Kinetic energy $= E = \frac{1}{2} I\omega^2 = \frac{1}{2} \cdot I \left(\frac{L}{I}\right)^2 = \frac{L^2}{2I}$.

If I decreases, E increases. Hence, energy must be expended.

47. TIP In sliding without friction and in rolling without slipping, no work is done against friction.

Hence, kinetic energy = loss in gravitational potential energy. In rolling with slipping, some energy is lost in doing work against friction, and kinetic energy is less than the loss in potential energy.

48. Angular momentum with respect to the origin is equal to the product of the linear momentum and the perpendicular distance of the line of motion from the origin.

49. $L = mvr$ = constant.

$$T = \frac{mv^2}{r} = \frac{m}{r} \cdot \frac{L}{m^2 r^2} = \frac{L}{m} r^{-3}.$$

51. $T = 2\pi/\omega = 2 \quad$ or $\quad \omega = \pi$.

For a particle undergoing SHM, starting from the origin,

$x = a \sin \omega t$.

For $x = a/2$,

$a/2 = a \sin \omega t$

or $\quad \omega t = \pi/6$

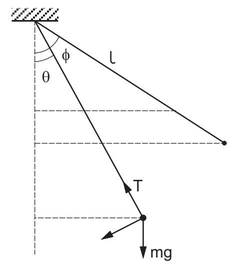
or $\quad t = \frac{\pi}{6\omega} = \frac{\pi}{6\pi} = \frac{1}{6}$ s.

53. $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t) = 2(1 + \cos t)\sin(1000t)$

$= 2\sin(1000t) + 2\sin(1000t)\cos t$

$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$.

54. $mg(l\cos\theta - l\cos\phi) = \frac{1}{2}mv^2$



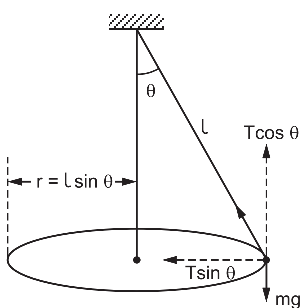
or $\frac{mv^2}{l} = 2mg(\cos\theta - \cos\phi)$.

Also, $T - mg\cos\theta = \frac{mv^2}{l} = 2mg(\cos\theta - \cos\phi)$

or $T = mg(3\cos\theta - 2\cos\phi)$.

55. As no forces act on the system, ω remains constant. Hence, $T = ml\omega^2$ remains constant. However, T is the sum of electrostatic force of attraction on the bob and the force exerted by the string due to elastic strain in it. As the positive charge on the bob increases due to photoemission, the electrostatic force on it increases, and the elastic strain on the string must decrease.

56.



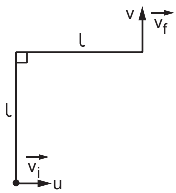
$T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$ or $T = m\omega^2 l$.

$T\cos\theta = mg$ or $m\omega^2 l\cos\theta = mg$ or $\omega^2 = \frac{g}{l\cos\theta}$.

$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$.

$T = m\left[\frac{4\pi^2}{t^2}\right]l$.

57.



$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl$$

or $$v^2 = u^2 - 2gl.$$

$$\vec{v}_i = u\hat{i}$$

$$\vec{v}_f = \hat{j}\sqrt{u^2 - 2gl}$$

Change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \hat{j}\sqrt{u^2 - 2gl} - u\hat{i}.$

Magnitude of $\Delta\vec{v} = \sqrt{(u^2 - 2gl) + u^2} = \sqrt{2(u^2 - gl)}.$

58. $\triangle$ABP is equilateral. Let T_1 and T_2 be the tensions in PA and PB respectively.

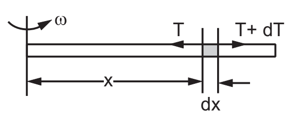
$T_1\cos 60° = T_2\cos 60° + mg$ or $T_1 - T_2 = 2mg.$

$T_1\cos 30° + T_2\cos 30° = m\omega^2 l\cos 30°$ or $T_1 + T_2 = m\omega^2 l.$

$$\therefore \quad T_2 = \frac{1}{2}m[\omega^2 l - 2g].$$

For $T_2 \geq 0$, $\omega \geq \sqrt{2g/l}$.

59.



The mass of the element $= dm = (m/l)\,dx.$

The force on the element towards the axis $= T - (T + dT) = -dT.$

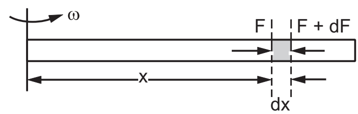
$$\therefore \quad -dT = (dm)\omega^2 x = \left(\frac{m}{l}\,dx\right)\omega^2 x.$$

$$T = -\frac{m\omega^2}{l}\cdot\frac{x^2}{2} + \text{constant}$$

For $x = l$, $T = 0$. $\therefore$ the constant $= \frac{1}{2}\cdot\frac{m\omega^2 l^2}{l}.$

$$\therefore \quad T = \frac{1}{2}\cdot\frac{m\omega^2}{l}(l^2 - x^2).$$

60.



The mass of the element = $dM = (M/L)\, dx$.

The force on the element towards the axis = $(F + dF) - F = dF$.

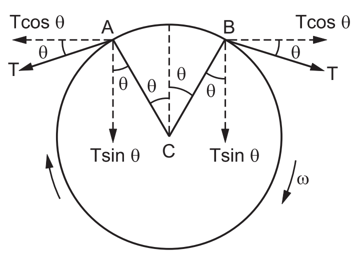
$$\therefore \quad dF = (dM)\,\omega^2 x = \left(\frac{M}{L}\,dx\right)\omega^2 x.$$

$$F = \frac{1}{2}\cdot\frac{M\omega^2}{L}x^2 + \text{constant}.$$

For $x = 0$, $F = 0$. $\quad\therefore$ the constant = 0.

For $x = L$, $F = ½\, M\omega^2 L$.

61.



Let T = tension in the ring. For the section AB, the net force is $2T\sin\theta$. If θ is small, $2T\sin\theta \simeq 2T\theta$, and the net force is towards C. Mass of section AB = $(2r\,\theta)\,m$. For circular motion,

$$2T\theta = (2r\,\theta m)\,r\omega^2 \qquad \text{or} \qquad T = mr^2\omega^2.$$

62. At C, $\frac{1}{2}mv^2 = mgr \qquad$ or $\qquad \frac{mv^2}{r} = 2mg$.

If N = the normal reaction of the track,

$$F - N = mv^2/r = 2mg \quad \text{or} \quad F = N + 2mg.$$

As $N \geq 0$, $F \geq 2mg$.

Taking the speeds at B and D to be the same as at C,

at B, $F - N = 2mg$ or $N = F - 2mg$ (Centre of curvature is O.)

at D, $N - F = 2mg$ or $N = F + 2mg$ (Centre of curvature is O′.)

For $\angle AOM = \theta$,

$$mg(r - r\cos\theta) = \frac{1}{2}mv^2 \qquad \text{or} \qquad \frac{mv^2}{r} = 2mg(1 - \cos\theta)$$

or $\quad 2mg(1 - \cos\theta) = mg\cos\theta + F - N.$

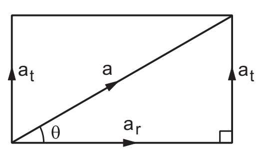
For $\quad F = N$, $\cos\theta = 2/3$.

63. TIP Due to the rotation of the earth about its axis, different points on its surface have the same angular velocity but different linear velocities, which is maximum at the equator and decreases at higher latitudes.

The earth rotating from west to east imparts an eastward velocity to the wind mass. The wind mass moving northward from the equator retains its eastward velocity, which is greater than the eastward velocity of the surface at higher latitudes. Hence, relative to the earth's surface the wind mass shifts to the east.

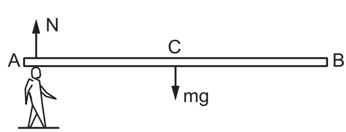
64. See the hint to Q. No. 63.

65.



The tangential acceleration a_t and the radial acceleration $a_r = v^2/r$ being mutually perpendicular, have a resultant acceleration $a = [a_t^2 + (v^2/r)^2]^{1/2}$. The only horizontal force on the body is the force of friction, $F = \mu mg = ma$, acting in the direction of a.

73.



When the beam is supported at A and B, the force exerted by each man $= mg/2$.

When the support at B is withdrawn, taking torque about A,

$$\tau = (mg)\,l/2 = I\alpha = (ml^2/3)\,\alpha$$

or $\quad \alpha = 3g/2l$.

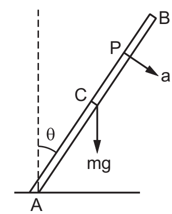
The instantaneous linear acceleration of the centre of mass is

$$a_{CM} = (\alpha)(AC) = (3g/2l)\, l/2 = 3g/4.$$

Let N = force exerted on the beam at A.

$$\therefore \quad mg - N = ma_{CM} = m(3g/4) \qquad \text{or} \quad N = \frac{1}{4}mg.$$

74. Taking torque about A, when the rod has fallen through an angle θ,



$$\tau = mg\frac{l}{2}\sin\theta = I\alpha = \left(\frac{1}{3}ml^2\right)\alpha$$

or $$\alpha = \frac{3g}{2l}\sin\theta$$

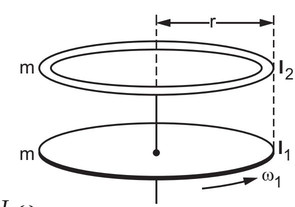
For any point P on the rod, at a distance r from A, the linear acceleration is

$$a = r\alpha = \frac{3gr}{2l}\sin\theta.$$

76. See the hint to Q. No. 45.

77. Let ω_1 = the initial angular velocity of the disc.

ω_2 = the final common angular velocity of the disc and the ring.



For the disc, $I_1 = \frac{1}{2}mr^2$.

For the ring, $I_2 = mr^2$.

By conservation of angular momentum,

$$L = I_1\omega_1 = (I_1 + I_2)\,\omega_2 \qquad \text{or} \quad \omega_2 = \frac{I_1\omega_1}{I_1 + I_2} = \omega_1/3.$$

Initial kinetic energy = $E_1 = \frac{1}{2}I_1\omega_1^2$.

Final kinetic energy = $E_2 = \frac{1}{2}(I_1 + I_2)\,\omega_2^2$.

Heat produced = loss in kinetic energy = $E_1 - E_2$.

Ratio of heat produced to initial kinetic energy = $\dfrac{E_1 - E_2}{E_1} = \dfrac{2}{3}$.

78.

The force of friction between the two surfaces in contact disappears when there is no relative (linear) motion between them. Angular momentum will not be conserved as the discs will have final angular velocities in opposite directions.

79. Let L = angular momentum.

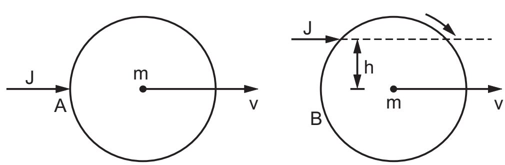
$$\tau = \frac{dL}{dt} \qquad \text{or} \quad dL = \tau\, dt.$$

For constant torque,

$$\Delta L = \tau\, \Delta t = I\, \Delta\omega \qquad \text{or} \quad I\omega \text{ if } \omega_i = 0.$$

Rotational kinetic energy $= \dfrac{1}{2} I\omega^2 = \dfrac{(\Delta L)^2}{2I}$.

80.



$J = mv$ for both. A has no angular motion. For B, angular momentum imparted by $J = L = Jh$.

82. Let P = external force F = force of friction between A and B.
a_1 = acceleration between A and B, a_2 = acceleration beyond B.

$$P - F = ma_1 \text{ and } P = ma_2. \qquad \therefore \quad a_2 > a_1.$$

Let α = angular acceleration between A and B. For one rotation,

$$\theta = 2\pi = \frac{1}{2}\alpha T^2$$

or $T = (4\pi/\alpha)^{1/2}$ = time of travel from A to B.

Angular velocity at B $= \omega_B = \alpha T$.

For one rotation to the right of B,

$$\theta = 2\pi = \omega_B t \qquad \text{or} \quad t = \frac{2\pi}{\alpha T} = \frac{\frac{1}{2}T^2}{T} = \frac{T}{2}.$$

83. TIP In rolling with slipping, the force of friction produces a torque which gives an angular acceleration to the body. Hence, part of the work done against friction is converted to rotational kinetic energy, which adds to the total kinetic energy. Only the remaining part of the work done against friction is converted to heat.

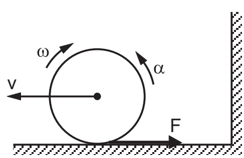
84. In rolling without slipping, no work is done against friction. Hence, loss in gravitational potential energy of a body is equal to its total kinetic energy, i.e., linear plus rotational kinetic energies. Also, $v = \omega r$.

$$\text{Total kinetic energy} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}[mv^2 + (mk^2)(v^2/r^2)]$$

$$= \frac{1}{2}mv^2(1 + k^2/r^2), \text{ where } k = \text{radius of gyration.}$$

As all the bodies have the same final total kinetic energy, their final velocities will depend only on the ratio k/r. Bodies with smaller values of k/r will have greater v and hence reach the bottom earlier.

85.

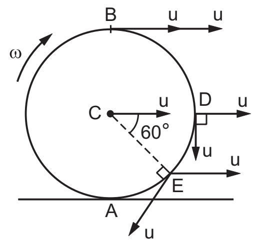


After impact, the force of friction will act in a direction opposite to that of the motion. The body will have retained its initial angular motion (clockwise). The force of friction will cause linear retardation, reducing v. It will also cause an anticlockwise angular acceleration, which will reduce ω to zero and then introduce anticlockwise ω till rolling without slipping begins. F will then disappear.

86. TIP In rolling without slipping, at constant speed, there is no force of friction between the surfaces.

Therefore, removing the pin causes no change to the system.

87. Every point on the ring has a horizontal velocity u due to its linear motion, and in addition a velocity u, tangential to the ring, due to its rotational motion. The resultant of these two is the velocity of the point with respect to the ground.

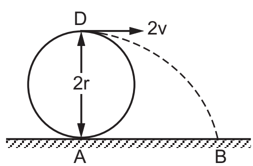


Hence,

$$v_A = 0,\ v_B = 2u,\ v_D = \sqrt{2}u,\ v_E = u.$$

88. See the hint to Q. No. 87.

89.



At the point of leaving the wheel, the blob of mud is at a height $2r$ above the road and has a horizontal velocity $2v$ (see the hint to Q. No. 87).

Let t = time of travel from D to B. Then, $2r = \frac{1}{2}gt^2$

or $\quad t = 2\sqrt{r/g}$ and $AB = (2v)\,t$.

90. TIP For a pulley with mass, the strings on its two sides do not have the same tension when in motion.

Also, the rotating pulley has some kinetic energy derived from the loss in potential energy of the block B as it moves down.

91. See the hint to Q. No. 90.

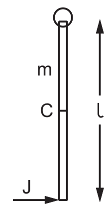
92. As all three particles are part of the system, any impact between them produces no change in the velocity of their centre of mass. Also, 'A + B' can exert a force on B only along the rod, i.e., normal to u.

95. Angular momentum = linear momentum × perpendicular distance from the point of rotation

or $L = Jl$

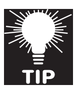
Also, $I = ml^2/3$.

$\therefore \quad \omega = L/I = 3J/ml$.



$$\text{Kinetic energy} = \frac{L^2}{2I} = \frac{J^2l^2}{2(ml^2/3)} = \frac{3J^2}{2m}$$

$$v_c = \omega \cdot \frac{l}{2} = \frac{3J}{2m}.$$

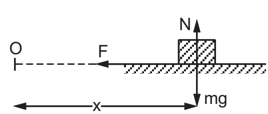
96. 

When a constant force is superimposed on a system undergoing SHM, along the line of SHM, the time period does not change. The mean position changes, as this is the position where the net force on the particle is zero.

97. The time period will change only when the additional electrostatic force has a component along the direction of the displacement, which is always perpendicular to the string.

98. The only horizontal force acting on the coin is the force of friction F. Therefore, its horizontal acceleration is always in the direction of F, and its magnitude is F/m. The magnitude and the direction of F can thus be obtained from the magnitude and the direction of the acceleration.

99. Let O be the mean position and x be the displacement from the mean position.



Acceleration $= a = \omega^2 x$.

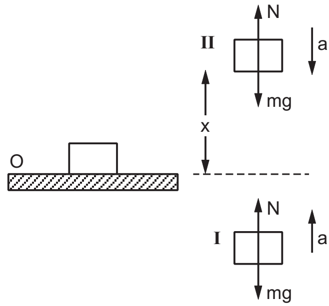
If F = the force of friction, $F = ma = m\omega^2 x$.

The coin will slip when $F = F_{\text{lim}} = \mu N = \mu mg$.

Let amplitude $= x_{\max} = A$.

$\therefore \quad \mu mg = m\omega^2 A \qquad$ or $\quad A = \dfrac{\mu g}{\omega^2}$.

100.



Let O be the mean position and a be the acceleration at a displacement x from O.

At position I, $N - mg = ma$. $\therefore \quad N \neq 0$

At position II, $mg - N = ma$.

For $N = 0$ (loss of contact), $g = a = \omega^2 x$.

Loss of contact will occur for amplitude $x_{max} = \dfrac{g}{\omega^2}$ at the highest point of the motion.

□

3

Properties of Matter, Fluids

[In all questions on gravitation, take G = universal constant of gravitation, R = radius of the earth and g = acceleration due to gravity at the surface of the earth, unless otherwise stated.]

• *Type 1* •

Choose the correct option (a, b, c or d).

1. The rotation of the earth about its axis speeds up such that a man on the equator becomes weightless. In such a situation, what would be the duration of one day?

(a) $2\pi\sqrt{R/g}$ (b) $\frac{1}{2\pi}\sqrt{R/g}$

(c) $2\pi\sqrt{Rg}$ (d) $\frac{1}{2\pi}\sqrt{Rg}$

2. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B, from west to east. Which train will exert greater force on the tracks?

(a) A

(b) B

(c) They will exert equal force.

(d) The mass and the speed of each train must be known to reach a conclusion.

3. A small body of superdense material, whose mass is twice the mass of the earth but whose size is very small compared to the size of the earth, starts from rest at a height $H << R$ above the earth's surface, and reaches the earth's surface in time t. Then t is equal to

(a) $\sqrt{2H/g}$ (b) $\sqrt{H/g}$

(c) $\sqrt{2H/3g}$ (d) $\sqrt{4H/3g}$

4. In the previous question, if the mass of the body is half the mass of the earth, and all other data remain the same, then t is equal to

(a) $\sqrt{2H/g}$ (b) $\sqrt{H/g}$

(c) $\sqrt{2H/3g}$ (d) $\sqrt{4H/3g}$

5. The time period of a simple pendulum of infinite length is

(a) infinite (b) $2\pi\sqrt{R/g}$

(c) $2\pi\sqrt{g/R}$ (d) $\frac{1}{2\pi}\sqrt{R/g}$

6. At what height above the earth's surface does the acceleration due to gravity fall to 1% of its value at the earth's surface?

(a) $9R$ (b) $10R$

(c) $99R$ (d) $100R$

7. At what height above the earth's surface is the acceleration due to gravity 1% less than its value at the surface? [$R = 6400$ km]

(a) 16 km (b) 32 km

(c) 64 km (d) $32\sqrt{2}$ km

8. Let the acceleration due to gravity be g_1 at a height h above the earth's surface, and g_2 at a depth d below the earth's surface. If $g_1 = g_2$, $h << R$ and $d << R$ then

(a) $h = d$

(b) $h = 2d$

(c) $2h = d$

(d) it is not possible for g_1 to be equal to g_2

9. If different planets have the same density but different radii then the acceleration due to gravity (g) on the surface of the planet will depend on its radius (R) as

(a) $g \propto \frac{1}{R^2}$ (b) $g \propto \frac{1}{R}$

(c) $g \propto R$ (d) $g \propto R^2$

10. P is a point at a distance r from the centre of a spherical shell of mass M and radius a, where $r < a$. The gravitational potential at P is

(a) $-\frac{GM}{r}$ (b) $-\frac{GM}{a}$

(c) $-GM\frac{r}{a^2}$ (d) $-GM\left(\frac{a-r}{a^2}\right)$

11. A particle of mass m is placed inside a spherical shell, away from its centre. The mass of the shell is M.

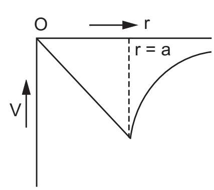
(a) The particle will move towards the centre.

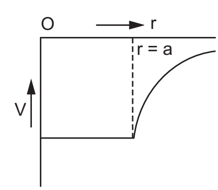
(b) The particle will move away from the centre, towards the nearest wall.

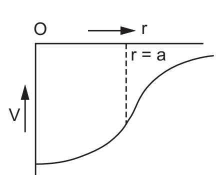
(c) The particle will move towards the centre if $m < M$, and away from the centre if $m > M$.

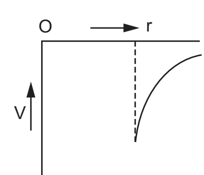
(d) The particle will remain stationary.

12. P is a point at a distance r from the centre of a solid sphere of radius a. The gravitational potential at P is V. If V is plotted as a function of r, which is the correct curve?

(a)


(b)


(c)


(d)


13. A point P lies on the axis of a ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be

(a) $\sqrt{\frac{2GM}{a}}$ (b) $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$

(c) $\sqrt{\frac{2GM}{a}(\sqrt{2}-1)}$ (d) zero

14. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $\frac{v_e}{\sqrt{2}}$

(c) $\frac{v_e}{2}$ (d) zero

15. In the previous question, if R is the radius of the planet and g is the acceleration due to gravity at its surface then the body will reach the centre of the planet in time

(a) $2\pi\sqrt{R/g}$ (b) $\pi\sqrt{R/g}$

(c) $\frac{\pi}{2}\sqrt{R/g}$ (d) $\sqrt{2R/g}$

16. If a small part separates from an orbiting satellite, the part will

(a) fall to the earth directly

(b) move in a spiral and reach the earth after a few rotations

(c) continue to move in the same orbit as the satellite

(d) move farther away from the earth gradually

17. A satellite going round the earth in a circular orbit loses some energy due to a collision. Its speed is v and distance from the earth is d.

(a) d will increase, v will increase.

(b) d will increase, v will decrease.

(c) d will decrease, v will decrease.

(d) d will decrease, v will increase.

18. The distances of two satellites from the surface of the earth are R and $7R$. Their time periods of rotation are in the ratio

(a) 1 : 7 (b) 1 : 8

(c) 1 : 49 (d) $1 : 7^{3/2}$

19. Inside a satellite orbiting very close to the earth's surface, water does not fall out of a glass when it is inverted. Which of the following is the best explanation for this?

(a) The earth does not exert any force on the water.

(b) The earth's force of attraction on the water is exactly balanced by the force created by the satellite's motion.

(c) The water and the glass have the same acceleration, equal to g, towards the centre of the earth, and hence there is no relative motion between them.

(d) The gravitational attraction between the glass and the water balances the earth's attraction on the water.

20. If a metal wire is stretched a little beyond its elastic limit (or yield point), and released, it will

(a) lose its elastic property completely

(b) not contract

(c) contract, but its final length will be greater than its initial length

(d) contract only up to its length at the elastic limit

21. A metal wire of length L, area of cross-section A and Young's modulus Y behaves as a spring of spring constant k.

(a) $k = YA/L$ (b) $k = 2YA/L$

(c) $k = YA/2L$ (d) $k = YL/A$

22. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y

respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(a) $2\pi\sqrt{m/k}$ (b) $2\pi\sqrt{m(YA+kL)/YAk}$

(c) $2\pi\sqrt{mYA/kL}$ (d) $2\pi\sqrt{mL/YA}$

23. A uniform rod of mass m, length L, area of cross-section A and Young's modulus Y hangs from the ceiling. Its elongation under its own weight will be

(a) zero (b) $\frac{mgL}{2AY}$

(c) $\frac{mgL}{AY}$ (d) $\frac{2mgL}{AY}$

24. A liquid drop at temperature t, isolated from its surroundings, breaks into a number of droplets. The temperature of the droplets will be

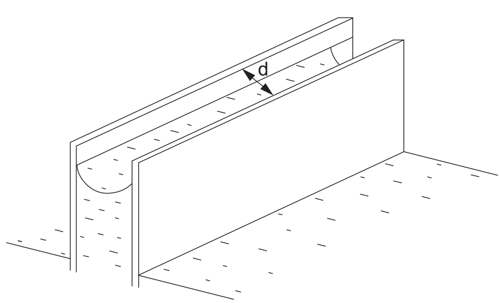
(a) equal to t

(b) greater than t

(c) less than t

(d) either (a), (b) or (c) depending on the surface tension of the liquid

25.



Two very wide parallel glass plates are held vertically at a small separation d, and dipped in water. Some water climbs up in the gap between the plates. Let S be the surface tension of water, p_0 = atmospheric pressure, p = pressure of water just below the water surface in the region between the plates.

(a) $p = p_0 - \dfrac{2S}{d}$

(b) $p = p_0 + \dfrac{2S}{d}$

(c) $p = p_0 - \dfrac{4S}{d}$

(d) $p = p_0 + \dfrac{4S}{d}$

26. Two soap bubbles with radii r_1 and r_2 $(r_1 > r_2)$ come in contact. Their common surface has a radius of curvature r.

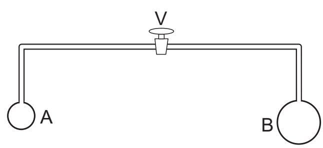
(a) $r = \dfrac{r_1 + r_2}{2}$

(b) $r = \dfrac{r_1 r_2}{r_1 - r_2}$

(c) $r = \dfrac{r_1 r_2}{r_1 + r_2}$

(d) $r = \sqrt{r_1 r_2}$

27.



The valve V in the bent tube is initially kept closed. Two soap bubbles A (smaller) and B (larger) are formed at the two open ends of the tube. V is now opened, and air can flow freely between the bubbles.

(a) There will be no change in the sizes of the bubbles.

(b) The bubbles will become of equal size.

(c) A will become smaller and B will become larger.

(d) The sizes of the two bubbles will become interchanged.

28. A liquid of density ρ and coefficient of viscosity η, flows with velocity v through a tube of diameter D. A quantity $R = \dfrac{\rho v D}{\eta}$ determines whether the flow will be streamlined or turbulent. R has the dimension of

(a) velocity

(b) acceleration

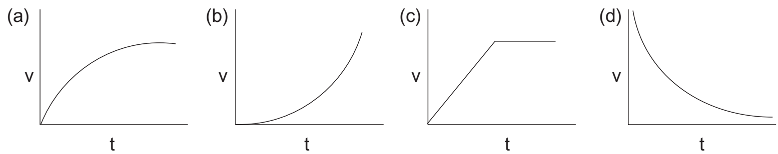
(c) force

(d) none of these

29. When cooking oil is heated in a frying pan, the oil moves around in the pan more easily when it is hot. The main reason for this is that with rise in temperature, there is a decrease in

(a) surface tension
(b) viscosity
(c) angle of contact
(d) density

30. A piece of cork starts from rest at the bottom of a lake and floats up. Its velocity v is plotted against time t. Which of the following best represents the resulting curve?



31. If F represents force, A represents area and t represents time, then which of the following quantities has the same dimensions as the coefficient of viscosity η?

(a) $\frac{FA}{t}$
(b) $\frac{Ft}{A}$
(c) FtA
(d) $\frac{F}{At}$

32. A large drop of oil, whose density is less than that of water, floats up through a column of water. Assume that the oil and the water do not mix. The coefficient of viscosity of the oil is η_0 and that of water is η_w. The velocity of the drop will depend on

(a) both η_0 and η_w
(b) η_0 only
(c) η_w only
(d) neither η_0 nor η_w

33. Which of the following processes will be least affected by the viscosity of water?

(a) Water flowing through a pipe
(b) Air bubble rising up through water
(c) A wide, shallow sheet of water flowing on a flat surface
(d) Water flowing out through a hole in the side of a tank

34. Raindrops of different radii are falling through air. The terminal velocity of a drop of radius r will be proportional to

(a) r^3 (b) r^2

(c) r (d) $\frac{1}{r}$

35. A raindrop reaching the ground with terminal velocity has momentum p. Another drop of twice the radius, also reaching the ground with terminal velocity, will have momentum

(a) $4p$ (b) $8p$

(c) $16p$ (d) $32p$

36. Which of the following phenomena does not involve the viscosity of air at all?

(a) A meteorite burns up on entering the earth's atmosphere.

(b) Raindrops falling from a great height reach the ground with a relatively small velocity.

(c) A ball spinning through air can move sideways.

(d) In air flowing through a tube of variable cross section, the pressure becomes different at different points.

37. A uniform rod of density ρ is placed in a wide tank containing a liquid of density ρ_0 ($\rho_0 > \rho$). The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position the rod makes an angle θ with the horizontal.

(a) $\sin\theta = \frac{1}{2}\sqrt{\rho_0/\rho}$ (b) $\sin\theta = \frac{1}{2}\cdot\frac{\rho_0}{\rho}$

(c) $\sin\theta = \sqrt{\rho/\rho_0}$ (d) $\sin\theta = \rho_0/\rho$

38. A beaker containing a liquid of density ρ moves up with an acceleration a. The pressure due to the liquid at a depth h below the free surface of the liquid is

(a) $h\rho g$ (b) $h\rho(g+a)$

(c) $h\rho(g-a)$ (d) $2h\rho g\left(\frac{g-a}{g+a}\right)$

39. A piece of wood floats in water kept in a beaker. If the beaker moves with a vertical acceleration a, the wood will

(a) sink deeper in the liquid if a is upward

(b) sink deeper in the liquid if a is downward, with $a < g$

(c) come out more from the liquid if a is downward, with $a < g$

(d) remain in the same position relative to the water

40. The weight of a balloon is W_1 when empty and W_2 when filled with air. Both are weighed in air by the same sensitive spring balance and under identical conditions.

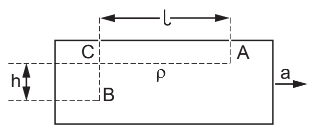
(a) $W_1 = W_2$, as the weight of air in the balloon is offset by the force of buoyancy on it.

(b) $W_2 < W_1$ due to the force of buoyancy acting on the filled balloon.

(c) $W_2 > W_1$, as the air inside is at a greater pressure and hence has greater density than the air outside.

(d) $W_2 = W_1$ + weight of the air inside it.

41.



A sealed tank containing a liquid of density ρ moves with a horizontal acceleration a, as shown in the figure. The difference in pressure between the points A and B is

(a) $h\rho g$ (b) $l\rho a$

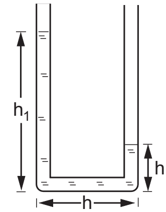
(c) $h\rho g - l\rho a$ (d) $h\rho g + l\rho a$

42. A U-tube containing a liquid moves with a horizontal acceleration a along a direction joining the two vertical limbs. The separation between these limbs is d. The difference in their liquid levels is

(a) ad/g (b) $2da/g$

(c) $da/2g$ (d) $d\tan(a/g)$

43. The U-tube shown has a uniform cross-section. A liquid is filled in the two arms up to heights h_1 and h_2, and then the liquid is allowed to move. Neglect viscosity and surface tension. When the levels equalize in the two arms, the liquid will



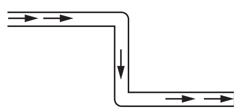
(a) be at rest

(b) be moving with an acceleration of $g\left(\dfrac{h_1 - h_2}{h_1 + h_2 + h}\right)$

(c) be moving with a velocity of $(h_1 - h_2)\sqrt{\dfrac{g}{2(h_1 + h_2 + h)}}$

(d) exert a net force to the right on the tube

44.



The tube shown is of uniform cross-section. Liquid flows through it at a constant speed in the direction shown by the arrows. The liquid exerts on the tube

(a) a net force to the right (b) a net force to the left

(c) a clockwise torque (d) an anticlockwise torque

45. Bernoulli's principle (or equation) is a consequence of

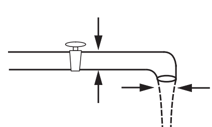
(a) conservation of energy only

(b) conservation of momentum only

(c) conservation of angular momentum only

(d) more than one of the above

46.



Water coming out of the mouth of a tap and falling vertically in streamline flow forms a tapering column, i.e., the area of cross-section of the liquid column decreases as it moves down. Which of the following is the most accurate explanation for this?

(a) As the water moves down, its speed increases and hence its pressure decreases. It is then compressed by the atmosphere.

(b) Falling water tries to reach a terminal velocity and hence reduces the area of cross-section to balance upward and downward forces.

(c) The mass of water flowing past any cross-section must remain constant. Also, water is almost incompressible. Hence, the rate of volume flow must remain constant. As this is equal to velocity × area, the area decreases as velocity increases.

(d) The surface tension causes the exposed surface area of the liquid to decrease continuously.

47. In the previous question, at the mouth of the tap the area of cross-section is 2.5 cm^2 and the speed of water is 3 m/s. The area of cross-section of the water column 80 cm below the tap is

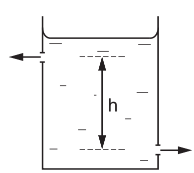
(a) 2 cm^2 (b) 1.5 cm^2 (c) 1 cm^2 (d) 0.5 cm^2

48. A cylindrical drum, open at the top, contains 30 litres of water. It drains out through a small opening at the bottom. 10 litres of water comes out in time t_1, the next 10 litres in a further time t_2 and the last 10 litres in a further time t_3. Then,

(a) $t_1 = t_2 = t_3$ (b) $t_1 > t_2 > t_3$

(c) $t_1 < t_2 < t_3$ (d) $t_2 > t_1 = t_3$

49.



There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in height between the two holes is h. As the liquid comes out of the two holes, the tank will experience a net horizontal force proportional to

(a) $\sqrt{h}$ (b) h (c) $h^{3/2}$ (d) h^2

• Type 2 •

Choose the correct options. One or more options may be correct.

50. An object is weighed at the North Pole by a beam balance and a spring balance, giving readings of W_B and W_S respectively. It is again weighed in the same manner at the equator, giving readings of W_B' and W_S' respectively. Assume that the acceleration due to gravity is the same everywhere and that the balances are quite sensitive.

(a) $W_B = W_S$ (b) $W_B' = W_S'$
(c) $W_B = W_B'$ (d) $W_S' < W_S$

51. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed by a spring balance gives the same reading at the equator as at a height h above the poles ($h << R$). The value of h is

(a) $\frac{\omega^2 R^2}{g}$ (b) $\frac{\omega^2 R^2}{2g}$
(c) $\frac{2\omega^2 R^2}{g}$ (d) $\frac{\sqrt{Rg}}{\omega}$

52. Use the assumptions of the previous question. An object weighed by a spring balance at the equator gives the same reading as a reading taken at a depth d below the earth's surface at a pole ($d << R$). The value of d is

(a) $\frac{\omega^2 R^2}{g}$ (b) $\frac{\omega^2 R^2}{2g}$
(c) $\frac{2\omega^2 R^2}{g}$ (d) $\frac{\sqrt{Rg}}{\omega}$

53. A double star is a system of two stars rotating about their centre of mass only under their mutual gravitational attraction. Let the stars have masses m and $2m$ and let their separation be l. Their time period of rotation about their centre of mass will be proportional to

(a) $l^{3/2}$ (b) l (c) $m^{1/2}$ (d) $m^{-1/2}$

54. Three point masses, m each, are at the corners of an equilateral triangle of side a. Their separations do not change when the system rotates about the centre of the triangle. For this, the time period of rotation must be proportional to

(a) $a^{3/2}$ (b) a

(c) m (d) $m^{-1/2}$

55. For a planet moving around the sun in an elliptical orbit, which of the following quantities remain constant?

(a) The total energy of the 'sun plus planet' system

(b) The angular momentum of the planet about the sun

(c) The force of attraction between the two

(d) The linear momentum of the planet

56. The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be

(a) v_e (b) $1.5v_e$

(c) $\sqrt{1.5}\,v_e$ (d) $2v_e$

57. The escape velocity for a planet is v_e. A particle is projected from its surface with a speed v. For this particle to move as a satellite around the planet,

(a) $\frac{v_e}{2} < v < v_e$ (b) $\frac{v_e}{\sqrt{2}} < v < v_e$

(c) $v_e < v < \sqrt{2}v_e$ (d) $\frac{v_e}{\sqrt{2}} < v < \frac{v_e}{2}$

58. If a satellite orbits as close to the earth's surface as possible,

(a) its speed is maximum

(b) time period of its rotation is minimum

(c) the total energy of the 'earth plus satellite' system is minimum

(d) the total energy of the 'earth plus satellite' system is maximum

59. For a satellite to orbit around the earth, which of the following must be true?

(a) It must be above the equator at some time.

(b) It cannot pass over the poles at any time.

(c) Its height above the surface cannot exceed 36,000 km.

(d) Its period of rotation must be $> 2\pi\sqrt{R/g}$.

60. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time

(a) 1.5 hours

(b) 1.6 hours if it is rotating from west to east

(c) 24/17 hours if it is rotating from west to east

(d) 24/17 hours if it is rotating from east to west

61. For a satellite to be geostationary, which of the following are essential conditions?

(a) It must always be stationed above the equator.

(b) It must rotate from west to east.

(c) It must be about 36,000 km above the earth.

(d) Its orbit must be circular, and not elliptical.

62. Two small satellites move in circular orbits around the earth, at distances r and $r + \Delta r$ from the centre of the earth. Their time periods of rotation are T and $T + \Delta T$. ($\Delta r << r$, $\Delta T << T$)

(a) $\Delta T = \frac{3}{2} T \frac{\Delta r}{r}$ (b) $\Delta T = -\frac{3}{2} T \frac{\Delta r}{r}$

(c) $\Delta T = \frac{2}{3} T \frac{\Delta r}{r}$ (d) $\Delta T = T \frac{\Delta r}{r}$

63. Let S be an imaginary closed surface enclosing mass m. Let $\vec{dS}$ be an element of area on S, the direction of $\vec{dS}$ being outward from S. Let $\vec{E}$ be the gravitational intensity at $\vec{dS}$. We define $\phi = \oint_S \vec{E} \cdot \vec{dS}$, the integration being carried out over the entire surface S.

(a) $\phi = -Gm$

(b) $\phi = -4\pi Gm$

(c) $\phi = -\dfrac{Gm}{4\pi}$

(d) No relation of the type (a), (b) or (c) can exist.

64. A small mass m is moved slowly from the surface of the earth to a height h above the surface. The work done (by an external agent) in doing this is

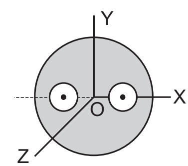
(a) mgh, for all values of h

(b) mgh, for $h << R$

(c) $\frac{1}{2} mgR$, for $h = R$

(d) $-\frac{1}{2} mgR$, for $h = R$

65. A solid sphere of uniform density and radius 4 units is located with its centre at the origin of coordinates, O. Two spheres of equal radii of 1 unit, with their centres at A(–2, 0, 0) and B(2, 0, 0) respectively, are taken out of the solid sphere, leaving behind spherical cavities as shown in the figure.



(a) The gravitational force due to this object at the origin is zero.

(b) The gravitational force at the point B(2, 0, 0) is zero.

(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.

(d) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

66. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $F_1/F_2 = r_1/r_2$, if $r_1 < R$ and $r_2 < R$

(b) $F_1/F_2 = r_2^2/r_1^2$, if $r_1 > R$ and $r_2 > R$

(c) $F_1/F_2 = r_1/r_2$, if $r_1 > R$ and $r_2 > R$

(d) $F_1/F_2 = r_1^2/r_2^2$, if $r_1 < R$ and $r_2 < R$

67. An elastic metal rod will change its length when it

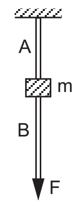
(a) falls vertically under its weight

(b) is pulled along its length by a force acting at one end

(c) rotates about an axis at one end

(d) slides on a rough surface

68. The wires A and B shown in the figure are made of the same material, and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is $mg/3$, one of the wires breaks.



(a) A will break before B if $r_A = r_B$.

(b) A will break before B if $r_A < 2r_B$.

(c) Either A or B may break if $r_A = 2r_B$.

(d) The lengths of A and B must be known to predict which wire will break.

69. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is l.

(a) Loss in gravitational potential energy of M is Mgl.

(b) The elastic potential energy stored in the wire is Mgl.

(c) The elastic potential energy stored in the wire is ½ Mgl.

(d) Heat produced is ½ Mgl.

70. A metal wire of length L, area of cross-section A and Young's modulus Y is stretched by a variable force F such that F is always slightly greater than the elastic forces of resistance in the wire. When the elongation of the wire is l,

(a) the work done by F is $\frac{YAl^2}{2L}$

(b) the work done by F is $\frac{YAl^2}{L}$

(c) the elastic potential energy stored in the wire is $\frac{YAl^2}{2L}$

(d) no heat is produced during the elongation

71. n drops of a liquid, each with surface energy E, join to form a single drop.

(a) Some energy will be released in the process.

(b) Some energy will be absorbed in the process.

(c) The energy released or absorbed will be $E(n - n^{2/3})$.

(d) The energy released or absorbed will be $nE\,[2^{2/3} - 1]$.

72. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble

(a) is greater than the pressure outside it

(b) is less than the pressure outside it

(c) increases as the bubble moves up

(d) decreases as the bubble moves up

73. When a capillary tube is dipped in a liquid, the liquid rises to a height h in the tube. The free liquid surface inside the tube is hemispherical in shape. The tube is now pushed down so that the height of the tube outside the liquid is less than h.

(a) The liquid will come out of the tube like in a small fountain.

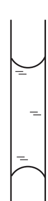
(b) The liquid will ooze out of the tube slowly.

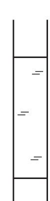
(c) The liquid will fill the tube but not come out of its upper end.

(d) The free liquid surface inside the tube will not be hemispherical.

74. A vertical glass capillary tube, open at both ends, contains some water. Which of the following shapes may be taken by the water in the tube?

(a) 

(b) 

(c) 

(d) 

75. A spring balance reads W_1 when a ball is suspended from it. A weighing machine reads W_2 when a tank of liquid is kept on it. When the ball is immersed in the liquid, the spring balance reads W_3 and the weighing machine reads W_4.

(a) $W_1 > W_3$ (b) $W_1 < W_3$

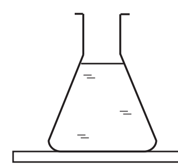
(c) $W_2 < W_4$ (d) $W_2 > W_4$

76. In the previous question,

(a) $W_1 + W_2 = W_3 + W_4$ (b) $W_1 + W_3 = W_2 + W_4$

(c) $W_1 + W_4 = W_2 + W_3$ (d) $W_1 + W_2 + W_3 = W_4$

77. A massless conical flask filled with a liquid is kept on a table in a vacuum. The force exerted by the liquid on the base of the flask is W_1. The force exerted by the flask on the table is W_2.



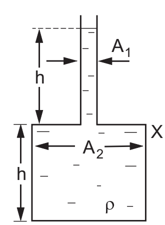
(a) $W_1 = W_2$

(b) $W_1 > W_2$

(c) $W_1 < W_2$

(d) The force exerted by the liquid on the walls of the flask is $(W_1 - W_2)$.

78. The vessel shown in the figure has two sections of areas of cross-section A_1 and A_2. A liquid of density ρ fills both the sections, up to a height h in each. Neglect atmospheric pressure.



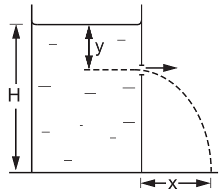
(a) The pressure at the base of the vessel is $2h\rho g$.

(b) The force exerted by the liquid on the base of the vessel is $2h\rho g A_2$.

(c) The weight of the liquid is $< 2h\rho g A_2$.

(d) The walls of the vessel at the level X exert a downward force $h\rho g(A_2 - A_1)$ on the liquid.

79.



A tank, which is open at the top, contains a liquid up to a height H. A small hole is made in the side of the tank at a distance y below the liquid surface. The liquid emerging from the hole lands at a distance x from the tank.

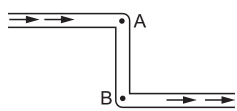
(a) If y is increased from zero to H, x will first increase and then decrease.

(b) x is maximum for $y = H/2$.

(c) The maximum value of x is H.

(d) The maximum value of x will depend on the density of the liquid.

80.

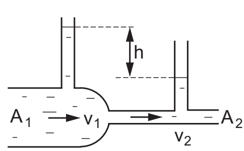


In the figure, an ideal liquid flows through the tube, which is of uniform cross-section. The liquid has velocities v_A and v_B, and pressures p_A and p_B at points A and B respectively.

(a) $v_A = v_B$ (b) $v_B > v_A$

(c) $p_A = p_B$ (d) $p_B > p_A$

81.



A liquid flows through a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross-section A_1 and A_2, are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is h.

(a) The volume of the liquid flowing through the tube in unit time is A_1v_1.

(b) $v_2 - v_1 = \sqrt{2gh}$

(c) $v_2^2 - v_1^2 = 2gh$

(d) The energy per unit mass of the liquid is the same in both sections of the tube.

82. A liquid of density ρ comes out with a velocity v from a horizontal tube of area of cross-section A. The reaction force exerted by the liquid on the tube is F.

(a) $F \propto v$ (b) $F \propto v^2$

(c) $F \propto A$ (d) $F \propto \rho$

83. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium, it undergoes oscillation with a time period T.

(a) $T \propto \sqrt{m}$ (b) $T \propto \sqrt{\rho}$

(c) $T \propto \frac{1}{\sqrt{A}}$ (d) $T \propto \frac{1}{\sqrt{\rho}}$

84. A vertical U-tube contains a liquid. The total length of the liquid column inside the tube is l. When the liquid is in equilibrium, the liquid surface in one of the arms of the U-tube is pushed down slightly and released. The entire liquid column will undergo a periodic motion.

(a) The motion is not simple harmonic motion.

(b) The motion is simple harmonic motion.

(c) If it undergoes simple harmonic motion, the time period will be $2\pi\sqrt{l/g}$.

(d) If it undergoes simple harmonic motion, the time period will be $2\pi\sqrt{l/2g}$.

Answers

1. a	**2.** a	**3.** c	**4.** d	**5.** b
6. a	**7.** b	**8.** c	**9.** c	**10.** b
11. d	**12.** c	**13.** b	**14.** b	**15.** c
16. c	**17.** d	**18.** b	**19.** c	**20.** c
21. a	**22.** b	**23.** b	**24.** c	**25.** a
26. b	**27.** c	**28.** d	**29.** b	**30.** a
31. b	**32.** c	**33.** d	**34.** b	**35.** d
36. d	**37.** a	**38.** b	**39.** d	**40.** c
41. d	**42.** a	**43.** c	**44.** c	**45.** a
46. c	**47.** b	**48.** c	**49.** b	**50.** a, c, d
51. b	**52.** a	**53.** a, d	**54.** a, d	**55.** a, b
56. c	**57.** b	**58.** a, b, c	**59.** a, d	**60.** b, d
61. a, b, c, d	**62.** a	**63.** b	**64.** b, c	**65.** a, c, d
66. a, b	**67.** b, c	**68.** a, b, c	**69.** a, c, d	**70.** a, c, d
71. a, c	**72.** a, d	**73.** c, d	**74.** d	**75.** a, c
76. a	**77.** b, d	**78.** a, b, c, d	**79.** a, b, c	**80.** a, d
81. a, c, d	**82.** b, c, d	**83.** a, c, d	**84.** b, d	

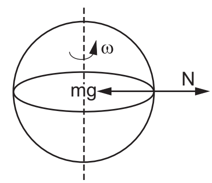
Hints and Solutions to Selected Questions

1. Let ω = angular velocity of the earth about its axis.

$$mg - N = m\omega^2 R$$

For $N = 0$, $\omega^2 = g/R$.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g}.$$



2. Let v = speed of each train relative to the earth's surface,
v_E = speed of earth's surface relative to the earth's axis,
v_A, v_E = speeds of A and B relative to the earth's axis.
Then, $v_A = v_E - v$, $v_B = v_E + v$.

$$N_A = mg - m\left(\frac{v_A^2}{R}\right),\ N_B = mg - m\left(\frac{v_B^2}{R}\right).$$

$\therefore\ N_A > N_B.$

3, 4. As the masses of the body and the earth are comparable, they will both move towards their centre of mass, which remains stationary. When they meet, the body will have moved through a distance $H/3$ (in Q. No. 3) and $2H/3$ (in Q. No. 4). In both cases, the body moves with acceleration g, as $H \ll R$, i.e., the body is close to the earth's surface.

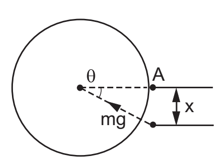
5. Force towards A $= mg\sin\theta = mg\,\frac{x}{R}\cdot$ (A = mean position)

Acceleration towards A $= g\,\frac{x}{R}$

or $\ddot{x} = -g\,\frac{x}{R}\cdot$

For $\omega^2 = g/R$, $\ddot{x} = -\omega^2 x$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g}.$$



6, 7. $g = \dfrac{g_0}{\left(1+\dfrac{h}{R}\right)^2} \cong g_0\left(1-\dfrac{2h}{R}\right)$, for $h << R$

$g = 0.01\, g_0$ for Q. No. 6 and $g = 0.99\, g_0$ for Q. No. 7.

9. $GM = R^2 g$ or $G\left(\dfrac{4}{3}\pi R^2 \rho\right) = R^2 g$ or $g = \left(\dfrac{4}{3}\pi G\rho\right)R.$

11. The gravitational intensity inside a spherical shell is zero.

12. $V = -\dfrac{GM}{2a^3}(3a^2 - r^2)$ inside and $-\dfrac{GM}{r}$ outside the sphere.

13. $V_P = -\dfrac{GM}{\sqrt{r^2+a^2}}$, $V_C = -\dfrac{GM}{a}$, $\dfrac{1}{2}mv^2 = m[V_P - V_C].$

14. $v_e = \sqrt{2Rg}$

At the surface of the planet, $V_s = -\dfrac{GM}{R}\cdot$

At the centre of the planet, $V_c = -\dfrac{3GM}{2R}\cdot$

$$\frac{1}{2}mv^2 = m(V_s - V_c]$$

or $$v^2 = 2\frac{GM}{R}\left(\frac{3}{2}-1\right) = \frac{GM}{R} = Rg = \frac{v_e^2}{2}\cdot$$

15. The time period of oscillation of a particle undergoing SHM in the tunnel, starting from rest at the surface $= 2\pi\sqrt{R/g}$. The journey from the surface to the centre is one-fourth of an oscillation.

17. $\dfrac{mv^2}{d} = G\dfrac{Mm}{d^2}$ or $GM = v^2 d$ or $v \propto \dfrac{1}{\sqrt{d}}$

Total energy $= \dfrac{1}{2}mv^2 - G\dfrac{Mm}{d} = -\dfrac{1}{2}G\dfrac{Mm}{d}\cdot$

18. $T^2 \propto r^3$, where r = distance from the centre of the earth.

Here, $r_1 = 2R$, $r_2 = 8R$.

21. Let the extension of wire be x for an external force F.

Stress $= \dfrac{F}{A}$ Strain $= \dfrac{x}{L}$ $Y = \dfrac{F/A}{x/L} = \dfrac{FL}{xA}$.

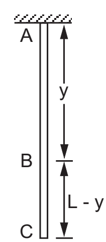
Equivalent force constant $= k = \dfrac{F}{x} = \dfrac{YA}{L}$.

22. For two springs in series, the equivalent force constant is

$k = \dfrac{k_1 k_2}{k_1 + k_2}$. Here, $k_1 = k$ and $k_2 = \dfrac{YA}{L}$ (See the hint to Q. No. 21.)

$$\therefore \quad k = \frac{\dfrac{kYA}{L}}{k + \dfrac{YA}{L}} = \frac{kYA}{kL + YA} \qquad \therefore \quad T = 2\pi\sqrt{\frac{m}{k}}.$$

23. Mass of section BC $= \dfrac{m}{L}(L - y)$.



$\therefore$ tension at B $= T = \dfrac{m}{L}(L - y)g$.

$\therefore$ elongation of element dy at B

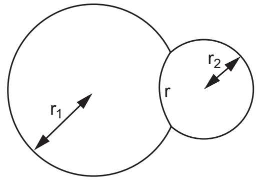
$$= dx = (dy)\frac{T}{AY} = \frac{m}{L}(L - y)g\frac{dy}{AY}.$$

$$\text{Total elongation} = \int dx = \frac{mg}{LAY}\int_0^L (L - y)\, dy = \frac{mgL}{2YA}.$$

24. The total surface area of the droplets is greater than that of the single drop. Hence, they have greater surface energy. This extra energy must be drawn from the internal energy of the drop.

25. The free liquid surface between the plates is cylindrical, and hence is curved along one axis only (parallel to the plates). The radius of curvature $r = d/2$. $p_0 - p = S/r$.

26.



Let p_0 = atmospheric pressure,

p_1 and p_2 = pressures inside the two bubbles.

$$p_1 - p_0 = \frac{4S}{r_1}, \qquad p_2 - p_0 = \frac{4S}{r_2}$$

or $p_2 - p_1 = \frac{4S}{r_2} - \frac{4S}{r_1}$ = pressure difference across the common surface.

Let r = radius of curvature of the common surface.

$$\therefore \quad p_2 - p_1 = \frac{4S}{r}. \qquad \therefore \quad \frac{4S}{r} = \frac{4S}{r_2} - \frac{4S}{r_1}.$$

27. The air pressure is greater inside the smaller bubble ($4S/r$). Hence, air flows from the smaller to the larger bubble.

28. R is dimensionless.

30. As the cork moves up, the force due to buoyancy remains constant. As its speed increases, the retarding force due to viscosity increases, being proportional to the speed. Thus. the acceleration gradually decreases. The acceleration is variable, and hence the relation between velocity and time is not linear.

31. η is defined through the relation $F = -\eta A \cdot \frac{dv}{dx}$.

$\frac{dv}{dx}$ has the dimension of $(\text{time})^{-1}$.

32. Forces of viscosity arise in a fluid when different sections of the fluid move with different velocities. Here, there is no such motion inside the oil drop and hence the viscosity of oil does not come into play.

33. When water flows out of a hole in a tank, it is in contact with the edges of the hole over a very small area, and therefore viscosity effects are small.

34. Terminal velocity is reached under the condition

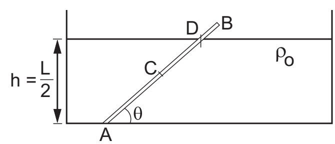
$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g,$$

where the symbols have their usual meanings. Thus, $v \propto r^2$.

35. As $v \propto r^2$ and the mass of the drop $m \propto r^3$, its momentum $p = mv \propto r^5$. Here, $2^5 = 32$.

36. The change in pressure with velocity is due to Bernoulli's principle, and is not affected by viscosity.

37.



Let AB = L, AC = $L/2$, AD = l, A = area of cross-section of the rod.

Weight of the rod = $AL\rho g$, acting through C.

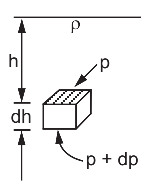
Buoyancy force = $Al\rho_0 g$, acting through the midpoint of AD.

Taking torque about A,

$$(lA\rho_0 g)\frac{l}{2}\cos\theta = (LA\rho g)\frac{L}{2}\cos\theta \quad \text{or} \quad \frac{l^2}{L^2} = \frac{\rho}{\rho_0}.$$

$$\text{Also, } \sin\theta = \frac{h}{l} = \frac{L}{2l} \quad \text{or} \quad \frac{l}{L} = \frac{1}{2\sin\theta} = \sqrt{\frac{\rho}{\rho_0}} \quad \text{or} \quad \sin\theta = \frac{1}{2}\sqrt{\frac{\rho_0}{\rho}}.$$

38.



Consider an element of the liquid of height dh and area of cross-section α at a depth h below the surface of the liquid. Let p and $p + dp$ be the liquid pressures at the upper and lower surfaces of the element.

Mass of the liquid in the element = $dm = (\alpha\, dh)\rho$.

Net upward force on the element = $[(p + dp)\alpha - p\alpha] - g\, dm$

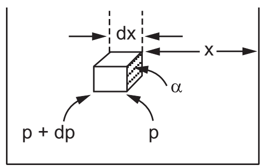
or $\quad \alpha\, dp - g\, dm = a\, dm,$ as the element moves up with an acceleration a

or $\quad \alpha\, dp = (g + a)(\alpha\, dh)\rho$

or $\quad \int dp = \int \rho(g + a)\, dh$

or $\quad p = \rho(g + a)h.$

41.



Consider an element of the liquid of width dx and area of cross-section α, at a distance x from the front of the tank.

Mass of the element = $dm = (\alpha\, dx)\rho$.

Net force to the right on the element = $(p + dp)\alpha - p\alpha = \alpha\, dp$.

$\therefore \quad \alpha\, dp = (\rho\alpha\, dx)a$

or $\quad \int_A^C dp = \int_A^C \rho a\, dx \qquad$ or $\quad p_C - p_A = \rho a l.$

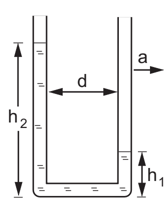
(Refer to the figure in the question to identify A, B, C.)

Also $p_B - p_C = \rho g h$

or $\quad p_B - (p_A + \rho a l) = \rho g h$

or $\quad p_B - p_A = h\rho g + l\rho a.$

42.



Let A = area of cross-section of the tube, ρ = density of the liquid.

Consider the section AB of the tube.

Mass of the liquid in AB = $dA\rho$.

Pressures at A and B $= h_2\rho g$ and $h_1\rho g$.

Net force to the right on AB $= (h_2\rho g - h_1\rho g)A$.

$\therefore \quad (h_2 - h_1)\rho g A = (dA\rho)a \quad$ or $\;(h_2 - h_1)\, g = da \quad$ or $\; h_2 - h_1 = ad/g$.

43. When the levels equalize,

the height of the liquid in each arm $= \dfrac{h_1 + h_2}{2}$.

We may then visualize that a length $h_1 - \dfrac{h_1 + h_2}{2} = \dfrac{h_1 - h_2}{2}$ of the liquid has been transferred from the left arm to the right arm. Then,

$$\text{mass of this liquid} = \left(\frac{h_1 - h_2}{2}\right) a\rho,$$

where, A = area of tube, ρ = density of the liquid.

Distance through which it moves down $= \dfrac{h_1 - h_2}{2}$.

$\therefore \quad$ loss in gravitational potential energy $= \left(\dfrac{h_1 - h_2}{2}\right)^2 A\rho$.

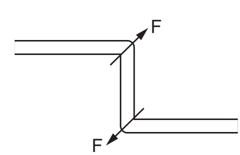
The mass of the entire liquid $= (h_1 + h_2 + h)\, A\rho$.

If this moves with a velocity v,

its kinetic energy $= \dfrac{1}{2}\,(h_1 + h_2 + h)\, A\rho v^2$.

Equating energies, we get v.

44. The forces exerted by the liquid at the bends are shown. (The liquid undergoes change of momentum only at these points, and hence the liquid and tube exert forces on each other.) The two forces form a couple exerting a clockwise torque.



47. $A_1 = 2.5\ \text{cm}^2, \quad v_1 = 3\ \text{m/s}, \quad h = 0.8\ \text{m}, \quad g = 10\ \text{m/s}^2$

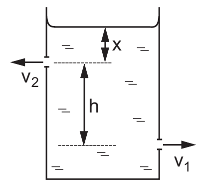
$v_2^2 = v_1^2 + 2gh = 25 \qquad$ or $\quad v_2 = 5\ \text{m/s}$.

$A_1 v_1 = A_2 v_2 \qquad$ or $\quad A_2 = \dfrac{A_1 v_1}{v_2} = 1.5\ \text{cm}^2$.

48. Velocity of efflux, $v = \sqrt{2gx}$, where x = height of liquid.

As the water drains out, x reduces, and hence v reduces. This reduces the rate of drainage. Hence, it requires a longer time to drain the same volume of water.

49.



$v_1 = \sqrt{2g(h+x)}$, $\quad v_2 = \sqrt{2gx}$

Let ρ = density of the liquid.

Let α = area of cross-section of each hole.

Volume of liquid discharged per second at a hole = αv.

$(v = v_1 \text{ or } v_2)$

Mass of liquid discharged per second = $\alpha v \rho$.

Momentum of liquid discharged per second = $\alpha v^2 \rho$.

$\therefore$ the force exerted at the upper hole (to the right) = $\alpha \rho v_2^2$

and the force exerted at the lower hole (to the left) = $\alpha \rho v_1^2$.

Net force on the tank = $\alpha\rho[v_1^2 - v_2^2] = \alpha\rho[2g(h+x) - 2gx] = 2\alpha\rho gh$.

50. Due to the rotation of the earth on its axis, the apparent weight of an object becomes slightly smaller at the equator than at the poles. This difference can be recorded on a spring balance, but not on a beam balance.

51. See the hints to Q. Nos. 1, 6, 7.

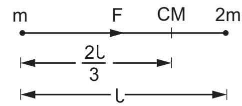
Apparent weight at the equator = $mg - m\omega^2 R$.

Weight at a height h above the pole = $mg\left(1 - \frac{2h}{R}\right)$.

Putting $mg - m\omega^2 R = mg\left(1 - \frac{2h}{R}\right)$

or $\quad \omega^2 R = \frac{2gh}{R} \qquad$ or $\quad h = \frac{\omega^2 R^2}{2g}$.

53. $F = G\dfrac{m(2m)}{l^2} = m\omega^2(2l/3)$



or $\dfrac{3mG}{l^3} = \omega^2.$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l^3}{3mg}}\,\cdot$$

56. Taking the potential at a large distance from the planet as zero, the potential at the centre of the planet $= -\dfrac{3GM}{2R}\cdot$

$\therefore \quad \dfrac{1}{2}mv^2 = m\left[0 - \left(-\dfrac{3GM}{2R}\right)\right]$

or $\quad v^2 = \dfrac{3GM}{R} = 3Rg = \dfrac{3}{2}(2RG) = \dfrac{3}{2}v_e^2$

or $\quad v = \sqrt{1.5}\,v_e.$

57. For a satellite orbiting very close to the earth's surface, the orbital velocity $= \sqrt{Rg}$ ($\because\ mg = mv^2/R$). This is equal to the velocity of projection and is the minimum velocity required to go into orbit. Also, the satellite would escape completely and not go into orbit for $v \geq v_e$.

$\therefore \quad v_e/\sqrt{2} < v < v_e.$

60. Let ω_0 = the angular velocity of the earth about its axis.

$\therefore \quad 24 = \dfrac{2\pi}{\omega_0} \quad$ or $\quad \omega_0 = \dfrac{2\pi}{24}\cdot$

Let ω = the angular velocity of the satellite.

$\therefore \quad 1.5 = \dfrac{2\pi}{\omega} \quad$ or $\quad \omega = \dfrac{2\pi}{1.5}\cdot$

For a satellite rotating from west to east (the same as the earth), the relative angular velocity, $\omega_1 = \omega - \omega_0$.

Time period of rotation relative to the earth $= \dfrac{2\pi}{\omega_1} = 1.6$ h.

For a satellite rotating from east to west (opposite to the earth), the relative angular velocity, $\omega_2 = \omega + \omega_0$.

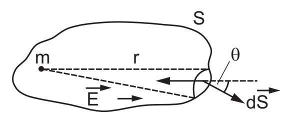
Time period of rotation relative to the earth $= \dfrac{2\pi}{\omega_2} = \dfrac{24}{17}$ h.

62. $T^2 \propto r^3$ or $T^2 = cr^3$

$\therefore \quad 2T\,\Delta T = 3cr^2\Delta r.$

Dividing, $\dfrac{2T\,\Delta T}{T^2} = \dfrac{3cr^2\Delta r}{cr^3}$.

63. Follow the method used to prove Gauss's law.



$$E = G\cdot\frac{m}{r^2}$$

$$\vec{E}\cdot d\vec{S} = EdS\cos(180° - \theta) = -EdS\cos\theta$$

$$\phi = \oint_S \vec{E}\cdot d\vec{S} = \oint_S -G\frac{m}{r^2}\,dS\cos\theta$$

$$= -Gm.\oint_S \frac{dS\cos\theta}{r^2} = -Gm.\oint_S d\omega = -4\pi Gm.$$

64. Work done $= m \times$ difference in gravitational potential.

65. Use arguments of symmetry as the yz plane divides the object symmetrically.

66. $F \propto r$ if $r < R$, and $F \propto \dfrac{1}{r^2}$ if $r > R$.

$\therefore \quad$ for $r_1, r_2 < R, \quad F_1/F_2 = r_1/r_2$

for $r_1, r_2 > R, \quad F_1/F_2 = r_2^2/r_1^2$.

68. Tension in B $= T_B = mg/3$.

Tension in A $= T_A = T_B + mg = 4mg/3$.

$\therefore \quad T_A = 4T_B$.

Stress $T/\pi r^2 = S$. Wire breaks when $S =$ breaking stress.

For $r_A = r_B$, $S_A = 4S_B$. $\quad\therefore\quad$ A breaks before B.

For $r_A = 2r_B$, $S_B = \dfrac{T_B}{\pi r_B^2}$.

$$S_A = \frac{T_A}{\pi r_A^2} = \frac{4T_B}{\pi(2r_B)^2} = \frac{T_B}{\pi r_B^2} = S_B.$$

As the stresses are equal, either may break.

71. Let S = surface tension = surface energy per unit area,

r = radius of each small drop, R = radius of a single drop.

$n \times 4/3\, \pi r^3 = 4/3\, \pi R^3$ or $R = rn^{1/3}$

Initial surface energy = $E_i = n \times 4\pi r^2 \times S = nE$.

Final surface energy = $E_f = 4\pi R^2 S = 4\pi r^2 n^{2/3}.\, S = n^{2/3}E$.

$\therefore$ energy released = $E_i - E_f = E(n - n^{2/3})$.

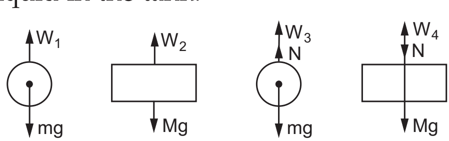
73. The angle of contact at the free liquid surface inside the capillary tube will change such that the vertical component of the surface tension forces just balance the weight of the liquid column.

74. The two free liquid surfaces must provide a net upward force due to surface tension to balance the weight of the liquid column.

75, 76. W_1, W_3 are the forces exerted by the ball and the spring balance on each other before and after immersion.

W_2, W_4 are the forces exerted by the tank and the weighing machine on each other before and after immersion.

Let m and M be the masses of the ball and the tank respectively, and let N be the force of interaction between the ball and the liquid in the tank.



$W_1 = mg$ $W_2 = Mg$

$W_3 + N = mg$

$W_4 = N + Mg$

77, 78. Pressure acts equally in all directions, and a liquid exerts forces on the walls of its container normal to the walls. Hence, the liquid exerting forces on the sloping walls of the conical flask will have a net upward component. Similarly, an upward force acts on the walls of the container at level X in question 69.

79. The velocity of efflux $= v = \sqrt{2gy}$.

The emerging liquid moves as a projectile and reaches the ground in time t, where,

$$H - y = \frac{1}{2} gt^2 \qquad \text{or} \quad t = \sqrt{\frac{2(H-y)}{g}}$$

$\therefore \quad x = vt = 2\sqrt{(H-y)y}$.

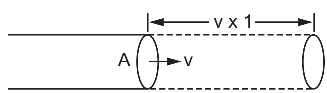
For x to be maximum, $\dfrac{dx}{dy} = 0 \quad$ or $\quad y = \dfrac{H}{2}$

$\therefore \quad x_{max} = H$.

81. $\dfrac{p_1}{\rho} + \dfrac{v_1^2}{2} = \dfrac{p_2}{\rho} + \dfrac{v_2^2}{2} \qquad$ or $\quad p_1 - p_2 = \dfrac{\rho}{2}(v_2^2 - v_1^2)$.

But $p_1 - p_2 = \rho g h = \dfrac{\rho}{2}(v_2^2 - v_1^2) \quad$ or $\quad v_2^2 - v_1^2 = 2gh$.

82.



Volume of liquid discharged per second $= v \times 1 \times A$.

Mass of this liquid $= vA\rho$.

Momentum of this liquid $= v^2 A\rho$.

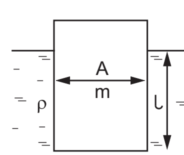
$\therefore$ rate of change of momentum = force $= v^2 A\rho$.

83. In equilibrium, let l = length of the block immersed.

$\therefore \quad mg = Al\rho g$.

If the block is given a further downward displacement x and its downward acceleration becomes a then

$$mg - A(l + x)\rho g = ma \quad \text{or} \quad a = -\left(\frac{A\rho g}{m}\right)x$$



Put $\omega^2 = \dfrac{A\rho g}{m}$.

$\therefore \quad a = -\omega^2 x$. This is SHM with time period $T = 2\pi\sqrt{\dfrac{m}{A\rho g}}$.

84. Let A = area of cross-section of the tube, ρ = density of liquid.

If one surface is pushed down by x, the other surface moves up by x.

Net unbalanced force on the liquid column = $2xA\rho g$.

Mass of the liquid column = $lA\rho$.

Let a = acceleration of liquid column.

$\therefore \quad -2xA\rho g = (lA\rho)a$

or $\quad a = -\left(\frac{2g}{l}\right)x$

Put $\omega^2 = \frac{2g}{l}$.

$\therefore \quad a = -\omega^2 x$. This is SHM with $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{2g}}$.

□

• *Revision Exercise 2* •

Choose the correct option in each of the following questions. Only one option is correct in each question.

R1. Two rods of identical dimensions, with Young's moduli Y_1 and Y_2 are joined end to end. The equivalent Young's modulus for the composite rod is

(a) $2Y_1Y_2/(Y_1+Y_2)$ (b) $Y_1Y_2/(Y_1+Y_2)$
(c) $1/2(Y_1+Y_2)$ (d) Y_1+Y_2

R2. A point starts from rest and moves along a circular path with a constant tangential acceleration. After one rotation, the ratio of its radial acceleration to its tangential acceleration will be equal to

(a) 1 (b) 2π
(c) $1/2\pi$ (d) 4π

R3. A simple pendulum swings with angular amplitude θ. The tension in the string when it is vertical is twice the tension in its extreme position. Then, $\cos\theta$ is equal to

(a) 1/3 (b) 1/2
(c) 2/3 (d) 3/4

R4. A rectangular block is kept on a rough horizontal surface and given some initial velocity. The coefficient of friction between the block and the surface is constant. The block will topple

(a) either at the very beginning or not at all
(b) just before coming to rest
(c) only if the coefficient of friction at the ground is ≥ 1
(d) only if some external force acts on it in addition to the frictional force

R5. A wheel of radius 10 cm rolls without slipping on the ground with velocity 50 cm/s. When it is at a point P on the ground, a small piece of mud separates from its highest point. How far from P will it hit the ground?

(a) 10 cm (b) 20 cm (c) 25 cm (d) 40 cm

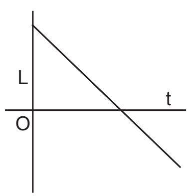
R6. A U-tube of constant cross-section A contains liquid of density ρ. Initially, the liquids in the two arms are held with a level difference h. After being released, the levels equalize after some time. The work done by gravity forces on the liquid in the process is ($W = \rho Agh$)

(a) zero (b) Wh

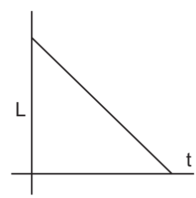
(c) $Wh/2$ (d) $Wh/4$

R7. A block slides on a rough horizontal ground from point A to point B. Point C is midway between A and B. The coefficient of friction between the block and the ground is constant. Its angular momentum L about C is plotted against time t. Which of the following curves is correct ?

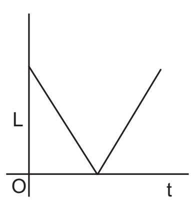
(a)



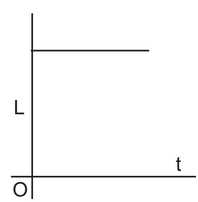
(b)



(c)



(d)



R8. A uniform rod of mass m is hinged at its upper end. It is released from a horizontal position. When it reaches the vertical position, what force does it exert on the hinge ?

(a) $1.5mg$ (b) $2mg$

(c) $2.5mg$ (d) $3.5mg$

R9. The escape velocity for a planet is V. A tunnel is dug along its diameter and a particle is dropped into the tunnel. At the centre of the planet, the speed of the particle will be

(a) V (b) $V/2$

(c) $V/\sqrt{2}$ (d) $V/2\sqrt{2}$

R10. A satellite orbits a planet, moving very close to its surface. By what maximum factor can its kinetic energy be increased suddenly, without change in direction, such that it still remains in orbit?

(a) 1.5 (b) $\sqrt{2}$

(c) 2 (d) 3

E11. The moment of inertia of a disc about an axis passing through its centre and normal to its plane is I. The disc is now folded along a diameter such that the two halves are mutually perpendicular. Its moment of inertia about this diameter will now be

(a) I (b) $I/\sqrt{2}$

(c) $I/2$ (d) $I/4$

R12. When an athlete runs with some acceleration, he leans forward. The line joining his centre of mass to his foot which is in contact with the ground makes an angle θ with the vertical. The coefficient of friction between his foot and the ground is k. His foot does not slip on the ground. His acceleration is

(a) kg (b) $kg\tan\theta$

(c) $g\tan\theta$ (d) $(g\tan\theta)/k$

R13. A uniform wooden stick of density d is completely immersed in water of density $D(D > d)$. One end of the stick is free to rotate about a hinge which is attached to a wall of the container. In the position of equilibrium the stick makes an angle θ with the vertical. Then, tan θ is equal to

(a) 0 (b) d/D

(c) $1 - d/D$ (d) D/d

R14. A liquid whose coefficient of viscosity is η flows on a horizontal surface. Let dx represent the vertical distance between two layers of liquid and dv represent the difference in the velocities of the two layers. Then the quantity $\eta(dv/dx)$ has the same dimensions as

(a) acceleration (b) force

(c) momentum (d) pressure

R15. When a trapeze artist, spinning in air, folds his body, his moment of inertia decreases from I_1 to I_2. The work done by him in this process is proportional to

(a) $I_1 - I_2$ (b) $1/I_2 - 1/I_1$

(c) $1/(I_1 - I_2)$ (d) $I_1 I_2/(I_1 - I_2)$

R16. A pipe ABCD of uniform cross-section is bent into three sections, viz., a horizontal section AB, a vertical section BC with C below B, and a horizontal section CD. Liquid flowing through the pipe has speed v_1 and pressure p_1 in section AB, and speed v_2 and pressure p_2 in section CD.

(a) $v_1 = v_2, p_1 = p_2$ (b) $v_1 = v_2, p_2 > p_1$

(c) $v_2 > v_1, p_2 > p_1$ (d) $v_2 > v_1, p_1 = p_2$

R17. A body of circular cross-section is placed on a smooth horizontal surface. When a horizontal impulse is given to its highest point, it begins to roll without slipping. The body is a

(a) ring (b) disc

(c) solid sphere (d) hollow sphere

R18. A uniform rod of length l is free to move and rotate in gravity-free space. When an impulse is given to one end of the rod, perpendicular to its length, its centre of mass moves with velocity v. It will rotate about its centre of mass with angular velocity

(a) $8v/3l$ (b) $6v/l$

(c) $3v/l$ (d) $3v/2l$

R19. A ring of mass m rolls down an inclined plane with acceleration a without slipping. The plane makes an angle θ with the horizontal. The force of friction acting on the ring is equal to

(a) ma (b) $mg \sin\theta$

(c) $m(g\sin\theta - a)$ (d) the limiting friction

R20. A sealed tank of length l and height h, completely filled with liquid of density ρ, moves with horizontal acceleration a. The maximum difference in pressure between any two points in the liquid is

(a) ρgh

(b) ρgl

(c) $\rho(gh + al)$

(d) all points in the liquid are at the same pressure

R21. A solid sphere of radius r rolls without slipping on the floor, with linear momentum p. Its angular momentum with respect to a point on the floor is

(a) $2pr/5$ (b) $3pr/5$

(c) $3pr/7$ (d) $7pr/5$

R22. A long glass capillary tube is dipped in water. It is known that water wets glass. The water level rises by h in the tube. The tube is now pushed down so that only a length $h/2$ is outside the water surface. The angle of contact of the water surface at the upper end of the tube will be

(a) 30° (b) 45°

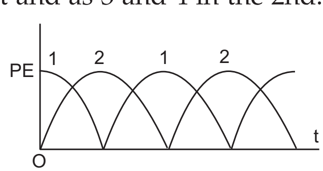
(c) $\tan^{-1} 2$ (d) 60°

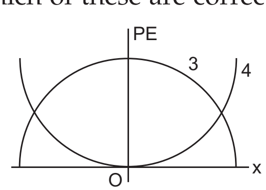
R23. A particle undergoes SHM of time period 2 seconds. In what time will it move from an extreme position to a displacement equal to half of its amplitude?

(a) 1/2 s (b) 1/3 s

(c) 1/4 s (d) 1/6 s

R24. For a particle undergoing SHM, the displacement x is related to time t as $x = A \cos \omega t$. In the graphs given below, its potential energy (PE) is plotted against t (1st graph) and x (2nd graph). Two options are shown in each graph, marked as 1 and 2 in the 1st and as 3 and 4 in the 2nd. Which of these are correct?





(a) 1 and 3 (b) 2 and 4
(c) 2 and 3 (d) 1 and 4

R25. A uniform rod of weight W is free to rotate about a hinge at one end. The hinge is fixed to a wall. If the rod is released from a horizontal position, what force will it exert on the hinge just after being released?

(a) W (b) $W/2$
(c) $W/3$ (d) $W/4$

R26. A vertical cylinder is filled with liquid. A small hole is made in the wall of the cylinder at a depth H below the free surface of the liquid. The force exerted on the cylinder by the liquid flowing out of the hole initially will be proportional to

(a) $\sqrt{H}$ (b) H
(c) $H^{3/2}$ (d) H^2

R27. A simple pendulum has time period T. A uniform rod, whose length is the same as that of the pendulum, undergoes small oscillations about its upper end. Its time period of oscillation will be

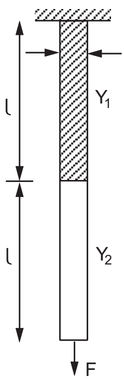
(a) $< T$
(b) T
(c) $> T$
(d) may be (a), (b) or (c) depending on whether T is <, equal to or > 2 seconds

Answers to Revision Exercise 2

R1. a	**R2.** d	**R3.** d	**R4.** a	**R5.** b
R6. d	**R7.** b	**R8.** c	**R9.** c	**R10.** c
R11. c	**R12.** c	**R13.** a	**R14.** d	**R15.** b
R16. b	**R17.** a	**R18.** b	**R19.** a	**R20.** c
R21. d	**R22.** d	**R23.** b	**R24.** d	**R25.** d
R26. b	**R27.** a			

Hints and Solutions to Selected Questions

R1.



Let δ_1 and δ_2 be the extensions of the two rods.

$$Y_1 = \frac{F/A}{\delta_1/l} = \frac{Fl}{A\delta_1}$$

or $$\delta_1 = \frac{Fl}{Y_1 A} \text{ and } \delta_2 = \frac{Fl}{Y_2 A}.$$

For equivalent Young's modulus Y,

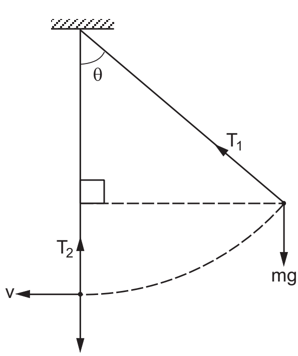
$$Y = \frac{F/A}{(\delta_1 + \delta_2)/2l} = \frac{2Fl}{A\left[\frac{Fl}{Y_1 A} + \frac{Fl}{Y_2 A}\right]} = \frac{2Y_1 Y_2}{Y_1 + Y_2}.$$

R2. Let a_t and a_r be the radial and tangental accelerations, and r be the radius of the circle.

After one rotation, $v^2 = 0 + 2a_t\,(2\pi r)$.

$$\therefore \quad a_r = \frac{v^2}{r} = 4\pi a_t \quad \text{or} \quad \frac{a_r}{a_t} = 4\pi.$$

R3.



$$T_1 = mg\cos\theta$$

$$\frac{1}{2}mv^2 = mg(l - l\cos\theta)$$

or $$\frac{v^2}{l} = 2g(1 - \cos\theta).$$

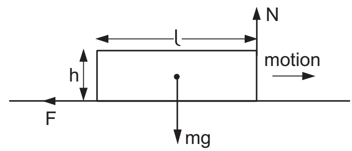
$$T_2 - mg = \frac{mv^2}{l} = 2mg(1 - \cos\theta)$$

or $$T_2 = 3mg - 2mg\cos\theta$$

$$= 2T_1 = 2mg\cos\theta$$

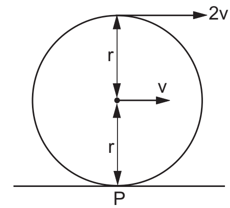
or $$\cos\theta = \frac{3}{4}.$$

R4.



The block will topple if the torques due to N and F about C do not balance, even when the line of action of N has reached the extreme position. This will depend on μ, l and h.

R5. Vertical motion: $-2r = 0 + \frac{1}{2}(-g)\,t^2$.

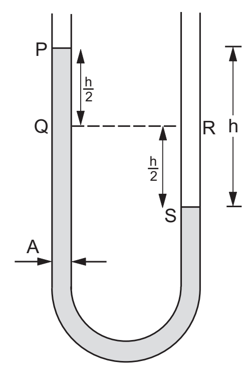


$$t = 2\sqrt{\frac{r}{g}} = 2\sqrt{\frac{0.1}{10}} = 0.2 \text{ s}.$$

Horizontal motion: $s = 2vt$

or $\quad s = 2(50 \text{ cm/s})(0.2 \text{ s}) = 20 \text{ cm}.$

R6. The liquid levels will equalize at Q and R. This is equivalent to the section PQ moving down to the position RS.



Mass of section PQ = $\rho Ah/2$.

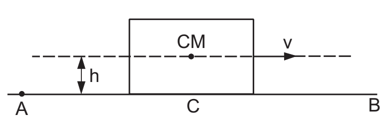
The centre of mass of this section moves down through a distance $h/2$.

Loss in potential energy

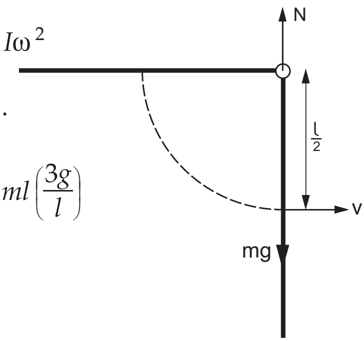
$$= \left(\frac{1}{2}\rho Ah\right)g \cdot \left(\frac{h}{2}\right) = \frac{1}{4}(\rho Agh)\,h = \frac{1}{4}Wh$$

= work done by gravity forces on the liquid.

R7.



At any position of the block, its angular momentum L about C is equal to its linear momentum (p) × the height of its centre of mass (h) above C. As its velocity v decreases linearly with time, L will also decrease linearly with time.



R8. Loss in PE $= mg\dfrac{l}{2} =$ gain in KE $= \dfrac{1}{2}I\omega^2$

or $\quad mgl = \left(\dfrac{ml^2}{3}\right)\omega^2 \quad$ or $\quad \omega^2 = \dfrac{3g}{l}\cdot$

$$N - mg = \frac{mv^2}{l/2} = \frac{2m}{l}\left(\omega\frac{l}{2}\right)^2 = \frac{1}{2}ml\left(\frac{3g}{l}\right)$$

or $\quad N = \dfrac{5}{2}mg.$

R9. Let M, R be the mass and radius of the planet, and g be the acceleration due to gravity on its surface.

Then, $V = \sqrt{2Rg}$ and $GM = R^2g$.

Gravitational potential at the surface is $-\dfrac{GM}{R}$ and at the centre is $-\dfrac{3GM}{2R}\cdot$ In going from the surface to the centre, loss in gravitational PE

$$= m\left[-\frac{GM}{R} - \left(-\frac{3}{2}\frac{GM}{R}\right)\right] = \frac{1}{2}\frac{GMm}{R} = \frac{1}{2}mv^2$$

or $\quad v^2 = \dfrac{GM}{R} = Rg = \dfrac{V^2}{2} \qquad$ or $\quad \dfrac{V}{\sqrt{2}}\cdot$

R10. For an orbit close to the planet's surface, $\dfrac{mv^2}{R} = mg.$

For escape velocity, $v_{max}^2 = 2Rg.$

$v_{max}^2/v^2 = 2.$

R11. $I = \dfrac{1}{2}mR^2.$

For a diameter, moment of inertia $= \dfrac{I}{2}\cdot$

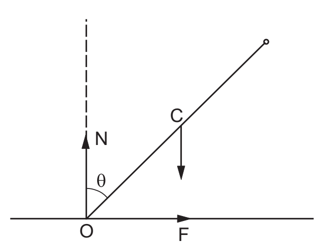
R12. $C =$ centre of mass.

Taking torques about C,

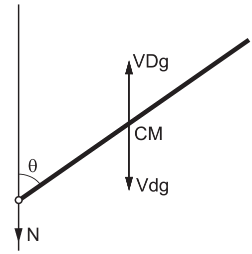
$F \times \text{OC}\cos\theta = N \times \text{OC}\sin\theta$

or $\quad F = mg\tan\theta = ma$

or $\quad a = g\tan\theta.$



R13.



Weight = Vdg.
Buoyancy force = $VDg > Vdg$.
Torques about hinge can balance only for $\theta = 0$.

R14. $F = \eta A \cdot \frac{dv}{dx}$ or $\frac{F}{A} = \eta \cdot \frac{dv}{dx}$

This has the dimension of pressure.

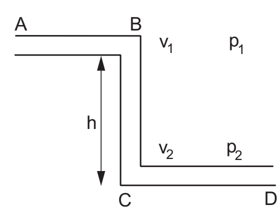
R15. The angular momentum L will be conserved.

Initial rotational KE = $E_i = \frac{L^2}{2I_1}$.

Final rotational KE = $E_f = \frac{L^2}{2I_2}$.

Work done = $E_f - E_i = \frac{L^2}{2}\left[\frac{1}{I_2} - \frac{1}{I_1}\right]$.

R16. From equation of continuity, $v_1 = v_2$.

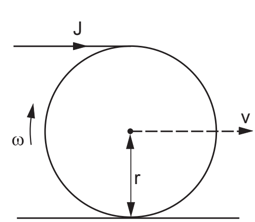


$$\frac{v_1^2}{2} + gh + \frac{p_1}{\rho} = \frac{v_2^2}{2} + 0 + \frac{p_2}{\rho}$$

$$p_2 - p_1 = h\rho g$$

or $p_2 > p_1$.

R17. $J = mv$.



$L = Jr = I\omega = (k^2)(v/r)$

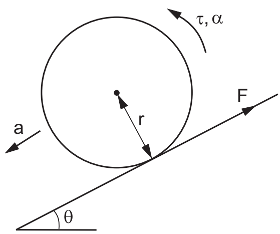
or $mvr = mk^2 v/r$

or $k^2 = r^2$.

R18. $J = mv$. $\quad L = J \cdot \dfrac{l}{2} = I\omega = \left(\dfrac{ml^2}{12}\right)\omega.$

$\therefore \quad mv\dfrac{l}{2} = \dfrac{ml^2}{12}\omega. \quad \omega = \dfrac{6v}{l}.$

R19.



$\tau = Fr = I\alpha = (mr^2)(a/r)$

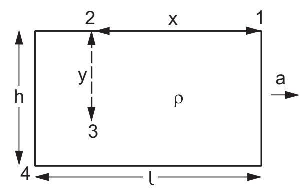
$\therefore \quad F = ma.$

R20. $p_2 - p_1 = x\rho a$

$p_3 - p_2 = y\rho g$

$\therefore \quad p_3 - p_1 = x\rho a + y\rho g$

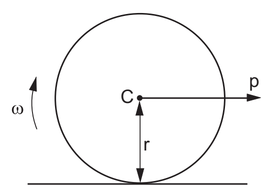
$p_4 - p_1 = \rho(gh + al)$



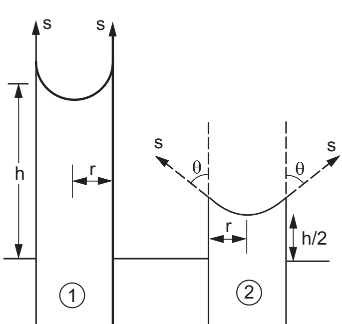
R21. Angular momentum with respect to a point on the floor

= angular momentum about C + pr

$= I\omega + pr = \left(\dfrac{2}{5}mr^2\right)\left(\dfrac{v}{r}\right) + mvr = \dfrac{7}{5}pr.$



R22.



In (1) $2\pi rs = \pi r^2 h\rho g.$

In (2) $2\pi rs\cos\theta = \pi r^2\left(\dfrac{h}{2}\right)\rho g.$

Dividing, $\cos\theta = \dfrac{1}{2}$

or $\quad \theta = 60°.$

R23. $T = \dfrac{2\pi}{\omega} = 2$ or $\omega = \pi.$

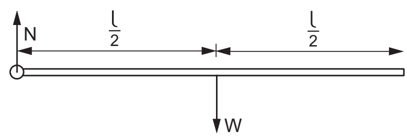
$$x = a\cos\omega t$$

or $\dfrac{a}{2} = a\cos\omega t$

or $\omega t = \dfrac{\pi}{3}$

or $t = \dfrac{\pi}{3\pi} = \dfrac{1}{3}$ s.

R25. $\tau = W \cdot \dfrac{l}{2} = I\alpha = \left(\dfrac{ml^2}{3}\right)\alpha$



or $\alpha = \dfrac{3W}{2ml}$.

$$a_{CM} = \alpha \cdot \frac{l}{2} = \frac{3W}{4m}$$

$W - N = ma_{CM} = \dfrac{3W}{4}$

or $N = \dfrac{W}{4}$.

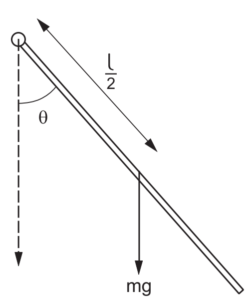
R26. $v = \sqrt{2gH}$. $F = \rho s v^2$ where s = area of cross-section of the hole.

$\therefore$ $F = \rho s(2gH) \propto H.$

R27. $T = 2\pi\sqrt{l/g}$ where l = length of simple pendulum = length of rod.

$\tau = (mg)\dfrac{l}{2}\sin\theta.$

For small θ, $\tau = \dfrac{1}{2}mgl\,\theta = -I\alpha = -\left(\dfrac{ml^2}{3}\right)\alpha$



or $\alpha = -\left(\dfrac{3g}{2l}\right)\theta.$

Time period $= 2\pi\sqrt{\dfrac{2l}{3g}} < T.$

□

Part 2

Heat and Thermodynamics

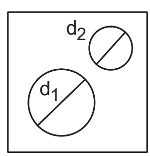
1

Heat and Thermodynamics

• *Type 1* •

Choose the correct option (a, b, c or d).

1.



Two holes of unequal diameters d_1 and d_2 ($d_1 > d_2$) are cut in a metal sheet. If the sheet is heated,

(a) both d_1 and d_2 will decrease

(b) both d_1 and d_2 will increase

(c) d_1 will increase, d_2 will decrease

(d) d_1 will decrease, d_2 will increase

2. In the previous question, the distance between the holes will

(a) increase

(b) decrease

(c) remain constant

(d) may either increase or decrease depending on the positions of the holes on the sheet and on the ratio d_1/d_2.

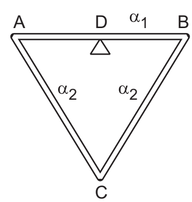
3. A metal wire of length l and area of cross-section A is fixed between rigid supports at negligible tension. If this is cooled, the tension in the wire will be

(a) proportional to l
(b) inversely proportional to l
(c) independent of l
(d) independent of A

4. Two metal rods of the same length and area of cross-section are fixed end to end between rigid supports. The materials of the rods have Young modulii Y_1 and Y_2, and coefficients of linear expansion α_1 and α_2. The junction between the rods does not shift if the rods are cooled.

(a) $Y_1\alpha_1 = Y_2\alpha_2$
(b) $Y_1\alpha_2 = Y_2\alpha_1$
(c) $Y_1\alpha_1^2 = Y_2\alpha_2^2$
(d) $Y_1^2\alpha_1 = Y_2^2\alpha_2$

5.



Three rods of equal length are joined to form an equilateral triangle ABC. D is the midpoint of AB. The coefficient of linear expansion is α_1 for AB, and α_2 for AC and BC. If the distance DC remains constant for small changes in temperature,

(a) $\alpha_1 = \alpha_2$
(b) $\alpha_1 = 2\alpha_2$
(c) $\alpha_1 = 4\alpha_2$
(d) $\alpha_1 = \frac{1}{2}\alpha_2$

6. When the temperature of a body increases from t to $t + \Delta t$, its moment of inertia increases from I to $I + \Delta I$. The coefficient of linear expansion of the body is α. The ratio $\frac{\Delta I}{I}$ is equal to

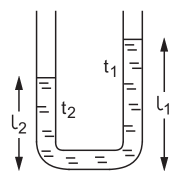
(a) $\frac{\Delta t}{t}$
(b) $\frac{2\Delta t}{t}$
(c) $\alpha\,\Delta t$
(d) $2\alpha\,\Delta t$

7. A horizontal tube, open at both ends, contains a column of liquid. The length of this liquid column does not change with temperature. Let γ = coefficient of volume expansion of the liquid and α = coefficient of linear expansion of the material of the tube.

(a) $\gamma = \alpha$ (b) $\gamma = 2\alpha$

(c) $\gamma = 3\alpha$ (d) $\gamma = 0$

8.



In a vertical U-tube containing a liquid, the two arms are maintained at different temperatures, t_1 and t_2. The liquid columns in the two arms have heights l_1 and l_2 respectively. The coefficient of volume expansion of the liquid is equal to

(a) $\dfrac{l_1 - l_2}{l_2 t_1 - l_1 t_2}$ (b) $\dfrac{l_1 - l_2}{l_1 t_1 - l_2 t_2}$

(c) $\dfrac{l_1 + l_2}{l_2 t_1 + l_1 t_2}$ (d) $\dfrac{l_1 + l_2}{l_1 t_1 + l_2 t_2}$

9. A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

(a) $\dfrac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$ (b) $\dfrac{f_1 - f_2}{f_1 t_1 - f_2 t_2}$

(c) $\dfrac{f_1 + f_2}{f_2 t_1 + f_1 t_2}$ (d) $\dfrac{f_1 + f_2}{f_1 t_1 + f_2 t_2}$

10. A solid with coefficient of linear expansion α just floats in a liquid whose coefficient of volume expansion is γ. If the system is heated, the solid will

(a) sink in all cases
(b) continue to float in all cases
(c) sink if $\gamma > 3\alpha$
(d) sink if $\gamma < 3\alpha$

11. A gas at absolute temperature 300 K has pressure

$$= 4 \times 10^{-10} \text{N/m}^2.$$

Boltzmann constant $= k = 1.38 \times 10^{-23}$ J/K. The number of molecules per cm^3 is of the order of

(a) 100
(b) 10^5
(c) 10^8
(d) 10^{11}

12. The root-mean-square (rms) speed of oxygen molecules (O_2) at a certain absolute temperature is v. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, the rms speed would be

(a) v
(b) $\sqrt{2}v$
(c) $2v$
(d) $2\sqrt{2}v$

13. The average translational kinetic energy of O_2 (molar mass 32) at a particular temperature is 0.048 eV. The average translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is

(a) 0.0015
(b) 0.003
(c) 0.048
(d) 0.768

14. A gas has volume V and pressure p. The total translational kinetic energy of all the molecules of the gas is

(a) $\frac{3}{2}pV$ only if the gas is monoatomic

(b) $\frac{3}{2}pV$ only if the gas is diatomic

(c) $> \frac{3}{2}pV$ if the gas is diatomic

(d) $\frac{3}{2}pV$ in all cases

15. A closed vessel is maintained at a constant temperature. It is first evacuated and then vapour is injected into it continuously. The pressure of the vapour in the vessel

(a) increases continuously

(b) first increases and then remains constant

(c) first increases and then decreases

(d) none of the above

16. When an air bubble rises from the bottom to the surface of a lake, its radius becomes double. Find the depth of the lake, given that the atmospheric pressure is equal to the pressure due to a column of water 10 m high. Assume constant temperature and disregard surface tension.

(a) 30 m (b) 40 m

(c) 70 m (d) 80 m

17. Two containers of equal volume contain the same gas at pressures p_1 and p_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure p and a common temperature T. The ratio p/T is equal to

(a) $\frac{p_1}{T_1}+\frac{p_2}{T_2}$ (b) $\frac{1}{2}\left[\frac{p_1}{T_1}+\frac{p_2}{T_2}\right]$

(c) $\frac{p_1T_2+p_2T_1}{T_1+T_2}$ (d) $\frac{p_1T_2-p_2T_1}{T_1-T_2}$

18. Two identical containers joined by a small pipe initially contain the same gas at pressure p_0 and absolute temperature T_0. One container is now maintained at the same temperature while the other is heated to $2T_0$. The common pressure of the gases will be

(a) $\frac{3}{2}p_0$ (b) $\frac{4}{3}p_0$

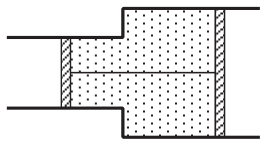
(c) $\frac{5}{3}p_0$ (d) $2p_0$

19. In the previous question, let V_0 be the volume of each container. All other details remain the same. The number of moles of gas in the container at temperature $2T_0$ will be

(a) $\frac{p_0V_0}{2RT_0}$ (b) $\frac{p_0V_0}{RT_0}$

(c) $\frac{2p_0V_0}{3RT_0}$ (d) $\frac{p_0V_0}{3RT_0}$

20.



A horizontal cylinder has two sections of unequal cross-sections, in which two pistons can move freely. The pistons are joined by a string. Some gas is trapped between the pistons. If this gas is heated, the pistons will

(a) move to the left
(b) move to the right
(c) remain stationary
(d) either (a) or (b) depending on the initial pressure of the gas

21. A gas expands from 1 litre to 3 litres at atmospheric pressure. The work done by the gas is about

(a) 2 J (b) 200 J
(c) 300 J (d) 2×10^5 J

22. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{c_p}{c_V}=\gamma$ for the gas is

(a) 2 (b) $\frac{3}{2}$
(c) $\frac{5}{3}$ (d) $\frac{4}{3}$

23. In the previous question, the gas may be

(a) monoatomic
(b) diatomic

(c) a mixture of monoatomic and diatomic gases

(d) a mixture of diatomic and triatomic gases

24. Each molecule of a gas has f degrees of freedom. The ratio $\frac{c_p}{c_V} = \gamma$ for the gas is

(a) $1 + \frac{f}{2}$ (b) $1 + \frac{1}{f}$

(c) $1 + \frac{2}{f}$ (d) $1 + \frac{(f-1)}{3}$

25. A mixture of n_1 moles of monoatomic gas and n_2 moles of diatomic gas has $\frac{c_p}{c_V} = \gamma = 1.5$.

(a) $n_1 = n_2$ (b) $2n_1 = n_2$

(c) $n_1 = 2n_2$ (d) $2n_1 = 3n_2$

26. The pressure p of a gas is plotted against its absolute temperature T for two different constant volumes V_1 and V_2, where $V_1 > V_2$. p is plotted on the y-axis and T on the x-axis.

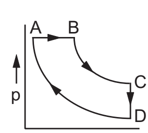
(a) The curve for V_1 has greater slope than the curve for V_2.

(b) The curve for V_2 has greater slope than the curve for V_1.

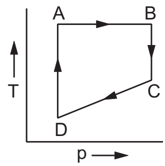
(c) The curves must intersect at some point other than $T = 0$.

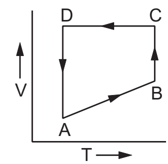
(d) The curves have the same slope and do not intersect.

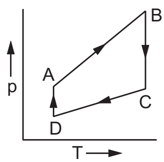
27.

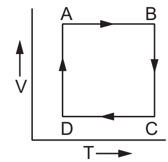


A cyclic process ABCD is shown in the p–V diagram. Which of the following curves represent the same process?

(a) 

(b) 

(c) 

(d) 

28. The ratio $\frac{c_p}{c_V} = \gamma$ for a gas. Its molecular weight is M. Its specific heat capacity at constant pressure is

(a) $\frac{R}{\gamma - 1}$

(b) $\frac{\gamma R}{\gamma - 1}$

(c) $\frac{\gamma R}{M(\gamma - 1)}$

(d) $\frac{\gamma RM}{(\gamma - 1)}$

29. The molar heat capacity of a gas at constant volume is C_V. If n moles of the gas undergo ΔT change in temperature, its internal energy will change by $nC_V\Delta T$

(a) only if the change of temperature occurs at constant volume

(b) only if the change of temperature occurs at constant pressure

(c) in any process which is not adiabatic

(d) in any process

30. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{3}{7}$

(d) $\frac{5}{7}$

31. The average degree of freedom per molecule for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. The heat absorbed by the gas is

(a) 75 J (b) 100 J
(c) 150 J (d) 125 J

32. Two cylinders A and B, fitted with pistons, contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K then the rise in temperature of the gas in B is

(a) 30 K (b) 18 K
(c) 50 K (d) 42 K

33. A system is taken from state A to state B along two different paths 1 and 2. The heat absorbed and work done by the system along these two paths are Q_1 and Q_2, and W_1 and W_2 respectively.

(a) $Q_1 = Q_2$ (b) $W_1 = W_2$
(c) $Q_1 - W_1 = Q_2 - W_2$ (d) $Q_1 + W_1 = Q_2 + W_2$

34. Equal masses of three liquids A, B and C have temperatures 10°C, 25°C and 40°C respectively. If A and B are mixed, the mixture has a temperature of 15°C. If B and C are mixed, the mixture has a temperature of 30°C. If A and C are mixed, the mixture will have a temperature of

(a) 16°C (b) 20°C
(c) 25°C (d) 29°C

35. If water at 0°C, kept in a container with an open top, is placed in a large evacuated chamber,

(a) all the water will vaporize
(b) all the water will freeze
(c) part of the water will vaporize and the rest will freeze
(d) ice, water and water vapour will be formed and reach equilibrium at the triple point

36. In the previous question, if the specific latent heat of vaporization of water at 0°C is η times the specific latent heat of freezing of water at 0°C, the fraction of water that will ultimately freeze is

(a) $\frac{1}{\eta}$ (b) $\frac{\eta}{\eta+1}$

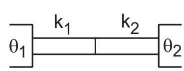
(c) $\frac{\eta-1}{\eta}$ (d) $\frac{\eta-1}{\eta+1}$

37. A substance of mass M kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The specific latent heat of fusion of the substance is

(a) Pt (b) $\frac{Pt}{M}$

(c) PtM (d) $\frac{PM}{t}$

38.



Two identical rods are made of different materials whose thermal conductivities are k_1 and k_2. They are placed end to end between two heat reservoirs at temperatures Q_1 and Q_2. The temperature of the junction of the rods is

(a) $\frac{\theta_1+\theta_2}{2}$ (b) $\frac{k_1\theta_1+k_2\theta_2}{k_1+k_2}$

(c) $\frac{k_1\theta_2+k_2\theta_1}{k_1+k_2}$ (d) $\frac{|k_1\theta_1+k_2\theta_2|}{|k_1-k_2|}$

39. A cylinder of radius R, made of a material of thermal conductivity k_1, is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The shell is made of a material of thermal conductivity k_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $k_1 + k_2$

(b) $\frac{k_1 k_2}{k_1 + k_2}$

(c) $\frac{k_1 + 3k_2}{4}$

(d) $\frac{3k_1 + k_2}{4}$

40. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watts would be

(a) 225

(b) 450

(c) 900

(d) 1800

41. Two spherical black bodies of radii r_1 and r_2 and with surface temperatures T_1 and T_2 respectively radiate the same power. r_1/r_2 must be equal to

(a) $(T_1/T_2)^2$

(b) $(T_2/T_1)^2$

(c) $(T_1/T_2)^4$

(d) $(T_2/T_1)^4$

42. A body cools from 50°C to 40°C in 5 minutes. The surrounding temperature is 20°C. In what further time (in minutes) will it cool to 30°C?

(a) 5

(b) $\frac{15}{2}$

(c) $\frac{25}{3}$

(d) 10

43. In the previous question, what will be its temperature 5 minutes after reaching 40°C?

(a) 35°C

(b) $\frac{100}{3}$ °C

(c) 32°C

(d) 30°C

• Type 2 •

Choose the correct options. One or more options may be correct.

44. A metal rod is shaped into a ring with a small gap. If this is heated,

(a) the length of the rod will increase

(b) the gap will decrease

(c) the gap will increase

(d) the diameter of the ring will increase in the same ratio as the length of the rod

45. The average translational energy and the rms speed of the molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m/s respectively. Assuming ideal gas behaviour, the corresponding values at 600 K are nearly

(a) 12.42×10^{-21} J, 968 m/s (b) 8.78×10^{-21} J, 684 m/s

(c) 6.21×10^{-21} J, 968 m/s (d) 12.42×10^{-21} J, 684 m/s

46. Let $\overline{v}, v_{\text{rms}}$ and v_{p} respectively denote the mean speed, the root-mean-square speed, and the most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m.

(a) No molecule can have speed greater than v_{rms}.

(b) No molecule can have speed less than $\dfrac{v_{\text{p}}}{\sqrt{2}}$.

(c) $v_{\text{p}} < \overline{v} < v_{\text{rms}}$.

(d) The average kinetic energy of a molecule is $\dfrac{3}{4} m v_{\text{p}}^2$.

47. Two identical containers A and B have frictionless pistons. They contain the same volume of an ideal gas at the same temperature. The mass of the gas in A is m_{A} and that in B is m_{B}. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. The changes in the pressure in A and B are found to be Δp and $1.5\,\Delta p$ respectively.

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$

(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

48. A gas undergoes a process in which its pressure p and volume V are related as Vp^n = constant. The bulk modulus for the gas in this process is

(a) np (b) $p^{\frac{1}{n}}$

(c) $\frac{p}{n}$ (d) p^n

49. n moles of a gas expands from volume V_1 to V_2 at constant temperature T. The work done by the gas is

(a) $nRT\left(\frac{v_2}{v_1}\right)$ (b) $nRT\left(\frac{v_2}{v_1} - 1\right)$

(c) $nRT \ln\left(\frac{v_2}{v_1}\right)$ (d) $nRT \ln\left(\frac{v_2}{v_1} + 1\right)$

50. A gas with $\frac{c_p}{c_V} = \gamma$ goes from an initial state (p_1, V_1,T_1) to a final state (p_2, V_2, T_2) through an adiabatic process. The work done by the gas is

(a) $\frac{nR(T_1 - T_2)}{\gamma - 1}$ (b) $\frac{p_1V_1 - p_2V_2}{\gamma - 1}$

(c) $\frac{p_1V_1 + p_2V_2}{\gamma + 1}$ (d) $n\gamma R(T_1 - T_2)$

51. A gas may expand either adiabatically or isothermally. A number of p–V curves are drawn for the two processes over different ranges of pressure and volume. It will be found that

(a) two adiabatic curves do not intersect

(b) two isothermal curves do not intersect

(c) an adiabatic curve and an isothermal curve may intersect

(d) the magnitude of the slope of an adiabatic curve is greater than the magnitude of the slope of an isothermal curve for the same values of pressure and volume

52. A gas expands such that its initial and final temperatures are equal. Also, the process followed by the gas traces a straight line on the *p*–*V* diagram.

(a) The temperature of the gas remains constant throughout.

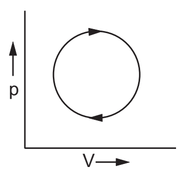
(b) The temperature of the gas first increases and then decreases.

(c) The temperature of the gas first decreases and then increases.

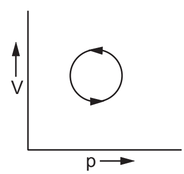
(d) The straight line has a negative slope.

53. The following are the *p*–*V* diagrams for cyclic processes for a gas. In which of these processes is heat absorbed by the gas?

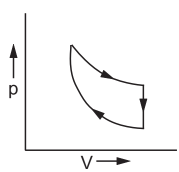
(a)



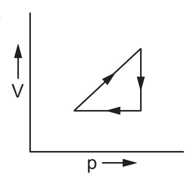
(b)



(c)



(d)



54. Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally.

(a) The final temperature is greater for the isothermal process.

(b) The final pressure is greater for the isothermal process.

(c) The work done by the gas is greater for the isothermal process.

(d) All the above options are incorrect.

55. In the previous question, if the two gases are compressed to the same final volume,

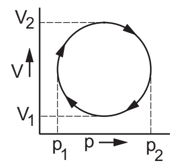
(a) the final temperature is greater for the adiabatic process

(b) the final pressure is greater for the adiabatic process

(c) the work done on the gas is greater for the adiabatic process

(d) all the above options are incorrect.

56.



In the cyclic process shown on the *V*–*p* diagram, the magnitude of the work done is

(a) $\pi\left(\frac{p_2 - p_1}{2}\right)^2$

(b) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$

(c) $\frac{\pi}{4}(p_2 - p_1)(V_2 - V_1)$

(d) $\pi(p_2V_2 - p_1V_1)$

57. In the previous question,

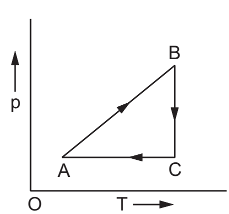
(a) work is done by the gas

(b) work is done on the gas

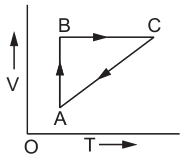
(c) heat is absorbed by the gas

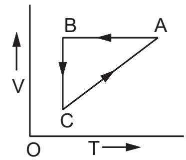
(d) heat is given out by the gas

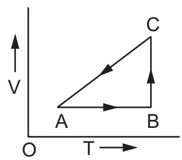
58.

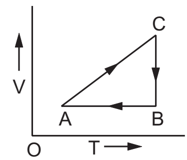


A cyclic process is shown in the *p*–*T* diagram. Which of the curves show the same process on a *V*–*T* diagram?

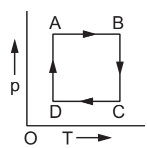
(a)


(b)


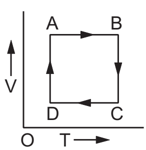
(c)


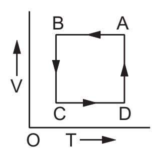
(d)


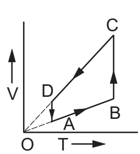
59.

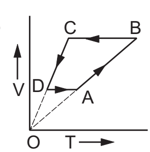


A cyclic process is shown on the *p*–*T* diagram. Which of the curves show the same process on a *V*–*T* diagram?

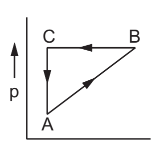
(a)


(b)


(c)


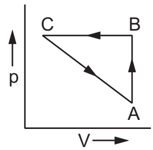
(d)


60.

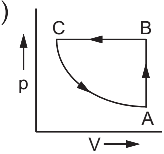


A cyclic process is shown on the *p*–*T* diagram. Which of the curves show the same process on a *p*–*V* diagram?

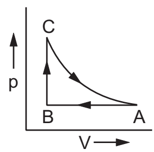
(a)



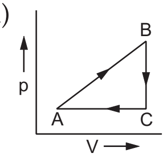
(b)



(c)



(d)



61. The first law of thermodynamics incorporates the concepts of

(a) conservation of energy
(b) conservation of heat
(c) conservation of work
(d) equivalence of heat and work

62. The internal energy of a system remains constant when it undergoes

(a) a cyclic process
(b) an isothermal process
(c) an adiabatic process
(d) any process in which the heat given out by the system is equal to the work done on the system

63. For an ideal gas,

(a) the change in internal energy in a constant-pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume and n is the number of moles of the gas
(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
(c) the internal energy does not change in an isothermal process
(d) no heat is added or removed in an adiabatic process

64. The molar heat capacity for an ideal gas

(a) cannot be negative

(b) must be equal to either C_V or C_p

(c) must lie in the range $C_V \leq C \leq C_p$

(d) may have any value between $-\infty$ and $+\infty$

65. The molar heat capacity for an ideal gas

(a) is zero for an adiabatic process

(b) is infinite for an isothermal process

(c) depends only on the nature of the gas for a process in which either volume or pressure is constant

(d) is equal to the product of the molecular weight and specific heat capacity for any process

66. c_p is always greater than c_V for a gas. Which of the following statements provide, party or wholly, the reason for this?

(a) No work is done by a gas at constant volume.

(b) When a gas absorbs heat at constant pressure, its volume must change.

(c) For the same change in temperature, the internal energy of a gas changes by a smaller amount at constant volume than at constant pressure.

(d) The internal energy of an ideal gas is a function only of its temperature.

67. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and the work done is W.

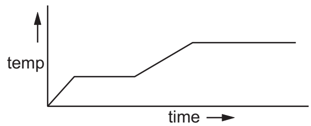
(a) $W = Q_1 - Q_2$

(b) $\eta = \dfrac{W}{Q_1}$

(c) $\eta = \dfrac{Q_2}{Q_1}$

(d) $\eta = 1 - \dfrac{Q_2}{Q_1}$

68.



Heat is supplied to a certain homogenous sample of matter, at a uniform rate. Its temperature is plotted against time, as shown. Which of the following conclusions can be drawn?

(a) Its specific heat capacity is greater in the solid state than in the liquid state.

(b) Its specific heat capacity is greater in the liquid state than in the solid state.

(c) Its latent heat of vaporization is greater than its latent heat of fusion.

(d) Its latent heat of vaporization is smaller than its latent heat of fusion.

69. The two ends of a uniform rod of thermal conductivity k are maintained at different but constant temperatures. The temperature gradient at any point on the rod is $\frac{d\theta}{dl}$ (equal to the difference in temperature per unit length). The heat flow per unit time per unit cross-section of the rod is I.

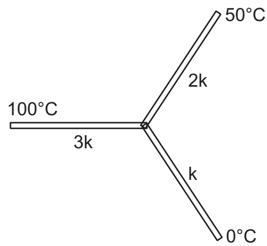
(a) $\frac{d\theta}{dl}$ is the same for all points on the rod.

(b) I will decrease as we move from higher to lower temperature.

(c) $I = k \cdot \frac{d\theta}{dl}$

(d) All the above options are incorrect.

70.

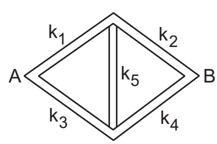


Three rods of the same dimensions have thermal conductivities $3k$, $2k$ and k. They are arranged as shown, with their ends at 100°C, 50°C and 0°C. The temperature of their junction is

(a) 75°C (b) $\frac{200}{3}$ °C

(c) 40°C (d) $\frac{100}{3}$ °C

71.



Five rods of the same dimensions are arranged as shown. They have thermal conductivities k_1, k_2, k_3, k_4 and k_5. When points A and B are maintained at different temperatures, no heat flows through the central rod. It follows that

(a) $k_1 = k_4$ and $k_2 = k_3$ (b) $k_1/k_4 = k_2/k_3$

(c) $k_1k_4 = k_2k_3$ (d) $k_1k_2 = k_3k_4$

72. Three rods A, B and C have the same dimensions. Their thermal conductivities are k_A, k_B and k_C respectively. A and B are placed end to end, with their free ends kept at a certain temperature difference. C is placed separately, with its ends kept at the same temperature difference. The two arrangements conduct heat at the same rate. k_C must be equal to

(a) $k_A + k_B$ (b) $\frac{k_A k_B}{k_A + k_B}$

(c) $\frac{1}{2}(k_A + k_B)$ (d) $2 \cdot \left(\frac{k_A k_B}{k_A + k_B}\right)$

73. The three rods described in the previous question are placed individually, with their ends kept at the same temperature difference. The rate of heat flow through C is equal to the rate of combined heat flow through A and B. k_C must be equal to

(a) $k_A + k_B$ (b) $\frac{k_A k_B}{k_A + k_B}$

(c) $\frac{1}{2}(k_A + k_B)$ (d) $2\left(\frac{k_A k_B}{k_A + k_B}\right)$

74. One end of a uniform rod of length 1 m is placed in boiling water while its other end is placed in melting ice. A point P on the rod is maintained at a constant temperature of 800°C. The mass of steam produced per second is equal to the mass of ice melted per second. If specific latent heat of steam is 7 times the specific latent heat of ice, the distance of P from the steam chamber must be

(a) $\frac{1}{7}$ m (b) $\frac{1}{8}$ m

(c) $\frac{1}{9}$ m (d) $\frac{1}{10}$ m

75. A and B are two points on a uniform metal ring whose centre is C. The angle ACB = θ. A and B are maintained at two different constant temperatures. When $\theta = 180°$, the rate of total heat flow from A to B is 1.2 W. When $\theta = 90°$, this rate will be

(a) 0.6 W (b) 0.9 W

(c) 1.6 W (d) 1.8 W

76. In a 10-metre-deep lake, the bottom is at a constant temperature of 4°C. The air temperature is constant at –4°C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be

(a) 7.5 m (b) 6 m

(c) 5 m (d) 2.5 m

77. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity k. If the temperature difference between the outer and inner surfaces of the shell is not to exceed T, the thickness of the shell should not be less than

(a) $\frac{4\pi k R^2 T}{P}$ (b) $\frac{4\pi k R^2}{TP}$

(c) $\frac{4\pi R^2 T}{kP}$ (d) $\frac{4\pi R^2 P}{kT}$

78. A spherical black body of radius r radiates power P, and its rate of cooling is R.

(a) $P \propto r$ (b) $P \propto r^2$

(c) $R \propto r^2$ (d) $R \propto \frac{1}{r}$

79. The temperature of an isolated black body falls from T_1 to T_2 in time t. Let c be a constant.

(a) $t = c\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$ (b) $t = c\left[\frac{1}{T_2^2} - \frac{1}{T_1^2}\right]$

(c) $t = c\left[\frac{1}{T_2^3} - \frac{1}{T_1^3}\right]$ (d) $t = c\left[\frac{1}{T_2^4} - \frac{1}{T_1^4}\right]$

80. A planet is at an average distance d from the sun, and its average surface temperature is T. Assume that the planet receives energy only from the sun, and loses energy only through radiation from its surface. Neglect atmospheric effects. If $T \propto d^{-n}$, the value of n is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

81. The solar constant for the earth is Σ. The surface temperature of the sun is T K. The sun subtends an angle θ at the earth.

(a) $\Sigma \propto T^4$ (b) $\Sigma \propto T^2$

(c) $\Sigma \propto \theta^2$ (d) $\Sigma \propto \theta$

82. The power radiated by a black body is P, and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around a wavelength $3\lambda_0/4$, the power radiated by it will increase by a factor of

(a) 4/3 (b) 16/9

(c) 64/27 (d) 256/81

83. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

(a) the temperature of B is 1934 K

(b) $\lambda_B = 1{\cdot}5$ μm

(c) the temperature of B is 11604 K

(d) the temperature of B is 2901 K

84. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant is $b = 2.88 \times 10^6$ nm K.

(a) $U_1 = 0$ (b) $U_3 = 0$

(c) $U_1 > U_2$ (d) $U_2 > U_1$

85. A body with an initial temperature θ_i is allowed to cool in a surrounding which is at a constant temperature of θ_0 $(\theta_0 < \theta_i)$. Assume that Newton's law of cooling is obeyed. Let k = constant. The temperature of the body after time t is best expressed by

(a) $(\theta_i - \theta_0)\, e^{-kt}$ (b) $(\theta_i - \theta_0) \ln (kt)$

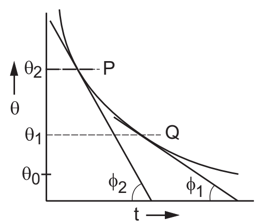
(c) $\theta_0 + (\theta_i - \theta_0)\, e^{-kt}$ (d) $\theta_i\, e^{-kt} - \theta_0$

86. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

(a) 100 J/°C (b) 300 J/°C

(c) 750 J/°C (d) 1500 J/°C

87.



A body cools in a surrounding which is at a constant temperature of θ_0. Assume that it obeys Newton's law of cooling. Its temperature θ is plotted against time *t*. Tangents are drawn to the curve at the points $P(\theta = \theta_1)$ and $Q(\theta = \theta_2)$. These tangents meet the time axis at angles of ϕ_2 and ϕ_1, as shown.

(a) $\dfrac{\tan \phi_2}{\tan \phi_1} = \dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (b) $\dfrac{\tan \phi_2}{\tan \phi_1} = \dfrac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

(c) $\dfrac{\tan \phi_1}{\tan \phi_2} = \dfrac{\theta_1}{\theta_2}$ (d) $\dfrac{\tan \phi_1}{\tan \phi_2} = \dfrac{\theta_2}{\theta_1}$

Answers

1. b	**2.** a	**3.** c	**4.** a	**5.** c
6. d	**7.** b	**8.** a	**9.** a	**10.** c
11. b	**12.** c	**13.** c	**14.** d	**15.** b
16. c	**17.** b	**18.** b	**19.** c	**20.** b
21. b	**22.** b	**23.** c	**24.** c	**25.** a
26. b	**27.** b	**28.** c	**29.** d	**30.** d
31. b	**32.** d	**33.** c	**34.** a	**35.** c
36. b	**37.** b	**38.** b	**39.** c	**40.** d
41. b	**42.** c	**43.** b	**44.** a, c, d	**45.** d
46. c, d	**47.** c	**48.** c	**49.** c	**50.** a, b
51. a, b, c, d	**52.** b, d	**53.** a, b, c	**54.** a, b, c	**55.** a, b, c
56. c	**57.** b, d	**58.** c	**59.** c	**60.** b
61. a, d	**62.** a, b, d	**63.** a, b, c, d	**64.** d	**65.** a, b, c, d
66. a, b	**67.** a, b, d	**68.** b, c	**69.** a, c	**70.** b
71. c	**72.** b	**73.** a	**74.** c	**75.** c
76. a	**77.** a	**78.** b, d	**79.** c	**80.** c
81. a, c	**82.** d	**83.** a, b	**84**. d	**85.** c
86. d	**87.** b			

Hints and Solutions to Selected Questions

1, 2. TIP When a body is heated, the distance between *any two points* on it *increases*. The increase is in the same ratio for any set of two points.

Choose diametrically opposite points across each hole in Q. No. 1, and points on the edges of the holes in Q. No. 2, and apply the above principle.

3. Let α = coefficient of thermal expansion, Y = Young modulus of the wire. If the wire were free to contract, its decrease in length would be $l\alpha t$, where t = decrease in temperature. To maintain constant length, $l\alpha t$ becomes the effective elongation.

$$\therefore \quad \text{strain} = \frac{l\alpha t}{l} = \alpha t.$$

Let T = tension $\qquad \therefore \quad \text{stress} = \dfrac{T}{A}\cdot$

Using $Y = \dfrac{\text{stress}}{\text{strain}}$, $\quad T = YA\alpha t.$

4. Tension must be the same in both the rods for their junction to be in equilibrium. Using the result of Q. No. 3,

$Y_1 A\alpha_1 t = Y_2 A\alpha_2 t.$

5. $DC^2 = l^2 - (l/2)^2 = [l(1+\alpha_2 t)]^2 - [l/2(1+\alpha_1 t)]^2.$

Neglect α^2 terms.

6. $I = \Sigma mr^2$

$I + \Delta I = \Sigma m[r(1+\alpha\,\Delta t)]^2$

or $\quad I + \Delta I = \Sigma[mr^2(1+2\alpha\,\Delta t)] = I(1+2\alpha\,\Delta t)$

or $\quad \Delta I/I = 2\alpha\,\Delta t.$

7. Let A_0 and A_t be the areas of cross-section of the tube at temperatures 0 and t respectively,

l = length of the liquid column (constant)

V_0 and V_t be the volumes of the liquid at temperatures 0 and t respectively,

$$V_0 = lA_0 \qquad V_t = lA_t$$

$$V_t = V_0(1+\gamma t) \qquad A_t = A_0(1+2\alpha t)$$

$$\therefore \quad V_t = lA_0(1+2\alpha t) = V_0(1+\gamma t) = lA_0(1+\gamma t)$$

or $\gamma = 2\alpha$.

8. Let ρ_0, ρ_1 and ρ_2 be the densities of the liquid at temperatures 0, t_1 and t_2 respectively

To balance pressure, $\rho_1 l_1 g = \rho_2 l_2 g$

or $$\left(\frac{\rho_0}{1+\gamma t_1}\right) l_1 = \left(\frac{\rho_0}{1+\gamma t_2}\right) l_2.$$

11. TIP Pressure $p = nkT$, where n = number of molecules per unit volume

$$n = \frac{p}{kT} = \frac{4 \times 10^{-10}}{1.38 \times 10^{-23} \times 300} \cong 10^{11} \text{ per m}^3 = 10^5/\text{cm}^3.$$

12. $c = \sqrt{\dfrac{3RT}{M}}$

Here, T becomes double and M becomes half.

17. TIP For a closed system, the total mass of gas or the number of moles remains constant.

$$p_1 V = n_1 RT_1, \quad p_2 V = n_2 RT_2, \quad p(2V) = (n_1 + n_2)RT$$

18, 19. See the hint to Q. No. 17.

20. The pressure of the gas remains constant, and is equal to the atmospheric pressure (for equilibrium of the 'gas plus pistons' system). If the temperature of the gas is increased, its volume must increase. For this, the pistons must move to the right.

21. At constant pressure, work = pressure × change in volume

22. $p^{1-\gamma}T^{\gamma} = \text{constant}$

or $\quad p \propto T^{[\gamma/(\gamma-1)]}$

Here, $p \propto T^3$

or $\quad \dfrac{\gamma}{\gamma - 1} = 3 \quad$ or $\quad \gamma = \dfrac{3}{2}$.

25. The average number of degrees of freedom per molecule,

$$f = \frac{\text{total number of degrees of freedom}}{\text{total number of molecules}} = \frac{n_1N_Af_1 + n_2N_Af_2}{n_1N_A + n_2N_A},$$

where N_A = Avogadro constant.

Here, $f_1 = 3,\ f_2 = 5$

$$\therefore \quad f = \frac{n_1f_1 + n_2f_2}{n_1 + n_2} = \frac{3n_1 + 5n_2}{n_1 + n_2}$$

Also, $\gamma = 1 + \dfrac{2}{f} = 1.5 \quad$ or $\quad f = 4$

$$\therefore \quad \frac{3n_1 + 5n_2}{n_1 + n_2} = 4.$$

26. $pV = nRT \qquad$ or $\quad p = \left(\dfrac{nR}{V}\right)T$

For a p–T graph, slope $= \dfrac{nR}{V} \propto \dfrac{1}{V}$.

27. AB → constant p, increasing V; $\quad \therefore$ increasing T

BC → constant T, increasing V, decreasing p

CD → constant V, decreasing p; $\quad \therefore$ decreasing T

DA → constant T, decreasing V, increasing p

Also, BC is at a higher temperature than AD.

28. Specific heat capacity $= \dfrac{\text{molar heat capacity}}{\text{molecular weight}}$.

30. $\Delta Q = nC_p\,\Delta T$ at constant pressure

$\Delta U = nC_V\,\Delta T$

Fraction $= \dfrac{\Delta U}{\Delta Q} = \dfrac{C_V}{C_p} = \dfrac{1}{\gamma} = \dfrac{5}{7}$ for diatomic gas.

31. $\gamma + 1 + \dfrac{2}{f} = 1 + \dfrac{2}{6} = \dfrac{4}{3}$

$$\frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q} = 1 - \frac{n\,C_V\,\Delta T}{n\,C_p\,\Delta T} = 1 - \frac{1}{\gamma} = 1 - \frac{3}{4} = \frac{1}{4}.$$

$\therefore \quad \Delta Q = 4\Delta W = 100$ J.

32. $\Delta T_{\text{A}} = \dfrac{\Delta Q}{C_p} \qquad \Delta T_{\text{B}} = \dfrac{\Delta Q}{C_V}$

$\therefore \quad \dfrac{\Delta T_{\text{B}}}{\Delta T_{\text{A}}} = \dfrac{C_p}{C_V} = \dfrac{7}{5}$ for diatomic gas

$\Delta T_{\text{B}} = \dfrac{7}{5} \times 30\text{ K} = 42\text{ K}.$

34. TIP When systems of masses $m_1, m_2, \ldots,$ specific heat capacities $s_1, s_2, \ldots$ and initial temperatures $\theta_1, \theta_2, \ldots$ are mixed, the temperature of the mixture is

$$\theta = \frac{\Sigma m s\theta}{\Sigma m s} = \frac{m_1 s_1 \theta_1 + m_2 s_2 \theta_2 + \ldots}{m_1 s_1 + m_2 s_2 + \ldots}.$$

For systems of equal mass, $\theta = \dfrac{\Sigma s\theta}{\Sigma s}$.

Let s_1, s_2 and s_3 be the specific heat capacities of A, B and C respectively.

For A + B, $15 = \dfrac{10s_1 + 25s_2}{s_1 + s_2} \qquad$ or $\quad 5s_1 = 10s_2 \qquad$ or $\quad s_1 = 2s_2$.

For B + C, $30 = \dfrac{25s_2 + 40s_3}{s_2 + s_3}$ or $5s_2 = 10s_3$ or $s_3 = 0.5s_2$.

For A + C, $\theta = \dfrac{10s_1 + 40s_3}{s_1 + s_3} = \dfrac{10(2s_2) + 40(0.5)s_2}{2s_2 + 0.5s_2} = \dfrac{40}{2.5} = 16$.

35. Let L_s = specific latent heat of vaporization,

L_I = specific latent heat of freezing.

Given, $L_s = \eta L_I$.

Let m = initial mass of water, f = fraction of water frozen.

Mass of vapour formed = $m(1-f)$, mass of ice formed = m_f.

$\therefore \quad m(1-f)L_s = mfL_I$

or $\quad \dfrac{1-f}{f} = \dfrac{L_I}{L_s} = \dfrac{1}{\eta}$

or $\quad \eta - \eta f = f$

or $\quad f = \dfrac{\eta}{\eta + 1}$.

37. For thermal equilibrium in the molten state, the rate of absorbing heat = P = the rate of losing heat due to radiation, etc.

When the power source is turned off, the system continues to lose heat at the same rate, as its temperature does not change.

Therefore, heat lost in time $t = Pt$ = required heat loss for complete solidification $= ML$, where L = specific latent heat of fusion.

38. Problems like this are best solved by using their electrical analogues.

TIP For a rod of length l, area of cross-section A and thermal conductivity k, we define the *thermal resistance* as

$$R = \frac{l}{kA}.$$

The given situation is like two resistances in series. We define the heat current $I = \dfrac{\theta_1 - \theta_2}{R}$.

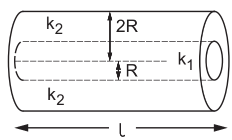
As the resistances are in series, they carry the same current. Let θ be the temperature of their junction.

$$I = \frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2}, \quad \text{where } R_1 = \frac{l}{k_1 A} \text{ and } R_2 = \frac{l}{k_2 A}$$

or $$\frac{\theta_1 - \theta}{\theta - \theta_2} = \frac{R_1}{R_2} = \frac{k_2}{k_1}.$$

Solve for θ.

39.



See the hint to Q. No. 38. For the inner and outer cylinders, thermal resistances are

$$R_1 = \frac{l}{k_1(\pi R^2)} \quad \text{and} \quad R_2 = \frac{l}{k_2(4\pi R^2 - \pi R^2)}.$$

For an equivalent conductor of length l, radius $2R$ and thermal conductivity k, thermal resistance $= \dfrac{l}{k(4\pi R^2)} = \dfrac{R_1 R_2}{R_1 + R_2}$, as the inner and outer cylinders are effectively in parallel between the same temperature difference.

40, 41. For spherical black body of radius r and absolute temperature T, the power radiated $= (4\pi r^2)(\sigma T^4)$.

42, 43. If a body cools from temperature θ_1 to θ_2 in time t, when the surrounding temperature is θ_0, by Newton's law of cooling,

$$\frac{\theta_1 - \theta_2}{t} = c\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right],$$

where c = constant.

44. See the hint to Q. Nos. 1 and 2.

45. Average translational energy of a molecule $= \frac{3}{2} kT \propto T$, and

rms speed $= \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$.

46. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ $\qquad \bar{v} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{2.55 \frac{RT}{M}}$ $\qquad v_{\text{p}} = \sqrt{\frac{2RT}{M}}$

47. Let p_{A}, p_{B} be the initial pressures in A and B respectively. When the gases double their volumes at constant temperature, their pressures fall to $\frac{p_{\text{A}}}{2}$ and $\frac{p_{\text{B}}}{2}$

$\therefore$ for A, $p_{\text{A}} - \frac{p_{\text{A}}}{2} = \Delta p$ or $p_{\text{A}} = 2\Delta p$

for B, $p_{\text{B}} - \frac{p_{\text{B}}}{2} = 1.5\Delta p$ or $p_{\text{B}} = 3\Delta p$

$\therefore \quad \frac{p_{\text{A}}}{p_{\text{B}}} = \frac{2}{3}$.

Also, $p_{\text{A}}V = \frac{m_{\text{A}}}{M}RT$ and $p_{\text{B}}V = \frac{m_{\text{B}}}{M}RT$.

$\therefore \quad \frac{p_{\text{A}}}{p_{\text{B}}} = \frac{m_{\text{A}}}{m_{\text{B}}}$

$\therefore \quad \frac{m_{\text{A}}}{m_{\text{B}}} = \frac{2}{3}$ or $3m_{\text{A}} = 2m_{\text{B}}$.

48. $Vp^n = (V + \Delta V)(p + \Delta p)^n = Vp^n\left(1 + \frac{\Delta V}{V}\right)\left(1 + n \cdot \frac{\Delta p}{p}\right)$

or $\frac{\Delta V}{V} = -n \cdot \frac{\Delta p}{p}$

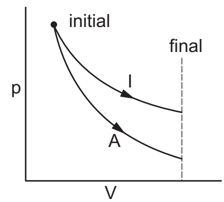
Now, $k = -\frac{\Delta p}{(\Delta V/V)} = \frac{p}{n}$.

53. In a cyclic process, heat is absorbed by a gas when the work done by it is positive, i.e., when work is done *by* the gas.

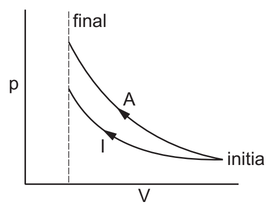
TIP

Work is done by the gas if the closed curve in a p–V diagram is clockwise, with p on the y-axis and V on the x-axis. If the axes for p and V are interchanged, work done is positive for an anticlockwise curve.

54.



55.



56. For a cyclic process plotted on the p–V diagram, the work done = area inside the closed curve. Treat the circle as an ellipse of semimajor axis $\frac{1}{2}(p_2 - p_1)$ and semiminor axis $\frac{1}{2}(V_2 - V_1)$.

58, 59, 60. See the hint to Q. No. 27.

Identify each of the processes AB, BC, CA, etc., in the given cyclic process. Then compare with the given curves.

64, 65. The molar heat capacity has the general definition

$$C = \frac{1}{n} \cdot \frac{\Delta Q}{\Delta T},$$

where n = number of moles, ΔQ = heat absorbed by the gas and ΔT = rise in temperature of gas.

It is possible to obtain almost any set of values for ΔQ and ΔT by proper selection of a process.

68. The horizontal parts of the curve, where the system absorbs heat at constant temperature, must depict changes of state. Here the latent heats are proportional to the lengths of the horizontal parts. In the sloping parts, specific heat capacity is inversely proportional to the slopes.

70. See the hint to Q. No. 38. Let θ = temperature of junction.

$$R_1 = \frac{l}{3kA}, \quad R_2 = \frac{l}{2kA}, \quad R_3 = \frac{l}{kA}.$$

Use Kirchhoff's first law to distribute current at the junction.

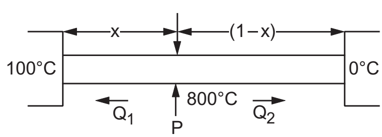
$$100 - \theta = R_1 I_1, \qquad \theta - 50 = R_2 I_2, \qquad \theta - 0 = R_3(I_1 - I_2)$$

71. This is analogous to a balanced Wheatstone bridge.

$R_1 = \frac{l}{k_1 A}$, etc., and $R_1 R_4 = R_2 R_3$ for balance.

72, 73. Use electrical analogues of resistances in series and parallel to find the thermal resistances.

74.



Let Q_1 and Q_2 amounts of heat flow from P in any time t. Let m be the masses of steam formed and ice melted in time t. Let k and A be the thermal conductivity and the area of cross-section respectively of the rod.

$$Q_1 = kA\left(\frac{800 - 100}{x}\right)t = mL_{\text{steam}}$$

$$Q_2 = kA\left(\frac{800 - 0}{1 - x}\right)t = mL_{\text{ice}}$$

Dividing, $\left(\frac{700}{x}\right)\left(\frac{1 - x}{800}\right) = 7$

or $\quad 1 - x = 8x \qquad$ or $\quad x = \frac{1}{9}$ m.

75. Let R = total thermal resistance of the ring,

ΔT = difference in temperature between A and B.

For $\theta = 180°$, two sections of resistance $R/2$ each are in parallel.

Equivalent resistance = $R/4$.

Rate of total heat flow $= I_1 = 1.2 = \dfrac{\Delta T}{R/4}$

or $\quad 0.3 = \dfrac{\Delta T}{R}$

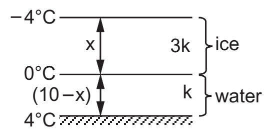
For $\theta = 90°$, two sections of resistances $R/4$ and $3R/4$ are in parallel.

Equivalent resistance $= \dfrac{(R/4)(3R/4)}{R/4 + 3R/4} = \dfrac{3R}{16}$.

Rate of total heat flow

$$I_2 = \frac{\Delta T}{3R/16}\,\text{W} = \frac{16}{3}\left(\frac{\Delta T}{R}\right)\text{W} = \frac{16}{3} \times 0.3\,\text{W} = 1.6\ \text{W}.$$

76. The rate of heat flow is the same through water and ice in the steady state.



$$I = kA\frac{4-0}{10-x} = 3kA\frac{0-(-4)}{x}$$

$$x = (10-x)\cdot 3$$

or $\quad x = 7.5$ m.

77. Area of spherical shell $= 4\pi R^2$.

Rate of heat flow $= P = k(4\pi R^2)\dfrac{T}{d}$, where d = thickness of shell.

78, 79. $P = (4\pi r^2)(\sigma T^4) = ms\left(-\dfrac{dT}{dt}\right) = -\dfrac{4}{3}\pi r^3 \rho s \cdot \dfrac{dT}{dt}$

Here, $R = \dfrac{dT}{dt}$.

$\therefore \quad P \propto r^2$ and $R \propto \dfrac{1}{r}\cdot$

$$\int_{T_1}^{T_2} -\frac{dT}{T^4} = (\text{constant}) \int_0^t dt$$

or $\quad c\left[\dfrac{1}{T_2^3} - \dfrac{1}{T_1^3}\right] = t.$

80. Let P = power radiated by the sun, R = radius of planet.

Energy received by planet $= \dfrac{P}{4\pi d^2} \times \pi R^2.$

Energy radiated by planet $= (4\pi R^2)\,\sigma T^4.$

For thermal equilibrium, $\dfrac{P}{4\pi d^2} \times \pi R^2 = 4\pi R^2 \sigma T^4.$

or $\quad T^4 \propto \dfrac{1}{d^2}$

or $\quad T \propto \dfrac{1}{d^{1/2}}$ $\qquad$ or $\quad T \propto d^{-1/2}.$

81. Let R = radius of the sun, d = distance of the earth from the sun.
Power radiated by the sun $= (4\pi R^2)\,\sigma T^4 = P.$
Energy received per unit area per second normally on the earth

$$= \Sigma = \frac{P}{4\pi d^2} = \frac{4\pi R^2 \sigma T^4}{4\pi d^2} = (\sigma T^4)\left(\frac{R}{d}\right)^2 = \frac{1}{4}\,\sigma T^4 \left(\frac{2R}{d}\right)^2$$

Angle subtended by the sun at the earth $= \theta = \dfrac{2R}{d}\cdot$

or $\quad \Sigma = \text{constant} \times T^4 \times \theta^2.$

82. Let T_0 = initial temperature of the black body.

$\therefore \quad \lambda_0 T_0 = b$ (constant)

Power radiated $= P_0 = c \cdot T_0^4 \qquad (c = \text{constant})$

Let T = new temperature of black body.

$\therefore \quad \dfrac{3\lambda_0}{4} T = b = \lambda_0 T_0$

or $\quad T = \dfrac{4T_0}{3}\cdot$

Power radiated $= c \cdot T^4 = (c\,T_0^4)\left(\dfrac{4}{3}\right)^4 = P_0\left(\dfrac{256}{81}\right)\cdot$

83. Let P and A be the power radiated and the surface area of both bodies respectively.

Let T_A and T_B be the absolute temperatures of A and B respectively.

$$P = 0.01A\sigma\, T_A^4 = 0.81A\sigma\, T_B^4$$

or $\quad T_A = 3T_B \qquad$ or $\quad T_B = \dfrac{T_A}{3} = \dfrac{5802\text{ K}}{3} = 1934\text{ K}.$

Also, $\lambda_A T_A = \lambda_B T_B$

or $\quad 3\lambda_A T_B = \lambda_B T_B \qquad$ or $\quad 3\lambda_A = \lambda_B.$

Given, $\lambda_B - \lambda_A = 1\ \mu\text{m} \qquad$ or $\quad 2\lambda_A = 1\ \mu\text{m}$

or $\quad \lambda_B = 3\lambda_A = 3\left(\dfrac{1}{2}\ \mu\text{m}\right) = 1.5\ \mu\text{m}.$

84. $\lambda_m T = b = 2.88 \times 10^6$ nm K $\qquad$ or $\quad \lambda_m = \dfrac{2.88 \times 10^6\text{ nm K}}{2880\text{ K}} = 1000\text{ nm}.$

The black body radiates maximum energy around λ_m.

$\therefore \quad U_2$ is greater than U_1 or U_3.

Also, energy is radiated at all wavelengths.

$\therefore \quad U_1, U_3 \neq 0$

85. $\dfrac{d\theta}{dt} = -k(\theta - \theta_0), \qquad$ where $k =$ constant.

$\therefore \quad \displaystyle\int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -\int_0^t k \cdot dt$

or $\quad [\ln(\theta - \theta_0)]_{\theta_i}^{\theta} = -kt$

or $\quad \ln(\theta - \theta_0) - \ln(\theta_i - \theta_0) = -kt$

or $\quad \dfrac{\theta - \theta_0}{\theta_i - \theta_0} = e^{-kt}$

or $\quad \theta = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}.$

86. Rate of loss of heat ∝ difference in temperature with the surroundings.

At 50°C, $\dfrac{dQ}{dt} = k(50 - 20) = 10,$ where k = constant

$\therefore \quad k = \dfrac{1}{3}$

At an average temperature of 35°C, $\dfrac{dQ}{dt} = \dfrac{1}{3}(35 - 20)$ J/s = 5 J/s.

Heat lost in 1 minute = $\dfrac{dQ}{dt} \times 60\text{ J} = 5 \times 60\text{ J} = 300\text{ J} = Q.$

Fall in temperature = 0.2°C = $\Delta\theta$.

$$Q = c\Delta\theta.$$

Heat capacity = $c = \dfrac{Q}{\Delta\theta} = \dfrac{300\text{ J}}{0.2°\text{C}} = 1500$ J/°C.

87. For θ–t plot, rate of cooling = $\dfrac{d\theta}{dt}$ = slope of the curve.

At P, $\dfrac{d\theta}{dt} = |\tan(180 - \phi_2)| = \tan\phi_2 = k(\theta_2 - \theta_0),$

where k = constant.

At Q, $\dfrac{d\theta}{dt} = |\tan(180 - \phi_1)| = \tan\phi_1 = k(\theta_1 - \theta_0).$

$\therefore \quad \dfrac{\tan\phi_2}{\tan\phi_1} = \dfrac{\theta_2 - \theta_0}{\theta_1 - \theta_0}.$

□

Part 3

Sound

1

Sound

• Type 1 •

Choose the correct option (a, b, c or d).

1. Two waves travelling in a medium in the x-direction are represented by $y_1 = A\sin(\alpha t - \beta x)$ and $y_2 = A\cos(\beta x + \alpha t - \pi/4)$, where y_1 and y_2 are the displacements of the particles of the medium, t is time, and α and β are constants. The two waves have different

(a) speeds
(b) directions of propagation
(c) wavelengths
(d) frequencies

2. A sine wave has an amplitude A and a wavelength λ. Let V be the wave velocity, and v be maximum velocity of a particle in the medium.

(a) V cannot be equal to v
(b) $V = v$, if $A = \lambda/2\pi$
(c) $V = v$, if $A = 2\pi\lambda$
(d) $V = v$, if $\lambda = A/\pi$

3. The equation $y = a\cos^2(2\pi nt - 2\pi x/\lambda)$ represents a wave with

(a) amplitude a, frequency n and wavelength λ
(b) amplitude a, frequency $2n$ and wavelength 2λ
(c) amplitude $a/2$, frequency $2n$ and wavelength λ
(d) amplitude $a/2$, frequency $2n$ and wavelength $\lambda/2$

4. The displacement due to a wave moving in the positive x-direction is given by $y = \frac{1}{(1+x^2)}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ seconds, where x and y are in metres. The velocity of the wave in m/s is

(a) 0.5 (b) 1
(c) 2 (d) 4

5. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for the other wave is

(a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$
(c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$

6. A wave travelling in a material medium is described by the equation $y = A\sin(kx - \omega t)$. The maximum particle velocity is

(a) $A\omega$ (b) ω/k
(c) $d\omega/dk$ (d) x/t

7. A metal string is fixed between rigid supports. It is initially at negligible tension. Its Young modulus is Y, density is ρ and coefficient of thermal expansion is α. If it is now cooled through a temperature $= t$, transverse waves will move along it with speed

(a) $Y\sqrt{\alpha t/\rho}$ (b) $\alpha t\sqrt{Y/\rho}$
(c) $\sqrt{Y\alpha t/\rho}$ (d) $t\sqrt{Y\alpha/\rho}$

8. Two identical strings are stretched at tensions T_A and T_B. A tuning fork is used to set them in vibration. A vibrates in its fundamental mode and B in its second harmonic mode.

(a) $T_A = 2T_B$ (b) $T_A = 4T_B$
(c) $2T_A = T_B$ (d) $4T_A = T_B$

9. The tension of a string is increased by 44%. If its frequency of vibration is to remain unchanged, its length must be increased by

(a) 44 % (b) $\sqrt{44}$ %
(c) 22 % (d) 20 %

10. In a sonometer wire, the tension is maintained by suspending a 50.7-kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become

(a) 200 Hz (b) 220 Hz
(c) 230 Hz (d) 240 Hz

11. A string of length 0.4 m and mass 10^{-2} kg is clamped at its ends. The tension in the string is 1.6 N. When a pulse travels along the string, the shape of the string is found to be the same at times t and $t + \Delta t$. The value of Δt is

(a) 0.05 s (b) 0.1 s
(c) 0.2 s (d) 0.4 s

12. A string A has double the length, double the tension, double the diameter and double the density as another string B. Their fundamental frequencies of vibration are n_A and n_B respectively. The ratio n_A/n_B is equal to

(a) 1/4 (b) 1/2
(c) 2 (d) 4

13. The extension in a string obeying Hooke's law is x. The speed of transverse waves in the stretched string is v. If the extension in the string is increased to 1.5x, the speed of transverse waves in it will be

(a) 1.22v (b) 0.61v
(c) 1.5v (d) 0.75v

14. A cylindrical resonance tube, open at both ends, has a fundamental frequency F in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency of the air column now is

(a) 4F (b) 2F

(c) F (d) $F/2$

15. An open pipe is suddenly closed at one end, as a result of which the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

(a) 200 Hz (b) 300 Hz

(c) 240 Hz (d) 480 Hz

16. The third overtone of an open organ pipe of length l_o has the same frequency as the third overtone of a closed pipe of length l_c. The ratio l_o/l_c is equal to

(a) 2 (b) 3/2

(c) 5/3 (d) 8/7

17. A pipe of length 1 m is closed at one end. The velocity of sound in air is 300 m/s. The air column in the pipe will not resonate for sound of frequency

(a) 75 Hz (b) 225 Hz

(c) 300 Hz (d) 375 Hz

18. Two closed organ pipes, A and B, have the same length. A is wider than B. They resonate in the fundamental mode at frequencies n_A and n_B respectively.

(a) $n_A = n_B$

(b) $n_A > n_B$

(c) $n_A < n_B$

(d) Either (b) or (c) depending on the ratio of their diameters.

19. An organ pipe filled with a gas at 27°C resonates at 400 Hz in its fundamental mode. If it is filled with the same gas at 90°C, the resonance frequency will be

(a) 420 Hz (b) 440 Hz

(c) 484 Hz (d) 512 Hz

20. A point source emits sound equally in all directions in a nonabsorbing medium. Two points P and Q are at distances of 9 m and 25 m respectively from the source. The ratio of the amplitudes of the waves at P and Q is

(a) 5 : 3 (b) 3 : 5

(c) 25 : 9 (d) 625 : 81

21. A source of sound is in the shape of a long narrow cylinder radiating sound waves normal to the axis of the cylinder. Two points P and Q are at perpendicular distances of 9 m and 25 m from the axis. The ratio of the amplitudes of the waves at P and Q is

(a) 5 : 3 (b) $\sqrt{5} : \sqrt{3}$

(c) 3 : 5 (d) 25 : 9

22. Two identical sounds A and B reach a point in the same phase. The resultant sound is C. The loudness of C is n dB higher than the loudness of A. The value of n is

(a) 2 (b) 3

(c) 4 (d) 6

23. Sound of wavelength λ passes through a Quincke's tube, which is adjusted to give a maximum intensity I_0. Through what distance should the sliding tube be moved to give an intensity $I_0/2$?

(a) $\lambda/2$ (b) $\lambda/3$

(c) $\lambda/4$ (d) $\lambda/8$

24. Two sources of sound of the same frequency produce sound intensities I and $4I$ at a point P when used individually. If they are used together such that the sounds from them reach P with a phase difference of $2\pi/3$, the intensity at P will be

(a) $2I$ (b) $3I$

(c) $4I$ (d) $5I$

25. If two waves of the form $y = a\sin(\omega t - kx)$ and $y = a\cos(kx - \omega t)$ are superposed, the resultant wave will have amplitude

(a) 0 (b) a

(c) $\sqrt{2}a$ (d) $2a$

26. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If V = the velocity of sound, the velocity of the car is

(a) $V/\sqrt{2}$ (b) $V/2$

(c) $V/3$ (d) $V/4$

• *Type 2* •

Choose the correct options. One or more options may be correct.

27. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A\sin(\alpha t - \beta x)$, where t = time, and α and β are constants.

(a) The frequency of the wave is α.

(b) The frequency of the wave is $\alpha/(2\pi)$.

(c) The wavelength is $(2\pi)/\beta$.

(d) The velocity of the wave is α/β.

28. A wave is represented by the equation

$$y = A\sin(10\pi x + 15\pi t + \pi/3),$$

where x is in metres and t is in seconds. The expression represents

(a) a wave travelling in the positive x-direction with a velocity of 1.5 m/s

(b) a wave travelling in the negative x-direction with a velocity of 1.5 m/s

(c) a wave travelling in the negative x-direction with a wavelength of 0.2 m

(d) a wave travelling in the positive x-direction with a wavelength of 0.2 m

29. For a sine wave passing through a medium, let y be the displacement of a particle, v be its velocity and a be its acceleration.

(a) y, v and a are always in the same phase.

(b) y and a are always in opposite phase.

(c) Phase difference between y and v is $\pi/2$.

(d) Phase difference between v and a is $\pi/2$.

30. P, Q and R are three particles of a medium which lie on the x-axis. A sine wave of wavelength λ is travelling through the medium in the x-direction. P and Q always have the same speed, while P and R always have the same velocity. The minimum distance between

(a) P and Q is $\lambda/2$ (b) P and Q is λ

(c) P and R is $\lambda/2$ (d) P and R is λ

31. A wave is represented by the equation

$$y = A\sin 314\left[\frac{t}{0.5\text{ s}} - \frac{x}{100\text{ m}}\right].$$

The frequency is n and the wavelength is λ.

(a) $n = 2$ Hz (b) $n = 100$ Hz

(c) $\lambda = 2$ m (d) $\lambda = 100$ m

32. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero moves along the negative x-direction with a velocity of 300 m/s. A and B are two points 6 m apart on the line of propagation of the wave. At any instant the phase difference between A and B is ϕ. The maximum difference in the displacements at A and B is Δ.

(a) $\phi = \pi$ (b) $\phi = 0$

(c) $\Delta = 0$ (d) $\Delta = 5 \times 10^{-5}$m

33. A sound wave passes from a medium A to a medium B. The velocity of sound in B is greater than that in A. Assume that there is no absorption or reflection at the boundary. As the wave moves across the boundary,

(a) the frequency of sound will not change
(b) the wavelength will increase
(c) the wavelength will decrease
(d) the intensity of sound will not change

34. In a stationary wave system, all the particles of the medium
(a) have zero displacement simultaneously at some instant
(b) have maximum displacement simultaneously at some instant
(c) are at rest simultaneously at some instant
(d) reach maximum velocity simultaneously at some instant

35. In the previous question, all the particles
(a) of the medium vibrate in the same phase
(b) in the region between two antinodes vibrate in the same phase
(c) in the region between two nodes vibrate in the same phase
(d) on either side of a node vibrate in opposite phase

36. A string of length L is stretched along the x-axis and is rigidly clamped at its two ends. It undergoes transverse vibration. If n is an integer, which of the following relations may represent the shape of the string at any time t?

(a) $y = A\sin\left(\frac{n\pi x}{L}\right)\cos\omega t$ (b) $y = A\sin\left(\frac{n\pi x}{L}\right)\sin\omega t$

(c) $y = A\cos\left(\frac{n\pi x}{L}\right)\cos\omega t$ (d) $y = A\cos\left(\frac{n\pi x}{L}\right)\sin\omega t$

37. The stationary waves set up on a string have the equation $y = (2\text{ mm})\sin[(6.28\text{ m}^{-1})x]\cos(\omega t)$. This stationary wave is created by two identical waves, of amplitude A each, moving in opposite directions along the string.
(a) $A = 2$ mm
(b) $A = 1$ mm
(c) The smallest length of the string is 50 cm.
(d) The smallest length of the string is 2 m.

38. When a stretched string of length L vibrates in its fundamental mode, the sound produced has wavelength $= L/2$ in air. The velocity of sound in air is V. The velocity of the transverse waves on the string is

(a) $V/4$
(b) $V/2$
(c) 2 V
(d) 4 V

39. When a stretched string of length L is vibrating in a particular mode, the distance between two nodes on the string is l. The sound produced in this mode of vibration constitutes the nth overtone of the fundamental frequency of the string.

(a) $L = (n + 1)l$
(b) $L = (n - 1)l$
(c) $L = nl$
(d) $L = (n + \frac{1}{2})l$

40. A transverse sinusoidal wave of amplitude A, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $A = 10^{-3}$ m and $v = 10$ m/s then λ and f are given by

(a) $\lambda = 2\pi \times 10^{-2}$ m
(b) $\lambda = 10^{-3}$ m
(c) $f = 10^3/(2\pi)$ Hz
(d) $f = 10^3$ Hz

41. A heavy uniform rope hangs vertically from the ceiling, with its lower end free. A disturbance on the rope travelling upward from the lower end has a velocity v at a distance x from the lower end.

(a) $v \propto 1/x$
(b) $v \propto x$
(c) $v \propto \sqrt{x}$
(d) $v \propto 1/\sqrt{x}$

42. When an open organ pipe resonates in its fundamental mode then at the centre of the pipe,

(a) the gas molecules undergo vitrations of maximum amplitude
(b) the gas molecules are at rest
(c) the pressure of the gas is constant
(d) the pressure of the gas undergoes maximum variation

43. In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level acts as a closed organ pipe. A vibrating tuning fork is held above the open end, and the water level is gradually pushed down. The first and the second resonances occur when the water level is 24.1 cm and 74.1 cm respectively below the open end. The diameter of the tube is

(a) 2 cm (b) 3 cm

(c) 4 cm (d) 5 cm

44. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is c. The velocity of sound in the gas is

(a) $c/\sqrt{2}$ (b) $3c/4$

(c) $2c/3$ (d) $c/\sqrt{3}$

45. The velocity of sound in dry air is V_d, and in moist air it is V_m. The velocities are measured under the same conditions of temperature and pressure. Which of the following statements is fully correct?

(a) $V_d > V_m$ because dry air has lower density than moist air.

(b) $V_d < V_m$ because moist air has lower density than dry air.

(c) $V_d > V_m$ because the bulk modulus of dry air is greater than that of moist air.

(d) $V_d < V_m$ because the bulk modulus of moist air is greater than that of dry air.

46. When we hear a sound, we can identify its source from

(a) the frequency of the sound

(b) the amplitude of the sound

(c) the wavelength of the sound

(d) the overtones present in the sound

47. Sounds from two identical sources S_1 and S_2 reach a point P. When the sounds reach directly, and in the same phase, the intensity at P is I_0. The power of S_1 is now reduced by 64%, and the phase difference between S_1 and S_2 is varied continuously. The maximum and minimum intensities recorded at P are now I_{max} and I_{min}.

(a) $I_{max} = 0.64\, I_0$
(b) $I_{min} = 0.36\, I_0$
(c) $I_{max}/I_{min} = 16$
(d) $I_{max}/I_{min} = 1.64/0.36$

48. A vibrating string produces 2 beats per second when sounded with a tuning fork of frequency 256 Hz. Slightly increasing the tension in the string produces 3 beats per second. The initial frequency of the string may have been

(a) 253 Hz
(b) 254 Hz
(c) 258 Hz
(d) 259 Hz

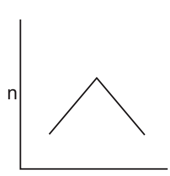
49. A whistle emitting a sound of frequency 450 Hz approaches a stationary observer at a speed of 33 m/s. Velocity of sound is 330 m/s. The frequency heard by the observer, in Hz, is

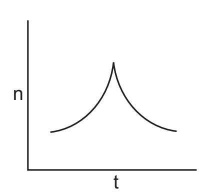
(a) 409
(b) 429
(c) 517
(d) 500

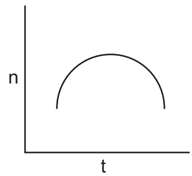
50. A bus is moving with a velocity of 5 m/s towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air = 335 m/s, the number of beats heard per second by a passenger on the bus will be

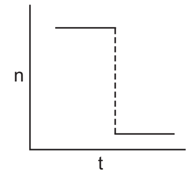
(a) 3
(b) 4
(c) 5
(d) 6

51. A railway engine whistling at a constant frequency moves with a constant speed. It goes past a stationary observer standing beside the railway track. The frequency (n) of the sound heard by the observer is plotted against time (t). Which of the following best represents the resulting curve?

(a)


(b)


(c) 

(d) 

52. Two stars P and Q have slightly different surface temperatures T_P and T_Q respectively, with $T_P > T_Q$. Both stars are receding from the earth with speeds v_P and v_Q relative to the earth. The wavelength of light at which they radiate the maximum energy is found to be the same for both.

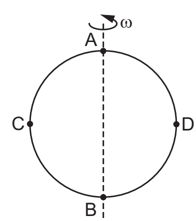
(a) $v_P > v_Q$

(b) $v_P < v_Q$

(c) $v_P = v_Q$, and the size of Q > the size of P

(d) Nothing can be said regarding v_P and v_Q from the given data.

53.



Assume that the sun rotates about an axis through its centre and perpendicular to the plane of rotation of the earth about the sun. The appearance of the sun, from any one point on the earth, is shown. Light belonging to a particular spectral line, as received from the points A, B, C and D on the edge of the sun, are analyzed.

(a) Light from all four points have the same wavelength.

(b) Light from C has greater wavelength than the light from D.

(c) Light from D has greater wavelength than the light from C.

(d) Light from A has the same wavelength as the light from B.

Answers

1. b	**2.** b	**3.** d	**4.** a	**5.** c
6. a	**7.** c	**8.** b	**9.** d	**10.** d
11. b	**12.** a	**13.** a	**14.** c	**15.** a
16. d	**17.** c	**18.** c	**19.** b	**20.** c
21. a	**22.** d	**23.** d	**24.** b	**25.** c
26. c	**27.** b, c, d	**28.** b, c	**29.** b, c, d	**30.** a, d
31. b, c	**32.** a, d	**33.** a, b, d	**34.** a, b, c, d	**35.** c, d
36. a, b	**37.** b, c	**38.** d	**39.** a	**40.** a, c
41. c	**42.** b, d	**43.** b	**44.** c	**45.** b
46. d	**47.** a, c	**48.** b, c	**49.** d	**50.** c
51. d	**52.** a	**53.** c, d		

Hints and Solutions to Selected Questions

2. $y = A\sin(\omega t - kx)$ $\qquad \omega = 2\pi n,\ k = \dfrac{2\pi}{\lambda}$, where n = frequency and λ = wavelength.

Wave velocity $= V = n\lambda = \omega/k$.

Particle velocity $= \dfrac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$.

$\therefore$ maximum particle velocity $= v = A\omega$.

For $V = v_1,\ \dfrac{\omega}{k} = A\omega$

or $A = \dfrac{1}{k} = \dfrac{\lambda}{2\pi}$.

3. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$

$$\therefore \quad y = a\cos^2\left(2\pi nt - \frac{2\pi x}{\lambda}\right) = \frac{1}{2}a\left[1 + \cos\left\{2\pi(2n)t - \frac{2\pi x}{(\lambda/2)}\right\}\right].$$

4. TIP: In a wave equation, x and t must be related in the form $(x - vt)$.

We rewrite the given equation as

$$y = \frac{1}{1 + (x - vt)^2}.$$

For $t = 0$, this becomes $y = \dfrac{1}{(1 + x^2)}$, as given.

For $t = 2$, this becomes $y = \dfrac{1}{[1 + (x - 2v)^2]} = \dfrac{1}{[1 + (x - 1)^2]}$.

$\therefore \quad 2v = 1 \qquad$ or $\quad v = 0.5$ m/s.

5. $y = a[\cos kx \cos \omega t + \sin kx \sin \omega t]$ is given.

As $y = 0$ at $x = 0$ for all values of t(node), the $\cos kx$ term must vanish in the sum of the two waves.

7. The tension in the string on cooling $= T = YA\alpha t$.

Also, mass per unit length $= m = A\rho$.

Wave velocity $= \sqrt{T/m} = \sqrt{Y\alpha t/\rho}$.

8. Let $n =$ frequency of tuning fork, $l =$ length of each string,

$m =$ mass per unit length of each string

$$n = \frac{1}{2l}\sqrt{\frac{T_A}{m}} = 2 \times \frac{1}{2l}\sqrt{\frac{T_B}{m}} .$$

10. Initially, $260 = \frac{1}{2l}\sqrt{\frac{T_1}{m}}$, $\quad T_1 = 50.7\ g = 507$ N.

When the mass is submerged, upthrust

$$= (0.0075\ m^3)(10^3\ kg/m^3)(10\ m/s^2)$$
$$= 75\ N.$$

New tension $= T_2 = (507 - 75)$ N $= 432$ N.

$$n = \frac{1}{2l}\sqrt{\frac{T_2}{m}}$$

or $\quad \frac{n}{260} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{432}{507}} = \sqrt{\frac{144}{169}} = \frac{12}{13} \qquad$ or $\quad n = 240$ Hz.

11. Wave velocity on string $= V = \sqrt{\frac{1.6}{(10^{-2}/0.4)}} = 8$ m/s.

For the string to regain its shape, the pulse must travel a distance which is twice the length of the string, as the pulse gets inverted at each reflection from a fixed end.

$\therefore \quad t = \frac{2 \times 0.4\ m}{8\ m/s} = 0.1$ s.

13. Tension $\propto$ extension, and speed $\propto \sqrt{\text{tension}}$.

15. Fundamental frequency of an open pipe $= n_o = \frac{V}{2l}$, and

of a closed pipe of the same length $= n_c = \frac{V}{4l} = \frac{1}{2} \cdot \frac{V}{2l} = \frac{n_o}{2}$.

Here, $3n_c = n_o + 100$

or $3\frac{n_o}{2} = n_o + 100$ or $n_o = 200$ Hz.

16. Let n_o and n_c be the fundamental frequencies of the open and closed pipes.

$\therefore \quad n_o = \frac{V}{2l_o}$ and $n_c = \frac{V}{4l_c}$.

Third overtone of the closed pipe $= 7n_c$.

Third overtone of the open pipe $= 4n_o$.

17. The tube will resonate for $l = (2N+1)\lambda/4$, $N = 0, 1, 2, \ldots$

Frequency $= n = \frac{V}{\lambda} = \frac{V(2N+1)}{4l} = \frac{300(2N+1)}{4 \times 1} = 75(2N+1)$.

18. Let l = length of pipe, d = diameter of pipe.

In the fundamental mode, $\lambda/4 = l + 0.3d$, with end correction.

Frequency $= \frac{V}{\lambda} = \frac{V}{4(l+0.3d)}$.

As l is the same, the wider tube will resonate at a lower frequency.

19. $n \propto V$ and $V \propto \sqrt{T}$.

Here, $n_1 = 400$ Hz $\quad T_1 = 273 + 27 \quad T_2 = 273 + 90$

$$\frac{n_2}{n_1} = \frac{\sqrt{363}}{\sqrt{300}} = \sqrt{1.21} = 1.1.$$

20. For a point source, intensity $\propto \frac{1}{(\text{distance})^2} \propto (\text{amplitude})^2$.

21. For a cylindrical source, intensity $\propto \frac{1}{\text{distance}} \propto (\text{amplitude})^2$.

22. Let a = amplitude due to A and B, individually.

Loudness due to A $= I_A = ka^2$ $\quad (k = \text{constant})$

Loudness due to A + B $= I_C = k(2a)^2 = 4I_A$.

$$n = 10\log_{10}(I_C/I_A) = 10\log_{10}4 = 10\times 0.6 = 6.$$

23. Let a = amplitude due to each wave.

$$I_0 = k(2a)^2 = 4ka^2.$$

Let ϕ = phase difference to obtain the intensity $I_0/2$.

Amplitude $= a' = \sqrt{a^2 + a^2 + 2a^2\cos\phi} = 2a\cos(\phi/2)$.

$\therefore \quad I_0/2 = k[4a^2\cos^2(\phi/2)] = I_0\cos^2(\phi/2)$.

or $\quad \cos(\phi/2) = 1/\sqrt{2} \quad$ or $\quad \phi/2 = \pi/4$

or $\quad \phi = \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda}\cdot\Delta$, where Δ = path difference.

$\therefore \quad \Delta = \lambda/4 = 2x$, where x = displacement of the sliding tube.

$\therefore \quad x = \lambda/8$.

25. The two waves are identical, with a phase difference of $\pi/2$.

26. Let u = speed of the car, $\quad n_s$ = frequency of the horn.

Frequency received, and reflected, by the cliff $= n_1 = \dfrac{n_s}{1 - u/V}\cdot$

Frequency observed by the driver $= n_2 = n_1(1 + u/V) = 2n_s$

or $\quad \dfrac{n_s(1 + u/V)}{1 - u/V} = 2n_s$

or $\quad u = V/3$.

31. The equation has to be reduced to the form $y = A\sin 2\pi\left(\dfrac{t}{T} - \dfrac{x}{\lambda}\right)\cdot$

$$y = A\sin 314\left[\frac{t}{0.5\text{ s}} - \frac{x}{100\text{ m}}\right] = A\sin 2\pi\left[\frac{50t}{0.5\text{ s}} - \frac{x\times 50}{100\text{ m}}\right]$$

$$= A\sin 2\pi\left[\frac{t}{0.01\text{ s}} - \frac{x}{2\text{ m}}\right]$$

$\therefore \quad n = \dfrac{1}{T} = \dfrac{1}{0.01\text{ s}} = 100\text{ Hz}, \quad \lambda = 2\text{ m}.$

32. $\lambda = \dfrac{300 \text{ m/s}}{25 \text{ Hz}} = 12 \text{ m}.$

Separation between A and B = 6 m = $\lambda/2$.

36. $y = 0$ at $x = 0$. This can be satisfied by the term $\sin\left(\dfrac{n\pi x}{L}\right)$.

37. Comparing with the equation $y = 2A\sin\left(\dfrac{n\pi x}{L}\right)\cos(\omega t)$,

$2A = 2 \text{ mm}$ or $A = 1 \text{ mm}$

$\dfrac{n\pi x}{L} = 6.28x = 2\pi x$ or $L = \dfrac{n}{2} \text{ m}.$

For $n = 1, L = 0.5 \text{ m}.$

38. Frequency $= \dfrac{V}{(L/2)} = \dfrac{2V}{L} = \dfrac{1}{2L}(v),$

where v = the velocity of transverse waves on the string.

39. $l = \lambda/2$ or $\lambda = 2l$.

Let v = velocity of the transverse waves on the string.

Frequency $= \dfrac{v}{\lambda} = \dfrac{v}{2l} = (n+1)\dfrac{v}{2L}$

or $L = (n+1)l.$

40. $y = A\sin\left(2\pi ft - \dfrac{2\pi}{\lambda}x\right)$

$\left(\dfrac{\partial y}{\partial t}\right)_{\max} = 2\pi fA = \dfrac{v}{10} = \dfrac{f\lambda}{10}$

or $\lambda = 20\pi A \text{ m} = 2\pi \times 10^{-2} \text{ m}.$

$f = \dfrac{v}{\lambda} = \dfrac{10 \text{ m/s}}{2\pi \times 10^{-2} \text{ m}} = \dfrac{10^3}{2\pi} \text{ Hz}.$

41. Let m = mass per unit length of the rope,

T = tension in the rope at a distance x from the lower end.

$\therefore \quad T = mxg$

$\therefore \quad v = \sqrt{T/m} = \sqrt{mxg/m} = \sqrt{gx}$

or $\quad v \propto \sqrt{x}.$

43. Let d = the diameter of the tube.

$\lambda/4 = 24.1 + 0.3d$, and

$3\lambda/4 = 74.1 + 0.3d$

or $\quad \lambda/2 = 50$ cm $\qquad$ or $\quad \lambda = 100$ cm.

$\therefore \quad 0.3d = (\lambda/4 - 24.1)$ cm $= (25 - 24.1)$ cm $= 0.9$ cm

or $\quad d = 3$ cm.

44. $\gamma = 1 + 2/f = 1 + 2/6 = 4/3$

$c = \sqrt{3p/\rho}$ and $v = \sqrt{\gamma p/\rho}$

$\therefore \quad \dfrac{v}{c} = \sqrt{\gamma/3} = \sqrt{4/9} = 2/3.$

47. Let a = initial amplitude due to S_1 and S_2 each.

$I_0 = k(4a^2)$, where k is a constant.

After reduction of power of S_1, amplitude due to $S_1 = 0.6a$.

Due to superposition, $a_{max} = a + 0.6a = 1.6a$, and

$a_{min} = a - 0.6a = 0.4a$

$I_{max}/I_{min} = (a_{max}/a_{min})^2 = (1.6a/0.4a) = 16.$

50. See the hint to Q. No. 26.

51. TIP Doppler effect depends on velocity, not on distance.

When the engine approaches the observer with *constant velocity*, the observer hears a frequency which is constant, and higher than the actual frequency. When the engine goes past the observer and recedes from him, he hears a frequency which is constant, and lower than the actual frequency.

52. Let λ_P and λ_Q be the actual wavelengths at which P and Q radiate maximum energy.

By Wien's law, $\lambda_P T_P = \lambda_Q T_Q$.

As $T_P > T_Q$, $\lambda_P < \lambda_Q$.

As these wavelengths appear equal on reaching the earth, λ_P has increased more than λ_Q. Hence, $v_P > v_Q$.

53. From the figure, C moves towards the earth and D moves away from the earth. The light from C will decrease in wavelength and the light from D will increase in wavelength.

□

Part 4

Optics

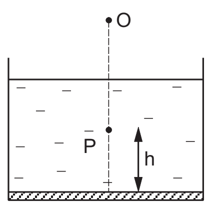
1

Optics

• *Type 1* •

Choose the correct option (a, b, c or d).

1.



A plane mirror is placed at the bottom of a tank containing a liquid of refractive index μ. P is a small object at a height h above the mirror. An observer O—vertically above P, outside the liquid—sees P and its image in the mirror. The apparent distance between these two will be

(a) $2\mu h$ (b) $\dfrac{2h}{\mu}$

(c) $\dfrac{2h}{\mu - 1}$ (d) $h\left(1 + \dfrac{1}{\mu}\right)$

2. P is a point on the axis of a concave mirror. The image of P, formed by the mirror, coincides with P. A rectangular glass slab of thickness t and refractive index μ is now introduced between P and the mirror. For the image of P to coincide with P again, the mirror must be moved

(a) towards P by $(\mu - 1)t$ (b) away from P by $(\mu - 1)t$
(c) towards P by $t(1 - 1/\mu)$ (d) away from P by $t(1 - 1/\mu)$

3. A converging lens forms a real image I on its optic axis. A rectangular glass slab of refractive index μ and thickness t is introduced between the lens and I. I will move

(a) away from the lens by $t(\mu - 1)$
(b) towards the lens by $t(\mu - 1)$
(c) away from the lens by $t(1 - 1/\mu)$
(d) towards the lens by $t(1 - 1/\mu)$

4. A ray of light incident on a slab of transparent material is partly reflected from the surface and partly refracted into the slab. The reflected and refracted rays are mutually perpendicular. The incident ray makes an angle i with the normal to the slab. The refractive index of the slab is

(a) $\tan^{-1}(i)$ (b) $\cot^{-1}(i)$
(c) $\sin^{-1}(i)$ (d) $\cos^{-1}(i)$

5. A ray of light travels from an optically denser to rarer medium. The critical angle for the two media is c. The maximum possible deviation of the ray will be

(a) $\pi - c$ (b) $\pi - 2c$
(c) $2c$ (d) $\pi/2 + c$

6. The light reflected by a plane mirror may form a real image,

(a) if the rays incident on the mirror are converging
(b) if the rays incident on the mirror are diverging
(c) if the object is placed very close to the mirror
(d) under no circumstances

7. A transparent sphere of radius R and refractive index μ is kept in air. At what distance from the surface of the sphere should a point object be placed so as to form a real image at the same distance from the sphere?

(a) R/μ (b) μR

(c) $\dfrac{R}{\mu - 1}$ (d) $\dfrac{R}{\mu + 1}$

8. An air bubble is inside water. The refractive index of water is 4/3. At what distance from the air bubble should a point object be placed so as to form a real image at the same distance from the bubble?

(a) $2R$

(b) $3R$

(c) $4R$

(d) The air bubble cannot form a real image.

9. A spherical surface of radius of curvature R separates air from glass (refractive index = 1.5). The centre of curvature is in the glass. A point object P placed in air is found to form a real image Q in the glass. The line PQ cuts the surface at a point O, and PO = OQ. The distance PO is equal to

(a) $5R$ (b) $3R$

(c) $2R$ (d) $1.5R$

10. A point source of light at the surface of a sphere causes a parallel beam of light to emerge from the opposite surface of the sphere. The refractive index of the material of the sphere is

(a) 1.5 (b) 5/3

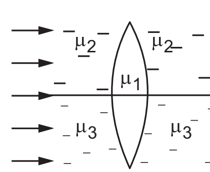
(c) 2 (d) 2.5

11. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become

(a) 30 cm (b) 45 cm

(c) 60 cm (d) 75 cm

12. A double convex lens, made of a material of refractive index μ_1, is placed inside two liquids of refractive indices μ_2 and μ_3, as shown. $\mu_2 > \mu_1 > \mu_3$. A wide, parallel beam



of light is incident on the lens from the left. The lens will give rise to

(a) a single convergent beam
(b) two different convergent beams
(c) two different divergent beams
(d) a convergent and a divergent beam

13. A convex lens of focal length 40 cm, a concave lens of focal length 40 cm and a concave lens of focal length 15 cm are placed in contact. The power of this combination in dioptres is

(a) +1.5 (b) –1.5
(c) +6.67 (d) –6.67

14. A short linear object of length *b* lies along the axis of a concave mirror of focal length *f*, at a distance *u* from the mirror. The size of the image is approximately

(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)$
(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

15. Half the surface of a transparent sphere of refractive index 2 is silvered. A narrow, parallel beam of light is incident on the unsilvered surface, symmetrically with respect to the silvered part. The light finally emerging from the sphere will be a

(a) parallel beam
(b) converging beam
(c) slightly divergent beam
(d) widely divergent beam

16. A boy of height 1 m stands in front of a convex mirror. His distance from the mirror is equal to its focal length. The height of his image is

(a) 0.25 m (b) 0.33 m
(c) 0.5 m (d) 0.67 m

17. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is

(a) 0° (b) 12.5°
(c) 15° (d) 22.5°

18. A ray of light is incident normally on one face of an equilateral prism of refractive index 1.5. The angle of deviation is

(a) 30° (b) 45°
(c) 60° (d) 75°

19. A thin prism P_1 of angle 4°, and made from a glass of refractive index 1.54, is combined with another thin prism P_2 made from a glass of refractive index 1.72, to produce dispersion without deviation. The angle of P_2 is

(a) 5.33° (b) 4°
(c) 3° (d) 2.6°

20. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm. The final image is formed at infinity. The focal lengths f_o of the objective and f_e of the eyepiece are

(a) 45 cm and –9 cm respectively
(b) 50 cm and 10 cm respectively
(c) 7.2 cm and 5 cm respectively
(d) 30 cm and 6 cm respectively

21. An astronomical telescope in normal adjustment receives light from a distant source S. The tube length is now decreased slightly.

(a) A virtual image of S will be formed at a finite distance.
(b) No image will be formed.

(c) A small, real image of S will be formed behind the eyepiece, close to it.

(d) A large, real image of S will be formed behind the eyepiece, far away from it.

22. In the previous question, if the tube length is increased slightly from its position of normal adjustment,

(a) a virtual image of S will be formed at a finite distance

(b) no image will be formed

(c) a small, real image of S will be formed behind the eyepiece, close to it

(d) a large, real image of S will be formed behind the eyepiece, far away from it

23. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is

(a) $\frac{L}{l}$ (b) $\frac{L}{l}+1$

(c) $\frac{L}{l}-1$ (d) $\frac{L+l}{L-l}$

24. In a compound microscope, maximum magnification is obtained when the final image

(a) is formed at infinity

(b) is formed at the least distance of distinct vision

(c) coincides with the object

(d) coincides with the objective lens

25. If ε_0 and μ_0 are the electric permittivity and magnetic permeability of free space respectively, and ε and μ are the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameter is

(a) $\frac{\varepsilon\mu}{\varepsilon_0\mu_0}$ (b) $\left(\frac{\varepsilon\mu}{\varepsilon_0\mu_0}\right)^{1/2}$

(c) $\left(\dfrac{\varepsilon_0\mu_0}{\varepsilon\mu}\right)$ (d) $\left(\dfrac{\varepsilon_0\mu_0}{\varepsilon\mu}\right)^{1/2}$

26. Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is

(a) $\mu \dfrac{2\pi}{\lambda} x$ (b) $\dfrac{1}{\mu} \cdot \dfrac{2\pi}{\lambda} x$

(c) $(\mu - 1) \dfrac{2\pi}{\lambda} x$ (d) $\dfrac{1}{(\mu - 1)} \dfrac{2\pi}{\lambda} x$

27. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are

(a) $5I$ and I (b) $5I$ and $3I$

(c) $9I$ and I (d) $9I$ and $3I$

28. In a Young's double-slit experiment using identical slits, the intensity at a bright fringe is I_0. If one of the slits is now covered, the intensity at any point on the screen will be

(a) I_0 (b) $I_0/2$

(c) $I_0/4$ (d) $I_0/(2\sqrt{2})$

29. In a Young's double-slit experiment, the central bright fringe can be identified

(a) as it has greater intensity than the other bright fringes

(b) as it is wider than the other bright fringes

(c) as it is narrower than the other bright fringes

(d) by using white light instead of monochromatic light

30. In a Young's double-slit experiment, if the slits are of unequal width,

(a) fringes will not be formed

(b) the positions of minimum intensity will not be completely dark

(c) bright fringe will not be formed at the centre of the screen
(d) distance between two consecutive bright fringes will not be equal to the distance between two consecutive dark fringes

31. In a Young's double-slit experiment, the fringe width is β. If the entire arrangement is now placed inside a liquid of refractive index μ, the fringe width will become

(a) $\mu\beta$ (b) $\frac{\beta}{\mu}$

(c) $\frac{\beta}{\mu+1}$ (d) $\frac{\beta}{\mu-1}$

32. In a Young's double-slit experiment, let S_1 and S_2 be the two slits, and C be the centre of the screen. If $\angle S_1CS_2 = \theta$ and λ is the wavelength, the fringe width will be

(a) $\frac{\lambda}{\theta}$ (b) $\lambda\theta$

(c) $\frac{2\lambda}{\theta}$ (d) $\frac{\lambda}{2\theta}$

• Type 2 •

Choose the correct options. One or more options may be correct.

33. A bird flies down vertically towards a water surface. To a fish inside the water, vertically below the bird, the bird will appear to
(a) be farther away than its actual distance
(b) be closer than its actual distance
(c) move faster than its actual speed
(d) move slower than its actual speed

34. A swimmer S inside water is vertically above a fixed point P. A rectangular glass slab B is placed between S and P. As seen by S, the position of P will appear to change, if

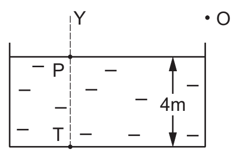
(a) B is moved horizontally (b) B is moved vertically
(c) S moves horizontally (d) S moves vertically

35. A stationary swimmer S, inside a liquid of refractive index μ_1, is at a distance d from a fixed point P inside the liquid. A rectangular block of width t and refractive index μ_2 ($\mu_2 < \mu_1$) is now placed between S and P. S will observe P to be at a distance

(a) $d - t\left(\dfrac{\mu_1}{\mu_2} - 1\right)$ (b) $d - t\left(1 - \dfrac{\mu_2}{\mu_1}\right)$

(c) $d + t\left(1 - \dfrac{\mu_2}{\mu_1}\right)$ (d) $d + t\left(\dfrac{\mu_1}{\mu_2} - 1\right)$

36.

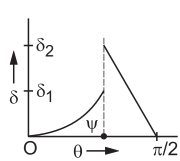


T is a point at the bottom of a tank filled with water, as shown. The refractive index of water is 4⁄3. YPT is the vertical line through T. To an observer at the position O, T will appear to be

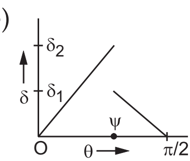
(a) to the left of YT
(b) somewhere on YT
(c) at a depth 3 m below T
(d) at a depth < 3 m below T

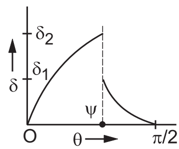
37. A ray of light travels from a medium of refractive index μ to air. Its angle of incidence in the medium is θ, measured from the normal to the boundary, and its angle of deviation is δ. δ is plotted against θ. Which of the following best represents the resulting curve?

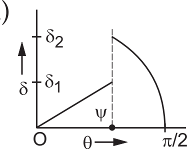
(a)



(b)



(c) 

(d) 

38. In the previous question,

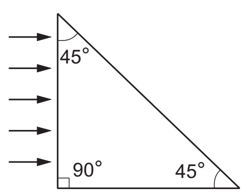
(a) $\psi = \sin^{-1}\left(\frac{1}{\mu}\right)$

(b) $\psi = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\mu}\right)$

(c) $\frac{\delta_2}{\delta_1} = \mu$

(d) $\frac{\delta_2}{\delta_1} = 2$

39.



A beam of light, consisting of red, green and blue colours, is incident on a right-angled prism, as shown. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will

(a) separate part of the red colour from the green and blue colours

(b) separate part of the blue colour from the red and green colours

(c) separate all the three colours from one another

(d) not separate even partially any colour from the other two colours

40. A ray of light travelling in a transparent medium falls on a surface separating the medium from air, at an angle of incidence of 45°. The ray undergoes total internal reflection. If *n* is the refractive index of the medium with respect to air, select the possible values of *n* from the following.

(a) 1.3

(b) 1.4

(c) 1.5

(d) 1.6

41. A solid, transparent sphere has a small, opaque dot at its centre. When observed from outside, the apparent position of the dot will be

(a) closer to the eye than its actual position

(b) farther away from the eye than its actual position

(c) the same as its actual position

(d) independent of the refractive index of the sphere

42. A watch glass has uniform thickness, and the average radius of curvature of its two surfaces is much larger than its thickness. It is placed in the path of a beam of parallel light. The beam will

(a) converge slightly

(b) diverge slightly

(c) be completely unaffected

(d) converge or diverge slightly depending on whether the beam is incident from the concave or the convex side

43. A thin concavo-convex lens has two surfaces of radii of curvature R and $2R$. The material of the lens has a refractive index μ. When kept in air, the focal length of the lens

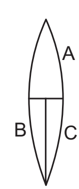
(a) will depend on the direction from which light is incident on it

(b) will be the same, irrespective of the direction from which light is incident on it

(c) will be equal to $\frac{R}{\mu - 1}$

(d) will be equal to $\frac{2R}{\mu - 1}$

44. A thin, symmetric double-convex lens of power P is cut into three parts A, B and C as shown. The power of



(a) A is P

(b) A is $2P$

(c) B is $\frac{P}{2}$

(d) B is $\frac{P}{4}$

45. If a convergent beam of light passes through a diverging lens, the result

(a) may be a convergent beam
(b) may be a divergent beam
(c) may be a parallel beam
(d) must be a parallel beam

46. Which of the following form virtual and erect images for all positions of the object?

(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror

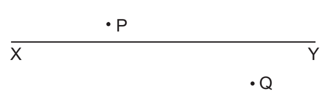
47. Two thin lenses, when in contact, produce a combination of power +10 dioptres. When they are 0.25 m apart, the power is reduced to +6 dioptres. The powers of the lenses in dioptres, are

(a) 1 and 9 (b) 2 and 8
(c) 4 and 6 (d) 5 each

48. An object and a screen are fixed at a distance d apart. When a lens of focal length f is moved between the object and the screen, sharp images of the object are formed on the screen for two positions of the lens. The magnifications produced at these two positions are M_1 and M_2.

(a) $d > 2f$ (b) $d > 4f$
(c) $M_1M_2 = 1$ (d) $|M_1| - |M_2| = 1$

49.



Two points P and Q lie on either side of an axis XY as shown. It is desired to produce an image of P at Q using a spherical mirror, with XY as the optic axis. The mirror must be

(a) converging (b) diverging
(c) positioned to the left of P (d) positioned to the right of Q

50. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C be its centre of curvature. A point object is placed at C, whose real image is also formed at C. If the mirror is now filled with water, the image will be

(a) real, and will remain at C

(b) real, and will be located above C

(c) virtual, and will be located below O

(d) real, and will be located between C and O

51. A diverging lens of focal length f_1 is placed in front of and coaxially with a concave mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement.

(a) The beam diameters of the incident and reflected beams must be the same.

(b) $d = 2|f_2| - |f_1|$

(c) $d = |f_2| - |f_1|$

(d) If the entire arrangement is immersed in water, the conditions will remain unaltered.

52. A converging lens of focal length f_1 is placed in front of and coaxially with a convex mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement.

(a) The beam diameters of the incident and reflected beams must be the same.

(b) $d = f_1 - 2|f_2|$

(c) $d = f_1 - |f_2|$

(d) If the entire arrangement is immersed in water, the conditions will remain unaltered.

53. If a converging beam of light is incident on a concave mirror, the reflected light

(a) may form a real image

(b) must form a real image

(c) may form a virtual image
(d) may be a parallel beam

54. A point object P moves towards a convex mirror with a constant speed V, along its optic axis. The speed of the image

(a) is always $< V$
(b) may be $>$, $=$ or $< V$ depending on the position of P
(c) increases as P comes closer to the mirror
(d) decreases as P comes closer to the mirror

55. A ray of white light passes through a rectangular glass slab, entering and emerging at parallel faces. The angle of incidence, measured from the normal to the glass surface, is large.

(a) White light will emerge from the slab.
(b) The light emerging from the slab will have a number of parallel, coloured rays.
(c) The emergent rays will not form a spectrum on a screen.
(d) Colours will be seen if the emergent rays enter the eye directly.

56. An astronomical telescope and a Galilean telescope use identical objective lenses. They have the same magnification, when both are in normal adjustment. The eyepiece of the astronomical telescope has a focal length f.

(a) The tube lengths of the two telescopes differ by f.
(b) The tube lengths of the two telescopes differ by $2f$.
(c) The Galilean telescope has shorter tube length.
(d) The Galilean telescope has longer tube length.

57. A single converging lens is used as a simple microscope. In the position of maximum magnification,

(a) the object is placed at the focus of the lens
(b) the object is placed between the lens and its focus
(c) the image is formed at infinity
(d) the object and the image subtend the same angle at the eye

58. A light of wavelength 6000 Å in air enters a medium of refractive index 1.5. Inside the medium, its frequency is ν and its wavelength is λ.

(a) $\nu = 5 \times 10^{14}$ Hz (b) $\nu = 7.5 \times 10^{14}$ Hz

(c) $\lambda = 4000$ Å (d) $\lambda = 9000$ Å

59. When lights of different colours move through water, they must have different

(a) wavelengths (b) frequencies

(c) velocities (d) amplitudes

60. In a Young's double-slit experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let β = fringe width. The central maximum will shift

(a) towards A (b) towards B

(c) by $t(\mu - 1)\dfrac{\beta}{\lambda}$ (d) by $\mu t \dfrac{\beta}{\lambda}$

61. In the previous question, films of thicknesses t_A and t_B and refractive indices μ_A and μ_B, are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will

(a) not shift

(b) shift towards A

(c) shift towards B

(d) option (b), if $t_B > t_A$; option (c), if $t_B < t_A$

62. If white light is used in a Young's double-slit experiment,

(a) bright white fringe is formed at the centre of the screen

(b) fringes of different colours are observed clearly only in the first order

(c) the first-order violet fringes are closer to the centre of the screen than the first-order red fringes

(d) the first-order red fringes are closer to the centre of the screen than the first-order violet fringes

63. In a Young's double-slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be

(a) $I_0 \cos\left(\frac{x}{\beta}\right)$

(b) $I_0 \cos^2\left(\frac{x}{\beta}\right)$

(c) $I_0 \cos^2\left(\frac{\pi x}{\beta}\right)$

(d) $\left(\frac{I_0}{4}\right)\cos^2\left(\frac{\pi x}{\beta}\right)$

Answers

1. b	**2.** d	**3.** c	**4.** a	**5.** b
6. a	**7.** c	**8.** d	**9.** a	**10.** c
11. c	**12.** d	**13.** d	**14.** d	**15.** a
16. c	**17.** c	**18.** c	**19.** c	**20.** d
21. a	**22.** d	**23.** a	**24**. b	**25.** b
26. a	**27.** c	**28.** c	**29.** d	**30.** b
31. b	**32.** a	**33.** a, c	**34.** c	**35.** d
36. a, d	**37.** a	**38.** a, d	**39.** a	**40.** c, d
41. c, d	**42.** b	**43.** b, d	**44.** a, c	**45.** a, b, c
46. b, c	**47.** b	**48.** b, c	**49.** a, c	**50.** d
51. a, b	**52.** a, b	**53.** a, c, d	**54.** a, c	**55.** a, b, c
56. b, c	**57.** b, d	**58.** a, c	**59.** a, b, c	**60.** a, c
61. d	**62.** a, b, c	**63.** c		

Hints and Solutions to Selected Questions

1. TIP Image formation by a mirror (either plane or spherical) does not depend on the medium.

The image of P will be formed at a distance h below the mirror. If d = depth of the liquid in the tank,

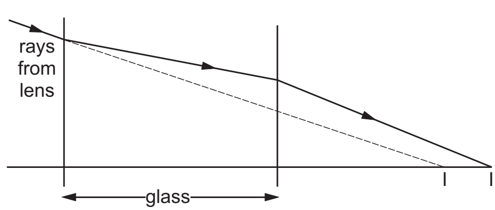
apparent depth of P $= x_1 = \dfrac{d-h}{\mu}$,

apparent depth of the image of P $= x_2 = \dfrac{d+h}{\mu}$,

apparent distance between P and its image $= x_2 - x_1 = 2h/\mu$.

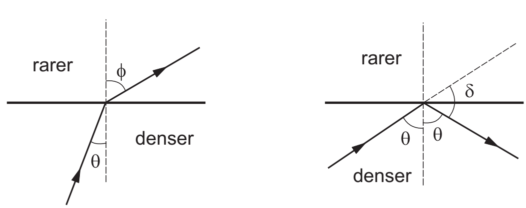
2. When the slab is introduced between P and the mirror, the apparent position of P shifts towards the mirror by $t(1 - 1/\mu)$. Hence, the mirror must be moved in the same direction through the same distance.

3.



Rays coming from the lens formed the image at I initially. Due to refraction in the slab, the rays would move as shown and form the image at I′.

5.



When the ray passes into the rarer medium, the deviation is $\delta = \phi - \theta$. This can have a maximum value of $(\pi/2 - c)$ for $\theta = c$ and $\phi = \pi/2$.

When total internal reflection occurs, the deviation is $\delta = \pi - 2\theta$, the minimum value of θ being c.

$\therefore$ the maximum value of $\delta = \pi - 2c$.

7. From symmetry, the rays must pass through the sphere, parallel to the optic axis.

Using $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{r}$,

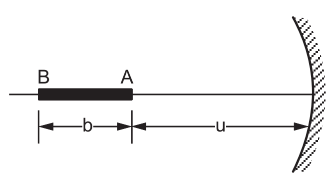
$$\frac{\mu}{\infty} - \frac{1}{u} = \frac{\mu - 1}{R}$$

$$u = -\frac{R}{\mu - 1}.$$

12. As $\mu_2 > \mu_1$, the upper half of the lens will become diverging.

As $\mu_1 > \mu_3$, the lower half of the lens will become converging.

14.



For the image of A, $\dfrac{1}{v_A} + \dfrac{1}{-u} = \dfrac{1}{-f}$.

For the image of B, $\dfrac{1}{v_B} + \dfrac{1}{-(u+b)} = \dfrac{1}{-f}$.

The image size is $v_A - v_B$.

18. Total internal reflection occurs at the second surface.

19. For no deviation, $A_1(\mu_1 - 1) = A_2(\mu_2 - 1)$.

$$A_2 = (4°)\left(\frac{1.54 - 1}{1.72 - 1}\right) = 3°.$$

20. Magnification = 5 = f_o/f_e.

Tube length = 36 = $f_o + f_e$.

21. When tube length is decreased, the (real) intermediate image formed by the objective will lie between the eyepiece and its focus. This will cause a virtual image to be formed.

22. When tube length is increased, the (real) intermediate image formed by the objective will lie beyond the focus of the eyepiece. This will produce a real, magnified image behind the eyepiece.

23. Let f_o and f_e be the focal lengths of the objective and eyepiece respectively. For normal adjustment, distance from the objective to the eyepiece (tube length) $= f_o + f_e$. Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$$\frac{1}{v} - \frac{1}{-(f_o + f_e)} = \frac{1}{f_e}$$

or $$\frac{1}{v} = \frac{1}{f_e} - \frac{1}{f_o + f_e} = \frac{f_o}{(f_o + f_e)\,f_e}$$

or $$v = \frac{(f_o + f_e)\,f_e}{f_o}.$$

$$\text{Magnification} = \left|\frac{v}{u}\right| = \frac{f_e}{f_o} = \frac{\text{image size}}{\text{object size}} = \frac{l}{L}.$$

$$\therefore \quad \frac{f_o}{f_e} = \frac{L}{l} = \text{magnification of telescope in normal adjustment.}$$

25. Velocity of light in vacuum $= c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

Velocity of light in the medium $= v = \dfrac{1}{\sqrt{\mu \varepsilon}}$.

Refractive index $= \dfrac{c}{v} = \left(\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}\right)^{1/2}$.

27, 28. TIP To find the intensity due to interference of two waves whose intensities are known, first find the amplitude of each wave ($\propto \sqrt{\text{intensity}}$). Then find the amplitude due to superposition, using the phase difference between the waves. The square of this amplitude gives the intensity due to interference.

31. $\beta \propto \lambda$. Inside water, λ decreases to λ/μ.

32. $\beta = \dfrac{\lambda D}{d}$ and $\theta = \dfrac{d}{D}$ $\quad\therefore\quad \beta = \dfrac{\lambda}{\theta}$.

33. TIP For refraction at plane surface, use $\dfrac{\mu_2}{v} = \dfrac{\mu_1}{u}$.

Let x = height of the bird above the water surface.

For light travelling from the bird to the fish,

$\mu_1 = 1, \quad \mu_2 = \mu$ (refractive index of water), $\quad u = -x$.

$$\frac{\mu}{v} = \frac{1}{-x}$$

or $\quad v = -\mu x \qquad$ or $\quad |v| = \mu x > x$.

Speed of bird $= \dot{x}$, apparent speed of bird $= |\dot{v}| = \mu\dot{x}$.

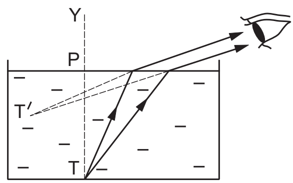
34. For normal viewing, apparent change in the position of P due to the slab does not depend on the position of the slab. When S moves horizontally, the condition of normal viewing does not apply, and the apparent position of P will change with the angle of viewing.

35. The refractive index of the block relative to the medium surrounding it is $\mu = \mu_2/\mu_1$. When the block is introduced, P will appear to move towards S through a distance

$$t' = t\left(1 - \frac{1}{\mu}\right) = t\left[1 - \frac{\mu_1}{\mu_2}\right] < 0.$$

Apparent distance from S to P $= d - t' = d + t\left(\frac{\mu_1}{\mu_2} - 1\right)$.

36.



The image of T is formed at T′. The position of T′ will change with the angle of viewing.

37, 38. See the hint to Q. No. 5.

For $\theta < c$, $\delta = \phi - \theta$ with $\frac{\sin\phi}{\sin\theta} = \mu$ or $\phi = \sin^{-1}(\mu\sin\theta)$.

$\therefore \quad \delta = \sin^{-1}(\mu\sin\theta) - \theta$

This is a nonlinear relation. The maximum value of δ is

$\delta_1 = \pi/2 - c$ for $\theta = c$ and $\phi = \pi/2$.

For $\theta > c$, $\delta = \pi - 2\theta$.

δ decreases linearly with θ.

$\delta_{max} = \pi - 2c = \delta_2 = 2\delta_1$.

39. The angle of incidence of all the rays is 45° at the hypotenuse. For a critical angle of 45°, the refractive index must be

$$(\sin 45°)^{-1} = \sqrt{2} = 1.414.$$

For red light, $\mu = 1.39 < 1.414$. Hence, its critical angle is $> 45°$.

Therefore, red light will pass through the surface into air.

For green and blue lights, $\mu > 1.414$. Hence, their critical angles are $< 45°$. They will be reflected internally and emerge from the surface at the bottom.

40. See the first part of the hint to Q. No. 39.

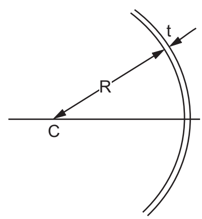
41. $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{r}$.

Here, $\mu_1=\mu=$ refractive index of glass, $\mu_2=1$, $u=-R=$ radius of sphere, $r=-R$.

$$\frac{1}{v}-\frac{\mu}{-R}=\frac{1-\mu}{-R}$$

or $\dfrac{1}{v}=\dfrac{1}{-R}$ or $v=-R$.

42.



As the watch glass is of uniform thickness, the two curved surfaces must have a common centre of curvature (C). Let R be the radius of curvature of the inner surface, $t=$ thickness of the watch glass. Let light parallel to the optic axis be incident from the left. For refraction at the first surface, let image be formed at a distance x.

$$\frac{\mu}{x}-\frac{1}{\infty}=\frac{\mu-1}{-R}$$

or $\dfrac{\mu}{x}=-\dfrac{\mu-1}{R}$, where $\mu=$ refractive index of glass.

For refraction at the second surface,

$$\frac{1}{v}-\frac{\mu}{x}=\frac{1-\mu}{-(R+t)}$$

or $\dfrac{1}{v}=(\mu-1)\left(\dfrac{1}{R+t}-\dfrac{1}{R}\right)=\dfrac{1}{f}<0.$

$\therefore$ $f<0$ (diverging).

43. See the hint to Q. No. 42.

47. Let the two lenses have focal lengths f_1 and f_2, in metres.

In contact, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \text{power} = 10$

or $\quad \frac{f_1 + f_2}{f_1 f_2} = 10 \qquad \ldots (1)$

For a separation of $d = 0.25$ m,

$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = 6$$

or $\quad 10 - \frac{d}{f_1 f_2} = 6$

or $\quad f_1 f_2 = \frac{1}{16} \qquad \ldots (2)$

In (1), $f_1 + f_2 = 10 f_1 f_2 = \frac{10}{16} = \frac{5}{8}$ or $f_2 = \frac{5}{8} - f_1$.

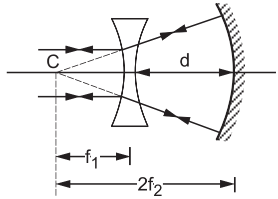
In (2), $f_1\left(\frac{5}{8} - f_1\right) = \frac{1}{16}$ or $f_1(10 - 16f_1) = 1$.

or $\quad 16f_1^2 - 10f_1 + 1 = 0$

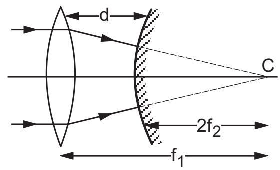
or $\quad f_1 = \frac{1}{2}$ or $\frac{1}{8}$.

50. When the mirror is filled with water, the apparent position of the object, as seen from the pole of the mirror, will be above C. [See the hint to Q. No. 33.] Hence, the image will be formed below C, i.e., between C and O.

51.



52.



54. As the object moves from infinity to the pole of the mirror, the virtual image moves from its focus to the pole.

56. In normal adjustment, tube length of an astronomical telescope is $(f_o + f_e)$ and that of a Galilean telescope is $(f_o - f_e)$, where f_o and f_e are the focal lengths of the objective and the eyepiece respectively.

Here, $f_e = f$.

Magnification $= \dfrac{f_o}{f_e}$ for both telescopes.

60. Let d = distance between the slits, λ = wavelength of light,

D = distance from the slits to the screen.

For a point P on the screen at a distance x from the centre of the screen, path difference $= \Delta = x\dfrac{d}{D}$.

Path difference introduced due to film $= t(\mu - 1)$.

For central maximum at P, $x\dfrac{d}{D} = t(\mu - 1)$

or $\quad x = t(\mu - 1)\dfrac{D}{d}$.

Now, $\beta = \dfrac{\lambda D}{d}$ $\qquad$ or $\dfrac{D}{d} = \dfrac{\beta}{\lambda}$

$\therefore \quad x = t(\mu - 1)\dfrac{\beta}{\lambda}$.

61. Additional path difference due to the two films

$$= (\mu_B - 1)t_B - (\mu_A - 1)t_A = t_A - t_B.$$

63. $\Delta = x\dfrac{d}{D}$

$\therefore$ phase difference $= \phi = \dfrac{2\pi}{\lambda}\Delta$.

Let a = amplitude at the screen due to each slit.

$\therefore$ $I_0 = k(2a)^2 = 4ka^2$, where k is a constant.

For phase difference ϕ, amplitude $= A = 2a\cos(\phi/2)$.

Intensity, $I = kA^2 = k(4a^2)\cos^2(\phi/2) = I_0\cos^2\left(\dfrac{\pi}{\lambda}\Delta\right)$

$$= I_0\cos^2\left(\frac{\pi}{\lambda}\cdot\frac{xd}{D}\right) = I_0\cos^2\left(\frac{\pi x}{\beta}\right).$$

□

Part 5

Electricity

1

Electrostatics

[In all questions in this chapter, $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2/\text{C}^2$,

$\varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{N m}^2$.]

• Type 1 •

Choose the correct option (a, b, c or d).

1. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

(a) $-Q/2$ (b) $-Q/4$

(c) $+Q/4$ (d) $+Q/2$

2. In the previous question, if e = electronic charge, the minimum magnitude of Q is

(a) e (b) $2e$

(c) $4e$ (d) none of these

3. Two particles, each of mass m and carrying charge Q, are separated by some distance. If they are in equilibrium under mutual gravitational and electrostatic forces then Q/m (in C/kg) is of the order of

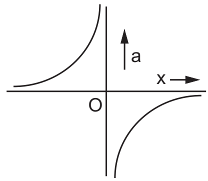
(a) 10^{-5} (b) 10^{-10}
(c) 10^{-15} (d) 10^{-20}

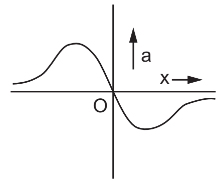
4. In the previous question, the equilibrium is
(a) stable
(b) unstable
(c) neutral
(d) may be any of the above depending on the separation of the particles

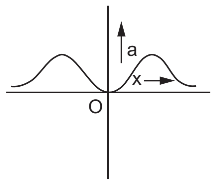
5. Three point charges are placed at the corners of an equilateral triangle. Assume that only electrostatic forces are acting.
(a) The system will be in equilibrium if the charges have the same magnitude but not all have the same sign.
(b) The system will be in equilibrium if the charges have different magnitudes and not all have the same sign.
(c) The system will be in equilibrium if the charges rotate about the centre of the triangle.
(d) The system can never be in equilibrium.

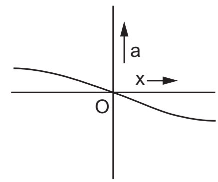
6. Charge Q is divided into two parts which are then kept some distance apart. The force between them will be maximum if the two parts are
(a) $Q/2$ each
(b) $Q/4$ and $3Q/4$
(c) $Q/3$ and $2Q/3$
(d) e and $(Q-e)$, where e = electronic charge

7. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the x-axis, passes through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-coordinate. Which of the following best represents the plot?

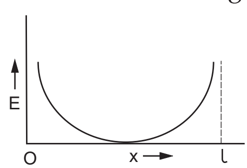
(a)


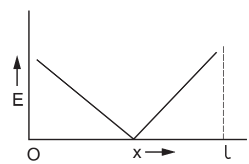
(b)


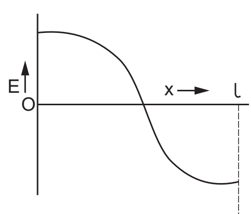
(c)


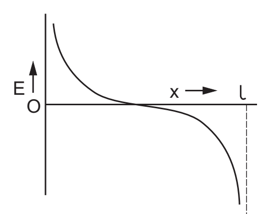
(d)


8. Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than l. Which of the following best represents the resulting curve?

(a)


(b)


(c)


(d)


9. Two identical pendulums, A and B, are suspended from the same point. The bobs are given positive charges, with A having more charge than B. They diverge and reach equilibrium, with A and B making angles θ_1 and θ_2 with the vertical respectively.

(a) $\theta_1 > \theta_2$

(b) $\theta_1 < \theta_2$

(c) $\theta_1 = \theta_2$

(d) The tension in A is greater than that in B.

10. A point charge Q is moved along a circular path around another fixed point charge. The work done is zero

(a) only if Q returns to its starting point

(b) only if the two charges have the same magnitude

(c) only if the two charges have the same magnitude and opposite signs

(d) in all cases

11. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges of magnitude Q are placed at $(n-1)$ corners. The field at the centre is

(a) $k\dfrac{Q}{r^2}$ (b) $(n-1)\,k\dfrac{Q}{r^2}$

(c) $\dfrac{n}{n-1}k\dfrac{Q}{r^2}$ (d) $\dfrac{n-1}{n}k\dfrac{Q}{r^2}$

12. A half ring of radius R has a charge of λ per unit length. The potential at the centre of the half ring is

(a) $k\dfrac{\lambda}{R}$ (b) $k\dfrac{\lambda}{\pi R}$

(c) $k\dfrac{\pi\lambda}{R}$ (d) $k\pi\lambda$

13. In the previous question, the field at the centre is

(a) zero (b) $k\dfrac{\lambda}{R}$

(c) $2k\dfrac{\lambda}{R}$ (d) $k\dfrac{\pi\lambda}{R}$

14. Two identical metal balls with charges $+2Q$ and $-Q$ are separated by some distance, and exert a force F on each other. They are joined by a conducting wire, which is then removed. The force between them will now be

(a) F (b) $F/2$

(c) $F/4$ (d) $F/8$

15. Charge Q is given a displacement $\vec{r} = a\hat{i} + b\hat{j}$ in an electric field $\vec{E} = E_1\hat{i} + E_2\hat{j}$. The work done is

(a) $Q(E_1a + E_2b)$ (b) $Q\sqrt{(E_1a)^2 + (E_2b)^2}$

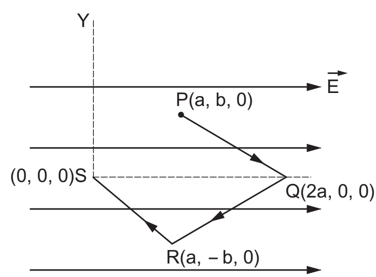
(c) $Q(E_1 + E_2)\sqrt{a^2 + b^2}$ (d) $Q(\sqrt{E_1^2 + E_2^2})\sqrt{a^2 + b^2}$

16. Let V_0 be the potential at the origin in an electric field $\vec{E} = E_x\hat{i} + E_y\hat{j}$. The potential at the point (x, y) is

(a) $V_0 - xE_x - yE_y$ (b) $V_0 + xE_x + yE_y$

(c) $xE_x + yE_y - V_0$ (d) $(\sqrt{x^2 + y^2})\sqrt{E_x^2 + E_y^2} - V_0$

17.



A point charge q moves from point P to point S along the path PQRS in a uniform electric field $\vec{E}$ pointing parallel to the positive direction of the x-axis. The coordinates of the points P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ respectively. The work done by the field in the above process is given by the expression

(a) qaE (b) $-qaE$

(c) $q(\sqrt{a^2 + b^2})E$ (d) $3qE\sqrt{a^2 + b^2}$

18. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. The electric field (in V/m) at the point (1 m, 0, 2 m) is

(a) $-8\hat{i}$ (b) $8\hat{i}$

(c) -16 (d) $8\sqrt{5}$

19. A nonconducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed nonuniformly on its circumference, producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E} \cdot \vec{dl}$ ($l = 0$ being the centre of the ring) in volts is

(a) +2 (b) –1

(c) –2 (d) 0

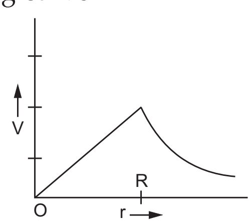
20. A charge $+q$ is placed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, … ad infinitum on the x-axis, and a charge $-q$ is placed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, … ad infinitum. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is

(a) 0 (b) $\dfrac{q}{8\pi \varepsilon_0 x_0 \ln 2}$

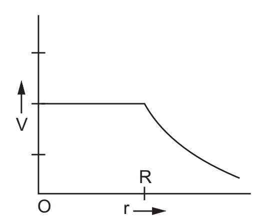
(c) ∞ (d) $\dfrac{q \ln 2}{4\pi \varepsilon_0 x_0}$

21. A solid sphere of radius R is charged uniformly. The electrostatic potential V is plotted as a function of distance r from the centre of the sphere. Which of the following best represents the resulting curve?

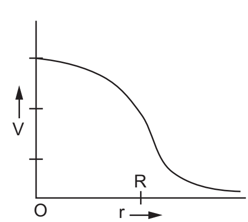
(a)



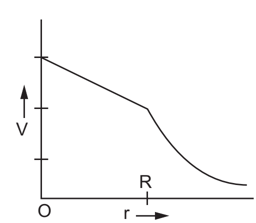
(b)



(c)



(d)



22. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

(a) R (b) $R/2$ (c) $R/3$ (d) $2R$

23. A large solid sphere with uniformly distributed positive charge has a smooth narrow tunnel through its centre. A small particle with negative charge, initially at rest far from the sphere, approaches it along the line of the tunnel, reaches its surface with a speed v, and passes through the tunnel. Its speed at the centre of the sphere will be

(a) 0 (b) v

(c) $\sqrt{2}v$ (d) $\sqrt{1.5}v$

24. Which of the following is not true for a region with a uniform electric field?

(a) It can have free charges.

(b) It may have uniformly distributed charge.

(c) It may contain dipoles.

(d) None of the above.

25. 'All charge on a conductor must reside only on its outer surface.' This statement is true

(a) in all cases

(b) for spherical conductors only (both solid and hollow)

(c) for hollow spherical conductors only

(d) for conductors which do not have any sharp points or corners

26. A point charge Q is placed outside a hollow spherical conductor of radius R, at a distance r ($r > R$) from its centre C. The field at C due to the induced charges on the conductor is

(a) zero (b) $k\dfrac{Q}{(r-R)^2}$

(c) $k\dfrac{Q}{r^2}$, directed towards Q (d) $k\dfrac{Q}{r^2}$, directed away from Q

27. A positive point charge, which is free to move, is placed inside a hollow conducting sphere with negative charge, away from its centre. It will

(a) move towards the centre

(b) move towards the nearer wall of the conductor

(c) remain stationary

(d) oscillate between the centre and the nearer wall

28. In a region of space, the electric field is in the x-direction and proportional to x, i.e., $\vec{E} = E_0 x\hat{i}$. Consider an imaginary cubical volume of edge a, with its edges parallel to the axes of coordinates. The charge inside this volume is

(a) zero (b) $\varepsilon_0 E_0 a^3$

(c) $\frac{1}{\varepsilon_0} E_0 a^3$ (d) $\frac{1}{6} \varepsilon_0 E_0 a^2$

29. A charge Q is placed at the mouth of a conical flask. The flux of the electric field through the flask is

(a) zero (b) Q/ε_0

(c) $\frac{Q}{2\varepsilon_0}$ (d) $< \frac{Q}{2\varepsilon_0}$

30. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

(a) $\lambda a/\varepsilon_0$ (b) $\sqrt{2}\lambda a/\varepsilon_0$

(c) $6\lambda a^2/\varepsilon_0$ (d) $\sqrt{3}\lambda a/\varepsilon_0$

31. A spherical conductor A of radius r is placed concentrically inside a conducting shell B of radius R $(R > r)$. A charge Q is given to A, and then A is joined to B by a metal wire. The charge flowing from A to B will be

(a) $Q\left(\frac{R}{R+r}\right)$ (b) $Q\left(\frac{r}{R+r}\right)$

(c) Q (d) zero

32. A spherical equipotential surface is not possible

(a) for a point charge

(b) for a dipole

(c) inside a uniformly charged sphere

(d) inside a spherical capacitor

33. In a certain charge distribution, all points having zero potential can be joined by a circle S. Points inside S have positive potential, and points outside S have negative potential. A positive charge, which is free to move, is placed inside S.

(a) It will remain in equilibrium.

(b) It can move inside S, but it cannot cross S.

(c) It must cross S at some time.

(d) It may move, but will ultimately return to its starting point.

34. If the earth's surface is treated as a conducting surface with some charge, what should be the order of magnitude of the charge per unit area, in C/m^2, so that a proton remains suspended in space near the earth's surface?

(a) 10^{-18} (b) 10^{-12}

(c) 10^{-6} (d) 1

35. A simple pendulum of time period T is suspended above a large horizontal metal sheet with uniformly distributed positive charge. If the bob is given some negative charge, its time period of oscillation will be

(a) $> T$

(b) $< T$

(c) T

(d) proportional to its amplitude

36. A spring–block system undergoes vertical oscillation above a large horizontal metal sheet with uniform positive charge. The time period of the oscillation is T. If the block is given a charge Q, its time period of oscillation will be

(a) T
(b) $> T$
(c) $< T$
(d) $> T$ if Q is positive and $< T$ if Q is negative

37. A large flat metal surface has a uniform charge density $+\sigma$. An electron of mass m and charge e leaves the surface at point A with speed u, and returns to it at point B. Disregard gravity. The maximum value of AB is

(a) $\dfrac{u^2 m \varepsilon_0}{\sigma e}$ (b) $\dfrac{u^2 e \varepsilon_0}{m\sigma}$ (c) $\dfrac{u^2 e}{\varepsilon_0 \sigma m}$ (d) $\dfrac{u^2 \sigma e}{\varepsilon_0 m}$

38. A and B are two points on the axis and the perpendicular bisector respectively of an electric dipole. A and B are far away from the dipole and at equal distances from it. The fields at A and B are $\vec{E}_A$ and $\vec{E}_B$.

(a) $\vec{E}_A = \vec{E}_B$
(b) $\vec{E}_A = 2\vec{E}_B$
(c) $\vec{E}_A = -2\vec{E}_B$
(d) $|E_B| = \dfrac{1}{2}|E_A|$, and $\vec{E}_B$ is perpendicular to $\vec{E}_A$

39. In the previous question, let V_A and V_B be the potentials at A and B respectively.

(a) $V_A = V_B$ (b) $V_A = 2V_B$
(c) $V_A \neq 0, V_B = 0$ (d) $V_A = 0, V_B = 0$

40. An electric dipole is placed at the origin and is directed along the x-axis. At a point P, far away from the dipole, the electric field is parallel to the y-axis. OP makes an angle θ with the x-axis.

(a) $\tan\theta = \sqrt{3}$ (b) $\tan\theta = \sqrt{2}$
(c) $\theta = 45°$ (d) $\tan\theta = \dfrac{1}{\sqrt{2}}$

41. An electric dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$, with $\vec{p}$ parallel to $\vec{E}$. It is then rotated by an angle θ. The work done is

(a) $pE\sin\theta$ (b) $pE\cos\theta$

(c) $pE(1-\cos\theta)$ (d) $pE(1-\sin\theta)$

42. If we treat the earth as a conducting sphere of radius 6400 km, its capacitance would be of the order of

(a) 1 μF (b) 1 mF

(c) 1 F (d) 10^{3} F

43. When two uncharged metal balls of radius 0.09 mm each collide, one electron is transferred between them. The potential difference between them would be

(a) 16 μV (b) 16 pV

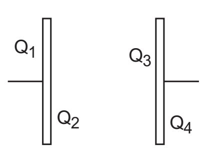
(c) 32 μV (d) 32 pV

44. A conducting sphere of radius R, and carrying a charge Q, is joined to an uncharged conducting sphere of radius $2R$. The charge flowing between them will be

(a) $Q/4$ (b) $Q/3$

(c) $Q/2$ (d) $2Q/3$

45.



In an isolated parallel-plate capacitor of capacitance C, the four surfaces have charges Q_1, Q_2, Q_3 and Q_4, as shown. The potential difference between the plates is

(a) $\dfrac{Q_1+Q_2+Q_3+Q_4}{2C}$ (b) $\dfrac{Q_2+Q_3}{2C}$

(c) $\dfrac{Q_2-Q_3}{2C}$ (d) $\dfrac{Q_1+Q_4}{2C}$

46. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now

(a) V (b) $V + \frac{Q}{C}$

(c) $V + \frac{Q}{2C}$ (d) $V - \frac{Q}{C}$, if $V < CV$

47. A capacitor is connected to a cell of emf $\mathcal{E}$ and some internal resistance. The potential difference across the

(a) cell is $\mathcal{E}$ (b) cell is $< \mathcal{E}$

(c) capacitor is $< \mathcal{E}$ (d) capacitor is $> \mathcal{E}$

48. In a parallel-plate capacitor of capacitance C, a metal sheet is inserted between the plates, parallel to them. The thickness of the sheet is half of the separation between the plates. The capacitance now becomes

(a) $4C$ (b) $2C$

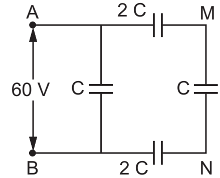
(c) $C/2$ (d) $C/4$

49. Two capacitors of capacitances 3 μF and 6 μF are charged to a potential of 12 V each. They are now connected to each other, with the positive plate of each joined to the negative plate of the other. The potential difference across each will be

(a) zero (b) 3 V

(c) 4 V (d) 6 V

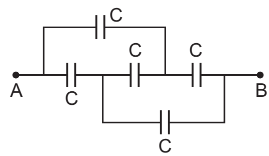
50.



In the circuit shown, a potential difference of 60 V is applied across AB. The potential difference between the points M and N is

(a) 10 V (b) 15 V (c) 20 V (d) 30 V

51.

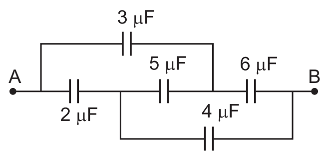


In the circuit shown, the equivalent capacitance between the points A and B is

(a) $C/5$ (b) $C/3$

(c) $C/2$ (d) C

52.



In the circuit shown, the equivalent capacitance between the points A and B is

(a) $\frac{10}{3}$ μF (b) $\frac{15}{4}$ μF

(c) $\frac{12}{5}$ μF (d) $\frac{25}{6}$ μF

53. Let u_a and u_d represent the energy density (energy per unit volume) in air and in a dielectric respectively, for the same field in both. Let K = dielectric constant. Then,

(a) $u_a = u_d$ (b) $u_a = Ku_d$

(c) $u_d = Ku_a$ (d) $u_a = (K - 1)u_d$

54. In a parallel-plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is connected to a cell and the slab is taken out.

(a) Some charge is drawn from the cell.

(b) Some charge is returned to the cell.

(c) The potential difference across the capacitor is reduced.

(d) No work is done by an external agent in taking the slab out.

55. In a parallel-plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is charged from a cell and then disconnected from it. The slab is now taken out.

(a) The potential difference across the capacitor is reduced.

(b) The potential difference across the capacitor is increased.

(c) The energy stored in the capacitor is reduced.

(d) No work is done by an external agent in taking the slab out.

• *Type 2* •

Choose the correct options. One or more options may be correct.

56. Three charged particles are in equilibrium under their electrostatic forces only.

(a) The particles must be collinear.

(b) All the charges cannot have the same magnitude.

(c) All the charges cannot have the same sign.

(d) The equilibrium is unstable.

57. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (–1, 0, 0) and (0, –1, 0).

(a) The potential at the origin is zero.

(b) The field at the origin is zero.

(c) The potential at all points on the z-axis, other than the origin, is zero.

(d) The field at all points on the z-axis, other than the origin, acts along the z-axis.

58. Four charges, all of the same magnitude, are placed at the four corners of a square. At the centre of the square, the potential is V and the field is E. By suitable choices of the signs of the four charges, which of the following can be obtained?

(a) $V = 0, E = 0$ (b) $V = 0, E \neq 0$

(c) $V \neq 0, E = 0$ (d) $V \neq 0, E \neq 0$

59. A deuteron and an α-particle are placed in an electric field. The forces acting on them are F_1 and F_2, and their accelerations are a_1 and a_2 respectively.

(a) $F_1 = F_2$ (b) $F_1 \neq F_2$

(c) $a_1 = a_2$ (d) $a_1 \neq a_2$

60. Two identical charges $+Q$ are kept fixed some distance apart. A small particle P with charge q is placed midway between them. If P is given a small displacement Δ, it will undergo simple harmonic motion if

(a) q is positive and Δ is along the line joining the charges.

(b) q is positive and Δ is perpendicular to the line joining the charges.

(c) q is negative and Δ is perpendicular to the line joining the charges.

(d) q is positive and Δ is along the line joining the charges.

61. A ring with a uniform charge Q and radius R, is placed in the yz plane with its centre at the origin.

(a) The field at the origin is zero.

(b) The potential at the origin is $k\frac{Q}{R}$.

(c) The field at the point $(x, 0, 0)$ is $k\frac{Q}{x^2}$.

(d) The field at the point $(x, 0, 0)$ is $k\frac{Q}{R^2 + x^2}$.

62. A positively charged thin metal ring of radius R is fixed in the xy plane, with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then the motion of P is

(a) periodic, for all value of z_0 satisfying $0 < z_0 < \infty$

(b) simple harmonic, for all values of z_0 satisfying $0 < z_0 \leq R$

(c) approximately simple harmonic, provided $z_0 << R$

(d) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$

63. A particle A of mass m and charge Q moves directly towards a fixed particle B, which has charge Q. The speed of A is v when it is far away from B. The minimum separation between the particles is proportional to

(a) Q^2 (b) $\frac{1}{v^2}$

(c) $\frac{1}{v}$ (d) $\frac{1}{m}$

64. Let V and E be the potential and the field respectively at a point. Which of the following assertions are correct?

(a) If $V = 0$, E must be zero. (b) If $V \neq 0$, E cannot be zero.

(c) If $E \neq 0$, V cannot be zero. (d) None of these.

65. In a uniform electric field,

(a) all points are at the same potential

(b) no two points can have the same potential

(c) pairs of points separated by the same distance must have the same difference in potential

(d) none of the above

66. Charges Q_1 and Q_2 lie inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.

(a) If Q_1 changes, both E and ϕ will change.

(b) If Q_2 changes, E will change but ϕ will not change.

(c) If $Q_1 = 0$ and $Q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.

(d) If $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

67. Charges Q_1 and Q_2 are placed inside and outside respectively of an uncharged conducting shell. Their separation is r.

(a) The force on Q_1 is zero. (b) The force on Q_1 is $k\frac{Q_1Q_2}{r^2}$.

(c) The force on Q_2 is $k\frac{Q_1Q_2}{r^2}$. (d) The force on Q_2 is zero.

68. A spherical conductor A lies inside a hollow spherical conductor B. Charges Q_1 and Q_2 are given to A and B respectively.

(a) Charge Q_1 will appear on the outer surface of A.

(b) Charge $-Q_1$ will appear on the inner surface of B.

(c) Charge Q_2 will appear on the outer surface of B.

(d) Charge $Q_1 + Q_2$ will appear on the outer surface of B.

69. A, B and C are three concentric metallic shells. Shell A is the innermost and shell C is the outermost. A is given some charge.

(a) The inner surfaces of B and C will have the same charge.

(b) The inner surfaces of B and C will have the same charge density.

(c) The outer surfaces of A, B and C will have the same charge.

(d) The outer surfaces of A, B and C will have the same charge density.

70. Two large, identical and parallel conducting plates have surfaces X and Y, facing each other. The charge per unit area on X is σ_1, and on Y it is σ_2.

(a) $\sigma_1 = -\sigma_2$ in all cases.

(b) $\sigma_1 = -\sigma_2$ only if a charge is given to one plate only.

(c) $\sigma_1 = \sigma_2 = 0$ if equal charges are given to both the plates.

(d) $\sigma_1 > \sigma_2$ if X is given more charge than Y.

71. P is a point on an equipotential surface S. The field at P is E.

(a) E must be perpendicular to S in all cases.

(b) E will be perpendicular to S only if S is a plane surface.

(c) E cannot have a component along a tangent to S.

(d) E may have a nonzero component along a tangent to S if S is a curved surface.

72. S_1 and S_2 are two equipotential surfaces on which the potentials are not equal.

(a) S_1 and S_2 cannot intersect.

(b) S_1 and S_2 cannot both be plane surfaces.

(c) In the region between S_1 and S_2, the field is maximum where they are closest to each other.

(d) A line of force from S_1 to S_2 must be perpendicular to both.

73. In a uniform electric field, equipotential surfaces must

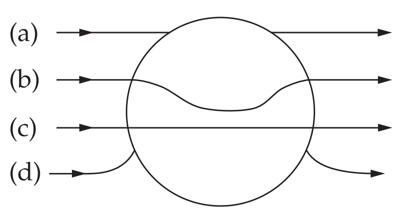
(a) be plane surfaces

(b) be normal to the direction of the field

(c) be spaced such that surfaces having equal differences in potential are separated by equal distances

(d) have decreasing potentials in the direction of the field

74. A solid metallic sphere is placed in a uniform electric field. Which of the curves shown in the figure represent the lines of force correctly?



75. The field at a distance r from a long string of charge per unit length λ is

(a) $k\dfrac{\lambda}{r^2}$

(b) $k\dfrac{\lambda}{r}$

(c) $k\dfrac{\lambda}{2r}$

(d) $k\dfrac{2\lambda}{r}$

76. Two points are at distances a and b $(a < b)$ from a long string of charge per unit length λ. The potential difference between the points is proportional to

(a) b/a (b) b^2/a^2

(c) $\sqrt{b/a}$ (d) $\ln(b/a)$

77. A charged particle moves with a speed v in a circular path of radius r around a long uniformly charged conductor.

(a) $v \propto r$ (b) $v \propto \frac{1}{r}$

(c) $v \propto \frac{1}{\sqrt{r}}$ (d) v is independent of r

78. In the previous question, if the conductor has a charge per unit length λ, the particle has mass m and charge q then

(a) $v \propto \sqrt{q}$ (b) $v \propto \sqrt{\lambda}$

(c) $v \propto \sqrt{m}$ (d) $v \propto \frac{1}{\sqrt{m}}$

79 A simple pendulum of length l has a bob of mass m, with a charge q on it. A vertical sheet of charge, with charge σ per unit area, passes through the point of suspension of the pendulum. At equilibrium, the string makes an angle θ with the vertical. Its time period of oscillation is T in this position.

(a) $\tan\theta = \frac{\sigma q}{2\varepsilon_0 m g}$ (b) $\tan\theta = \frac{\sigma q}{\varepsilon_0 m g}$

(c) $T < 2\pi\sqrt{l/g}$ (d) $T > 2\pi\sqrt{l/g}$

80. A dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. The force on the dipole is $\vec{F}$ and the torque is $\vec{\tau}$.

(a) $\vec{F} = 0$ (b) $\vec{F} = |\vec{p}|\,\vec{E}$

(c) $|\vec{\tau}| = \vec{p} \cdot \vec{E}$ (d) $\vec{\tau} = \vec{p} \times \vec{E}$

81. A dipole is placed in an electric field whose direction is fixed but whose magnitude varies with distance. It is possible that the dipole experiences

(a) no net force and no torque

(b) a net force but no torque

(c) a net force and a torque

(d) no net force but a torque

82. Two large, parallel conducting plates are placed close to each other. The inner surfaces of the two plates have surface charge densities $+\sigma$ and $-\sigma$. The outer surfaces are without charge. The electric field has a magnitude of

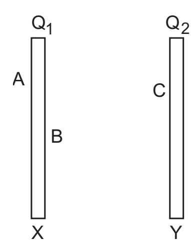
(a) $2\sigma/\varepsilon_0$ in the region between the plates

(b) σ/ε_0 in the region between the plates

(c) σ/ε_0 in the region outside the plates

(d) zero in the region outside the plates

83.



Two large, parallel conducting plates X and Y, kept close to each other, are given charges Q_1 and Q_2 ($Q_1 > Q_2$). The four surfaces of the plates are A, B, C and D, as shown.

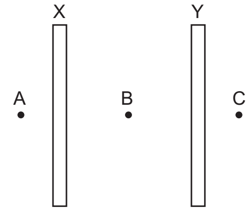
(a) The charge on A is $\frac{1}{2}(Q_1 + Q_2)$.

(b) The charge on B is $\frac{1}{2}(Q_1 - Q_2)$.

(c) The charge on C is $\frac{1}{2}(Q_2 - Q_1)$.

(d) The charge on D is $\frac{1}{2}(Q_1 + Q_2)$.

84. X and Y are large, parallel conducting plates close to each other. Each face has an area A. X is given a charge Q. Y is without any charge. Points A, B and C are as shown in the figure.



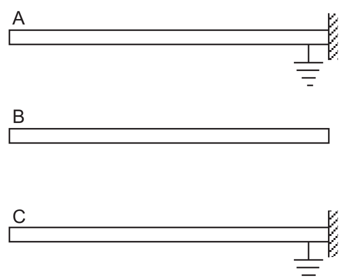
(a) The field at B is $\frac{Q}{2\varepsilon_0 A}$.

(b) The field at B is $\frac{Q}{\varepsilon_0 A}$.

(c) The fields at A, B and C are of the same magnitude.

(d) The fields at A and C are of the same magnitude, but in opposite directions.

85,



A, B and C are three large, parallel conducting plates, placed horizontally. A and C are rigidly fixed and earthed. B is given some charge. Under electrostatic and gravitational forces, B may be

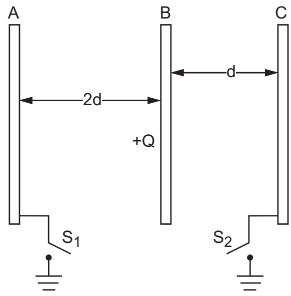
(a) in equilibrium midway between A and C

(b) in equilibrium if it is closer to A than to C

(c) in equilibrium if it is closer to C than to A

(d) B can never be in stable equilibrium

86. Three identical, parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. $+Q$ charge is given to B.



(a) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1.

(b) If S_2 is closed with S_1 open, a charge of amount Q will pass through S_2.

(c) If S_1 and S_2 are closed together, a charge of amount $Q/3$ will pass through S_1, and a charge of amount $2Q/3$ will pass through S_2.

(d) All the above statements are incorrect.

87. A parallel-plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased.

(a) The force of attraction between the plates will decrease.

(b) The field in the region between the plates will not change.

(c) The energy stored in the capacitor will increase.

(d) The potential difference between the plates will decrease.

88. When a charge of amount Q is given to an isolated metal plate X of surface area A, its surface charge density becomes σ_1. When an isolated identical plate Y is brought close to X, the surface charge density on X becomes σ_2. When Y is earthed, the surface charge density becomes σ_3.

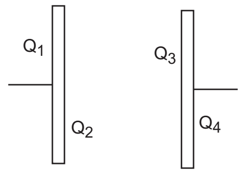
(a) $\sigma_1 = \dfrac{Q}{A}$

(b) $\sigma_1 = \dfrac{Q}{2A}$

(c) $\sigma_1 = \sigma_2$

(d) $\sigma_3 = \dfrac{Q}{A}$

89.

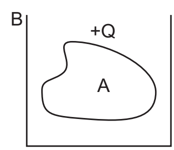


In an isolated parallel-plate capacitor of capacitance C, the four surfaces have charges Q_1, Q_2, Q_3 and Q_4, as shown. The potential difference between the plates is

(a) $\frac{Q_1 + Q_2}{C}$ (b) $\left|\frac{Q_2}{C}\right|$

(c) $\left|\frac{Q_3}{C}\right|$ (d) $\frac{1}{C}[(Q_1 + Q_2) - (Q_3 - Q_4)]$

90.



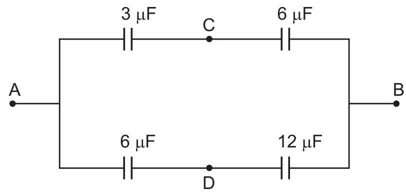
A conductor A is given a charge of amount $+Q$ and then placed inside a deep metal can B, without touching it.

(a) The potential of A does not change when it is placed inside B.
(b) If B is earthed, $+Q$ amount of charge flows from it into the earth.
(c) If B is earthed, the potential of A is reduced.
(d) Either (b) or (c) are true, or both are true only if the outer surface of B is connected to the earth and not its inner surface.

91. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius $2R$. If they are joined by a metal wire,

(a) $Q/3$ amount of charge will flow from the sphere to the shell
(b) $2Q/3$ amount of charge will flow from the sphere to the shell
(c) Q amount of charge will flow from the sphere to the shell
(d) $k\frac{Q^2}{4R}$ amount of heat will be produced

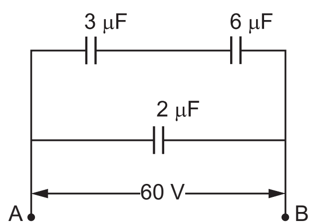
92.



In the circuit shown, some potential difference is applied between A and B. If C is joined to D,

(a) no charge will flow between C and D
(b) some charge will flow between C and D
(c) the equivalent capacitance between C and D will not change
(d) the equivalent capacitance between C and D will change

93.



In the circuit shown, the potential difference across the 3-μF capacitor is V, and the equivalent capacitance between A and B is C_{AB}.

(a) $C_{AB} = 4$ μF
(b) $C_{AB} = \frac{18}{11}$ μF
(c) $V = 20$ V
(d) $V = 40$ V

94. The two plates X and Y of a parallel-plate capacitor of capacitance C are given a charge of amount Q each. X is now joined to the positive terminal and Y to the negative terminal of a cell of emf $\mathcal{E} = Q/C$.

(a) Charge of amount Q will flow from the positive terminal to the negative terminal of the cell through the capacitor.
(b) The total charge on the plate X will be $2Q$.
(c) The total charge on the plate Y will be zero.
(d) The cell will supply $C\mathcal{E}^2$ amount of energy.

95. The separation between the plates of a parallel-plate capacitor is made double while it remains connected to a cell.

(a) The cell absorbs some energy.

(b) The electric field in the region between the plates becomes half.

(c) The charge on the capacitor becomes half.

(d) Some work has to be done by an external agent on the plates.

96. A parallel-plate capacitor is charged from a cell and then disconnected from the cell. The separation between the plates is now doubled.

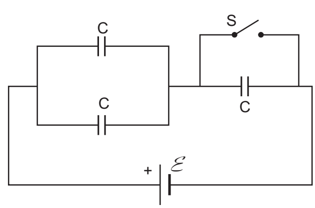
(a) The potential difference between the plates will become double.

(b) The field between the plates will not change.

(c) The energy of the capacitor doubles.

(d) Some work will have to be done by an external agent on the plates.

97.



In the circuit shown, each capacitor has a capacitance C. The emf of the cell is $\mathcal{E}$. If the switch S is closed,

(a) some charge will flow out of the positive terminal of the cell

(b) some charge will enter the positive terminal of the cell

(c) the amount of charge flowing through the cell will be $C\mathcal{E}$.

(d) the amount of charge flowing through the cell will be $\frac{4}{3}C\mathcal{E}$.

98. In a parallel-plate capacitor of plate area A, plate separation d and charge Q, the force of attraction between the plates is F.

(a) $F \propto Q^2$ (b) $F \propto \frac{1}{A}$ (c) $F \propto d$ (d) $F \propto \frac{1}{d}$

99. The capacitance of a parallel-plate capacitor is C_0 when the region between the plates has air. This region is now filled with a dielectric slab of dielectric constant K. The capacitor is connected to a cell of emf $\mathcal{E}$, and the slab is taken out.

(a) Charge $\mathcal{E}C_0(K-1)$ flows through the cell.

(b) Energy $\mathcal{E}^2C_0(K-1)$ is absorbed by the cell.

(c) The energy stored in the capacitor is reduced by $\mathcal{E}^2C_0(K-1)$.

(d) The external agent has to do $\frac{1}{2}\mathcal{E}^2C_0(K-1)$ amount of work to take the slab out.

100. A parallel-plate air capacitor of capacitance C_0 is connected to a cell of emf $\mathcal{E}$ and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it.

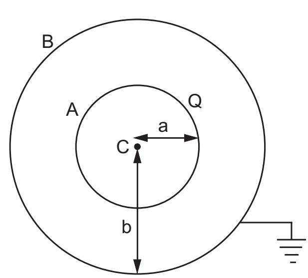
(a) The potential difference between the plates decreases K times.

(b) The energy stored in the capacitor decreases K times.

(c) The change in energy is $\frac{1}{2}C_0\mathcal{E}^2(K-1)$.

(d) The change in energy is $\frac{1}{2}C_0\mathcal{E}^2\left(1-\frac{1}{K}\right)$.

101.



A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common centre of A and B.

(a) The field at a distance r from C, where $a \le r \le b$, is $k\dfrac{Q}{r^2}\cdot$

(b) The potential at a distance r from C, where $a \le r \le b$, is $k\dfrac{Q}{r}\cdot$

(c) The potential difference between A and B is $kQ\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\cdot$

(d) The potential at a distance r from C, where $a \le r \le b$, is $kQ\left(\dfrac{1}{r}-\dfrac{1}{b}\right)\cdot$

Answers

1. b	**2.** c	**3.** b	**4.** c	**5.** d
6. a	**7.** b	**8.** d	**9.** c	**10.** d
11. a	**12.** d	**13.** c	**14.** d	**15.** a
16. a	**17.** b	**18.** a	**19.** a	**20.** d
21. c	**22.** c	**23.** d	**24.** d	**25.** a
26. c	**27.** c	**28.** b	**29.** c	**30.** d
31. c	**32.** b	**33.** c	**34.** a	**35.** b
36. a	**37.** a	**38.** c	**39.** c	**40.** b
41. c	**42.** b	**43.** c	**44.** d	**45.** c
46. c	**47.** a	**48.** b	**49.** c	**50.** d
51. d	**52.** a	**53.** c	**54.** b	**55.** b
56. a, b, c, d	**57.** b, d	**58.** a, b, c, d	**59.** b, c	**60.** a, c
61. a, b,	**62.** a, c, d	**63.** a, b, d	**64.** d	**65.** d
66. a, b, c	**67.** a, c	**68.** a, b, d	**69.** a, c	**70.** a, c
71. a, c	**72.** a, c, d	**73.** a, b, c, d	**74.** d	**75.** d
76. d	**77.** d	**78.** a, b, d	**79.** a, c	**80.** a, d
81. b, c, d	**82.** b, d	**83.** a, b, c, d	**84.** a, c, d	**85.** b, d
86. a, b, c	**87.** b, c	**88.** b, c, d	**89.** b, c	**90.** a, b, c
91. c, d	**92.** a, c	**93.** a, d	**94.** a, b, c, d	**95.** a, b, c, d
96. a, b, c, d	**97.** a, d	**98.** a, b	**99.** a, b, d	**100.** a, b, d
101. a, c, d				

Hints and Solutions to Selected Questions

2. The minimum possible value of a charge is e. Here,

for $q = -\dfrac{Q}{4} = -e, \quad Q = 4e.$

3. The attractive gravitational force $= G\dfrac{m^2}{d^2}$ (d = separation)

and the repulsive electrostatic force $= k\dfrac{Q^2}{d^2}\cdot$

For equilibrium, $Gm^2 = kQ^2$

$$\text{or} \quad \frac{Q}{m} = \sqrt{\frac{G}{k}} = \left[\frac{6.6\times10^{-11}}{9\times10^{9}}\right]^{1/2} \sim 10^{-10}.$$

4. As the equilibrium condition is independent of the separation d, any displacement of the particles causes no change in equilibrium.

6. The force of interaction $= F = k\dfrac{q(Q-q)}{d^2}\cdot$

For F to be maximum, $\dfrac{dF}{dq} = 0 = (Q-2q)\dfrac{k}{d^2}$

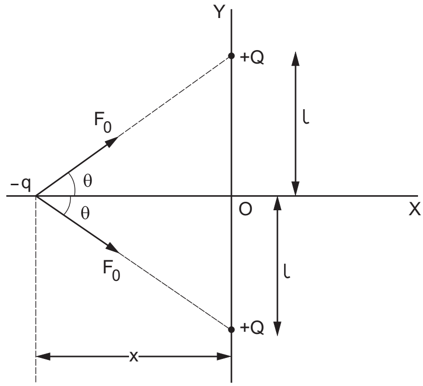
$$\text{or} \quad q = \frac{Q}{2}\cdot$$

7. See the figure in the next page. When the particle is to the left of the origin O, the net force F on it is towards O, and hence causes acceleration a.

$$F = 2F_0 \cos\theta$$

$$= 2\cdot k\frac{qQ}{(x^2+l^2)}\cdot\frac{x}{\sqrt{x^2+l^2}} = 2kQq\frac{x}{(x^2+l^2)^{3/2}}\cdot$$

F is zero for large values of x and also for $x = 0$.

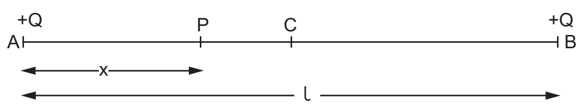


Thus it must increase to a maximum and fall to zero at O.

The acceleration $a = F/m$, must have the same nature.

To the right of O, the net force is to the left while motion is to the right. Thus, the direction of a is opposite to the particle's direction of motion and is taken as negative. The variation of a with x will follow the same pattern.

8.



C is the midpoint of AB. The field is directed to the right (positive) in the region between A and C, zero at C, and to the left (negative) between C and B. The magnitude of E will increase sharply for $x \sim 0$ and $x \sim l$.

9. Equal electrostatic forces act on both the bobs. The weights are also the same. Hence, they have identical free-body diagrams.

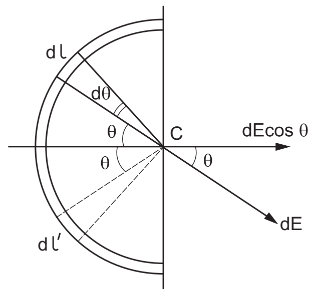
10. The electrostatic force on Q is always perpendicular to its displacement.

11. If charges were placed at all the corners, the field at the centre would be zero. Hence, the field at the centre due to any one charge is equal (and opposite) to the field due to all the other $(n - 1)$ charges.

12. The total charge on the half ring is $\pi R\lambda$. As all points on the ring are at a distance R from the centre, the potential at the centre is

$$V = k \cdot \frac{\pi R\lambda}{R} = k\pi\lambda.$$

13.



$dl = Rd\theta$.

Charge on $dl = \lambda R d\theta$.

Field at C due to $dl = k \cdot \dfrac{\lambda R d\theta}{R^2} = dE$.

We need to consider only the component $dE\cos\theta$, as the component $dE\sin\theta$ will cancel out because of the field at C due to the symmetrical element dl'.

The total field at C is $= 2\displaystyle\int_0^{\pi/2} dE\cos\theta$

$$= 2\frac{k\lambda}{R}\int_0^{\pi/2} \cos\theta\, d\theta = 2k\frac{\lambda}{R}\cdot$$

14. $F = k\dfrac{2Q^2}{d^2}$, where d = separation.

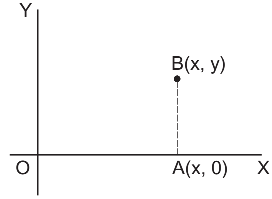
Since charge is shared, the charge on each ball $= \dfrac{1}{2}(2Q - Q) = \dfrac{Q}{2}\cdot$

The force now becomes $F' = k\dfrac{(Q/2)^2}{d^2} = \dfrac{1}{4}\cdot k\dfrac{Q^2}{d^2} = \dfrac{F}{8}\cdot$

15. The force is $\vec{F} = Q\vec{E}$.

The work is $W = \vec{F}\cdot\vec{r} = Q(\vec{E}\cdot\vec{r})$

16. $E_x = -\dfrac{\partial V}{\partial x} \qquad E_y = -\dfrac{\partial V}{\partial y}$



If y is constant (= 0), $\displaystyle\int_{V_0}^{V_A} dV = -\int_0^x E_x\, dx$

or $\quad V_A - V_0 = -E_x x.$

When x is constant (= x), $\displaystyle\int_{V_A}^{V_B} dV = -\int_0^y E_y\, dy$

or $\quad V_B - V_A = -E_y\, dy.$

Adding, $V_B - V_0 = -xE_x - yE_y$

or $\quad V_B = V_0 - xE_x - yE_y.$

17. **TIP** The work done is independent of the path followed, and is equal to $(q\vec{E}) \cdot \vec{r}$, where $\vec{r}$ = displacement from P to S.

Here, $\vec{r} = -a\hat{i} - b\hat{j}$, while $\vec{E} = E\hat{i}$.

$\therefore \quad$ work $= -(qE\hat{i}) \cdot (a\hat{i} + b\hat{j}) = -qaE$

18. $E_x = -\dfrac{\partial V}{\partial x} = -8x.$

$E_y = E_z = 0$ as V is independent of y and z.

For $x = 1$, $\quad \vec{E} = -8\hat{i}.$

19. The line integral gives the work done in bringing a unit positive charge, on which the force is $\vec{E}$, from ∞ to the centre of the ring. This is equal to the potential at the centre, taking the potential at ∞ to be zero. Its value is $V = k\dfrac{Q}{R} = 9 \times 10^9 \times \dfrac{1.11 \times 10^{-10}}{0.5} = 2$ V.

20. $V = K\left[\dfrac{q}{x_0} - \dfrac{q}{2x_0} + \dfrac{q}{3x_0} - \dfrac{q}{4x_0} \ldots\right]$

$= k\dfrac{q}{x_0}\left[1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} \ldots\right]$

$= \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{x_0} \cdot \ln 2.$

21. $V = k\dfrac{Q}{2R^3}(3R^2 - r^2).$

22. Using the expression of the previous question's answer, for $r = 0$, potential at the centre $= V_c = k\dfrac{3Q}{2R}\cdot$

We require a point where $V = \dfrac{V_c}{2} = k\dfrac{3Q}{4R}\cdot$

This point cannot lie inside the sphere where $V \geq k\dfrac{Q}{R}\cdot$

Let the point lie outside the sphere, at a distance r from the centre. Then,

$$V = k\frac{Q}{r} = k\frac{3Q}{4R} \qquad \text{or} \quad r = \frac{4}{3}R$$

Distance from the surface $= r - R = \dfrac{R}{3}\cdot$

23. Potential at $\infty = V_\infty = 0.$

Potential at the surface of the sphere $= V_s = k\dfrac{Q}{R}\cdot$

Potential at the centre of the sphere $= V_c = \dfrac{3}{2}k\dfrac{Q}{R}\cdot$

Let m and $-q$ be the mass and the charge of the particle respectively.

Let v_0 = speed of the particle at the centre of the sphere.

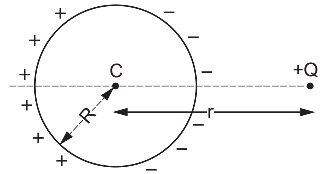
$$\frac{1}{2}mv^2 = -q[V_\infty - V_s] = qk\frac{Q}{R}\cdot$$

$$\frac{1}{2}mv_0^2 = -q[V_\infty - V_c] = q\cdot\frac{3}{2}k\frac{Q}{R}\cdot$$

Dividing, $\dfrac{v_0^2}{v^2} = \dfrac{3}{2} = 1.5 \qquad$ or $\quad v_0 = \sqrt{1.5}v.$

26. See the figure given in the next page. The total field at C must be zero.

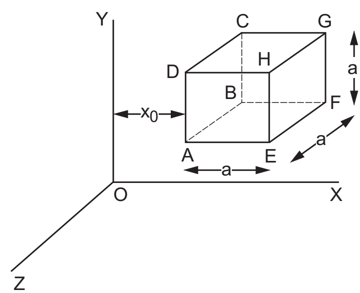
The field at C due to the point charge is $E = k\dfrac{Q}{r^2}$, to the left.



∴ the field at C due to the induced charges must be $k\,Q/r^2$ to the right, directed towards Q.

27. The field inside a conductor is zero. Hence, the point charge experiences no force.

28.



The field at the face ABCD = $E_0 x_0 \hat{i}$.

∴ flux over the face ABCD = $-(E_0 x_0)a^2$.

The negative sign arises as the field is directed into the cube.

The field at the face EFGH = $E_0(x_0 + a)\hat{i}$.

∴ flux over the face EFGH = $E_0(x_0 + a)a^2$.

The flux over the other four faces is zero as the field is parallel to the surfaces.

∴ total flux over the cube = $E_0 a^3 = \dfrac{1}{\varepsilon_0} q$,

where q is the total charge inside the cube.

∴ $q = \varepsilon_0 E_0 a^3$.

30. The maximum length of the string which can fit into the cube is $\sqrt{3}a$, equal to its body diagonal. The total charge inside the cube is $\sqrt{3}a\lambda$, and hence the total flux through the cube is $\sqrt{3}a\lambda/\varepsilon_0$.

31. ***Method 1***

When the two are joined by a metal wire, they become a single conductor. As charge can reside only on the outer surface of a conductor, the entire charge Q must flow to the outer sphere.

Method 2

When charge of amount q has flown from A to B, the charge on A is $(Q - q)$.

The potentials of A and B are

$$V_A = k\frac{Q-q}{r} + k\frac{q}{R}$$

$$V_B = k\frac{Q-q}{R} + k\frac{q}{R}$$

$$\therefore \quad V_A - V_B = k(Q-q)\left(\frac{1}{r} - \frac{1}{R}\right) > 0$$

$$\therefore \quad V_A > V_B \text{ for all values of } q.$$

Charge will flow from A to B till $q = Q$.

33. **TIP** A positive charge which is free to move will always move from higher to lower potential.

34. Let σ = charge per unit area on the earth's surface.

Then, the field near the surface $= E = \dfrac{\sigma}{\varepsilon_0}$.

The upward force on a proton $= eE = \dfrac{e\sigma}{\varepsilon_0}$.

The downward force $= mg$.

For equilibrium, $mg = \dfrac{e\sigma}{\varepsilon_0}$ or $\sigma = \dfrac{\varepsilon_0 mg}{e}$.

$$\sigma = \frac{8.85 \times 10^{-12} \times 1.67 \times 10^{-27} \times 10}{1.6 \times 10^{-19}}\ \text{C/m}^2 \sim 10^{-18}\ \text{C/m}^2.$$

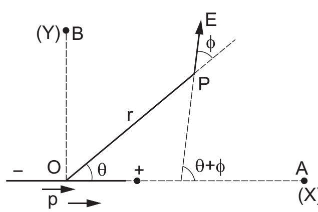
35. The metal sheet produces a uniform upward electric field, causing a constant downward force on the bob in all positions. This increases the effective value of g, reducing T.

36. As in the previous question, the effective value of g will change. However, the time period of a spring–block system does not depend on g.

37. As in Q. No. 34, the force on the electron is $e\sigma/\varepsilon_0$ and its acceleration towards the metal sheet is $e\sigma/m\varepsilon_0$. The electron will move as a projectile with an effective value of $g = e\sigma/m\varepsilon_0$.

Its maximum range will then be $\dfrac{u^2}{e\sigma/m\varepsilon_0} = \dfrac{u^2 m\varepsilon_0}{e\sigma}$.

38, 39, 40.



$$E_P = k\frac{2p}{r^3}\sqrt{3\cos^2\theta + 1}.$$

$$V_P = k\frac{p}{r^2}\cos\theta.$$

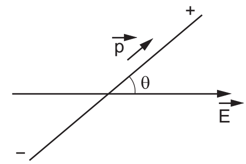
For A, $\theta = 0$, for B, $\theta = 90°$, $r = OA = OB$, $\tan\phi = \dfrac{1}{2}\tan\theta$.

In Q. No. 40, $\theta + \phi = 90°$

or $\quad \tan(90 - \theta) = \dfrac{1}{2}\tan\theta$

or $\quad 2\cot\theta = \tan\theta \qquad$ or $\tan\theta = \sqrt{2}$.

41. $\tau = pE\sin\theta$.



$$\text{Work} = \int \tau d\theta = \int_0^\theta pE\sin\theta = pE[\cos\theta]_\theta^0$$

$$= pE(1 - \cos\theta).$$

42. $C = 4\pi\varepsilon_0 R = \dfrac{6.4\times 10^6}{9\times 10^9}\text{ F} \sim 10^{-3}\text{ F}.$

43. Potentials of the two spheres $= \pm\dfrac{e}{C}\cdot$

$$\therefore \quad \text{potential difference} = \frac{2e}{C} = \frac{2e}{4\pi\varepsilon_0 r} = 9\times 10^9\,\frac{2\times 1.6\times 10^{-19}}{9\times 10^{-5}}\text{ V}$$

$$= 3.2\times 10^{-5}\text{ V} = 32\ \mu\text{V}.$$

44. The capacitances are $C_1 = 4\pi\varepsilon_0 R$ and $C_2 = 4\pi\varepsilon_0(2R) = 2C_1$.

The common potential $= V = \dfrac{Q}{C_1 + C_2}\cdot$

The charge flowing = charge on C_2

$$= C_2 V = Q\frac{C_2}{C_1 + C_2} = Q\frac{2C_1}{C_1 + 2C_1} = \frac{2}{3}Q.$$

45. TIP Plane conducting surfaces facing each other must have equal and opposite charge densities. Here, as the plate areas are equal, $Q_2 = -Q_3$.

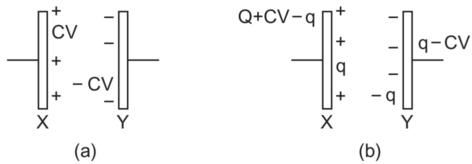
The charge on a capacitor means the charge on the inner surface of the positive plate—in this case, Q_2.

Potential difference between the plates

= charge on the capacitor ÷ capacitance.

$$\therefore \quad \text{potential difference} = \frac{Q_2}{C} = \frac{2Q_2}{2C} = \frac{Q_2 - (-Q_2)}{2C} = \frac{Q_2 - Q_3}{2C}\cdot$$

46. In the figure given in the next page, let X and Y be positive and negative plates. After charging from the cell, the inner faces of X and Y have charges $\pm CV$, as shown in (a). The outer surfaces have no charge.



When charge Q is given to X, let the inner faces of X and Y have charges $\pm q$ (see Q. No. 45). Then, by the principle of charge conservation, the outer faces have charges $(Q + CV - q)$ for X and $(q - CV)$ for Y, as shown in (b). Now, the outer faces must have equal charges.

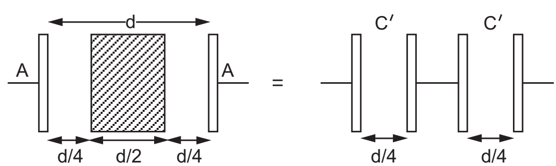
$$\therefore \quad Q + CV - q = q - CV$$

or $\quad 2q = 2CV + Q \qquad$ or $\quad q = CV + \dfrac{Q}{2}$.

Potential difference $= \dfrac{q}{C} = V + \dfrac{Q}{2C}$.

47. TIP When a capacitor is fully charged, it draws no current. When no current flows in the circuit, potential difference across the cell = emf of cell = potential difference across the capacitor.

48.



Method 1

Before the metal sheet is inserted, $C = \dfrac{\varepsilon_0 A}{d}$.

After the sheet is inserted, the system is equivalent to two capacitors in series, each of capacitance $C' = \dfrac{\varepsilon_0 A}{(d/4)} = 4C$.

The equivalent capacity is now $2C$.

Method 2

If a dielectric slab of dielectric constant K and thickness t is inserted in the air gap of a capacitor of plate separation d and plate area A, its capacitance becomes

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}.$$

Here, the initial capacitance, $C = \frac{\varepsilon_0 A}{d}$.

For the metal sheet, $t = \frac{d}{2}$, $K = \infty$. The new capacitance is

$$C' = \frac{\varepsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{\frac{d}{2}} = 2\frac{\varepsilon_0 A}{d} = 2C.$$

49. Charges on the two capacitors are 36 μC and 72 μC. When they are connected in opposition,
the total charge on the system = (72 – 36) μC = 36 μC.

This is shared between the capacitors so that they acquire the same potential difference. Let this be V.

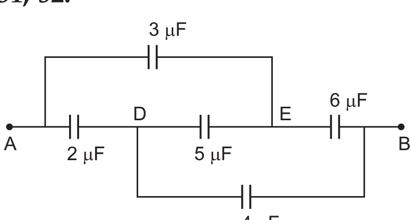
$(3\ \mu F)V + (6\ \mu F)V = 36\ \mu C$ or $V = 4$ V.

50. Let Q amount of charge flow through the MN branch.

$V = 60\ \text{V} = \frac{Q}{2C} + \frac{Q}{C} + \frac{Q}{2C} = 2\frac{Q}{C}$ or $Q = 30C$ V.

Potential difference between M and N $= \frac{Q}{C} = \frac{30C}{C}$ V = 30 V.

51, 52.



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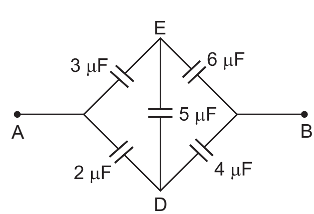


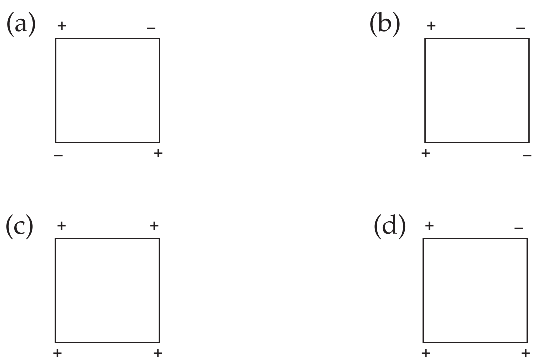
Rearranging the circuit, the points E and D are at the same potential (by symmetry). The 5-μF capacitor can therefore be removed.

54. **TIP** When a capacitor remains connected to a cell, its potential difference remains constant and is equal to the emf of the cell. Any change in the capacitor may change its capacitance, its charge and the energy stored in it.

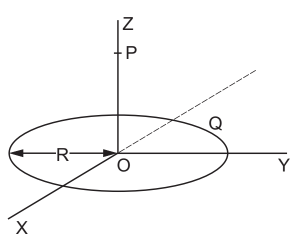
When the dielectric slab is taken out, the capacitance decreases. Hence charge decreases, and the difference in charge is returned to the cell.

55. **TIP** When a charged capacitor is disconnected from the cell to which it was connected, its charge remains constant. Any change in the capacitor may change its capacitance, its stored energy and its potential difference.

58.



62.



Let OP = z_0.

Field at P = $E = kQ \dfrac{z_0}{(R^2 + z_0^2)^{3/2}}$

E is always directed away from O. Hence a negatively charged particle is accelerated towards O and undergoes periodic motion.

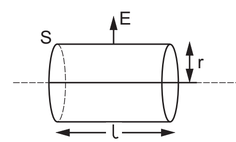
For $z_0 << R$, the acceleration is proportional to the z-coordinate, and the particle undergoes approximate SHM.

63. By conservation of energy, $\frac{1}{2}mv^2 = k \cdot \frac{Q^2}{x_{\min}}$,

where $x_{\min}$ = minimum separation.

74. The field is zero inside a conductor and hence lines of force cannot exist inside it. Also, due to induced charges on its surface, the field is distorted close to its surface, and a line of force must deviate near the surface outside the sphere.

75. $\oint_s \vec{E} \cdot d\vec{S} = 2\pi r l E = \frac{1}{\varepsilon_0}(\lambda l)$



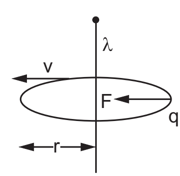
or $\quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

76. $E = \frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\partial V}{\partial r}$

or $\quad \int_{V_a}^{V_b} dV = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{dr}{r}$

or $\quad V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\left(\frac{b}{a}\right)$.

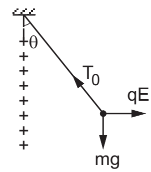
77, 78.



$F = \frac{mv^2}{r} = qE = q \cdot \frac{\lambda}{2\pi\varepsilon_0 r}$

or $\quad v^2 = \frac{q\lambda}{2\pi\varepsilon_0 m}$.

79. $E = \dfrac{\sigma}{2\varepsilon_0}$, $T_0 \cos\theta = mg$, $T_0 \sin\theta = qE$



$\tan\theta = \dfrac{q}{mg}\left(\dfrac{\sigma}{2\varepsilon_0}\right)$.

As $T_0 > mg$, the effective value of g is increased.

Hence, time period of oscillation decreases.

85. As A and C are earthed, they are connected to each other. Hence, 'A + B' and 'B + C' are two capacitors with the same potential difference. If B is closer to A than to C then the capacitance C_{AB} is $> C_{BC}$. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upward force on B. This can balance its weight.

86. When either A or C is earthed (but not both together), a parallel-plate capacitor is formed with B, with $\pm Q$ charges on the inner surfaces. [The other plate, which is not earthed, plays no role]. Hence charge of amount $+Q$ flows to the earth.

When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio 1 : 2, and hence share charge Q in the same ratio.

91. The capacitances of the two are $C_1 = 4\pi\varepsilon_0 R$ and $C_2 = 4\pi\varepsilon_0(2R)$.

The initial energy, $E_i = \dfrac{Q^2}{2C_1}$.

The final energy, $E_f = \dfrac{Q^2}{2C_2}$.

The heat produced $= E_i - E_f = \dfrac{Q^2}{2}\left[\dfrac{1}{4\pi\varepsilon_0 R} - \dfrac{1}{2 \times 4\pi\varepsilon_0 R}\right]$

$$= k\frac{Q^2}{2R}\left[1 - \frac{1}{2}\right] = \frac{kQ^2}{4R}.$$

□

• *Revision Exercise 3* •

Choose the correct option in each of the following questions. Only one option is correct in each question.

R1. A nonconducting ring of radius R has uniformly distributed positive charge. A small part of the ring, of length d, is removed ($d << R$). The electric field at the centre of the ring will now be

(a) directed towards the gap, inversely proportional to R^3

(b) directed towards the gap, inversely proportional to R^2

(c) directed away from the gap, inversely proportional to R^3

(d) directed away from the gap, inversely proportional to R^2

R2. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.

(a) If Y is fixed, both p and E are conserved.

(b) If Y is fixed, E is conserved, but not p.

(c) If both are free to move, p is conserved but not E.

(d) If both are free, E is conserved, but not p.

R3. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time, finally

(a) X will stop, Y will move with velocity u

(b) X and Y will both move with velocities $u/2$ each

(c) X will stop, Y will move with velocity $< u$

(d) both will move with velocities $< u/2$

R4. In a uniform electric field, the potential is 10 V at the origin of coordinates, and 8 V at each of the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). The potential at the point (1, 1, 1) will be

(a) 0 (b) 4 V

(c) 8 V (d) 10 V

R5. Two conducting, concentric, hollow spheres A and B have radii a and b respectively, with A inside B. Their common potential is V. A is now given some charge such that its potential becomes zero. The potential of B will now be

(a) 0 (b) $V(1 - a/b)$

(c) Va/b (d) $V(b - a)/(b + a)$

R6. A positive charge is fixed at the origin of coordinates. An electric dipole, which is free to move and rotate, is placed on the positive x-axis. Its moment is directed away from the origin. The dipole will

(a) move towards the origin (b) move away from the origin

(c) rotate by $\pi/2$ (d) rotate by π

R7. An electric dipole is fixed at the origin of coordinates. Its moment is directed in the positive x-direction. A positive charge is moved from the point $(r, 0)$ to the point $(-r, 0)$ by an external agent. In this process, the work done by the agent is

(a) positive and inversely proportional to r

(b) positive and inversely proportional to r^2

(b) negative and inversely proportional to r

(d) negative and inversely proportional to r^2

R8. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges are placed at $(n - 1)$ corners. At the centre, the intensity is E and the potential is V. The ratio V/E has magnitude

(a) rn (b) $r(n - 1)$

(c) $(n - 1)/r$ (d) $r(n - 1)/n$

R9. A point charge Q is placed at the centre of an uncharged conducting shell. Let r be the distance of a point from Q. The point may lie either inside or outside the shell. The electric intensity at the point will be $Q/4\pi\varepsilon_0 r^2$ if the point lies

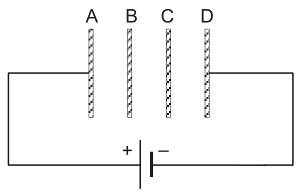
(a) inside the shell but not outside it

(b) outside the shell but not inside it

(c) either inside or outside the shell

(d) close to either of the surfaces of the shell only

R10.



A, B, C, and D are identical, parallel, conducting plates arranged as shown, with equal separations between consecutive plates. A and D are connected to a cell. If B is now connected to C, which of the following will occur?

(a) Only that some charge will flow through the cell.

(b) Only that some charge will flow from B to C.

(c) Only that there will be no electric field between B and C.

(d) More than one of the above

R11. A is a nonconducting ring and B is a conducting ring. They have the same radius and equal amounts of charge. This is distributed nonuniformly on A and uniformly on B. X and Y are points on the axes of A and B respectively, at equal distances from their centres. X and Y will have

(a) the same potential and intensity

(b) the same potential but different intensities

(c) different potentials but the same intensity

(d) different potentials and different intensities

R12. A drop of mercury has some charge. If it breaks up into a number of identical droplets, the electrostatic energy of the system will

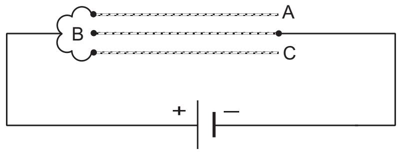
(a) increase

(b) decrease

(c) remain constant

(d) any of the above depending on the number of droplets

R13.



A, B and C are identical, parallel, conducting plates arranged at equal separations. B lies between A and C. A and C are joined together and connected to one terminal of a cell. B is joined to the other terminal of the cell. A and C now have charge Q each. B has charge q.

(a) $Q = -2q$ (b) $q = -2Q$

(c) $q = -Q$ (d) $q = -3Q/2$

R14. A and B are two hollow, concentric, conducting spheres, with A inside B. A and B are given charges Q_1 and Q_2 respectively. If they are now joined by a thin wire, what charge will flow from A to B?

(a) $Q_1 - Q_2$ (b) $(Q_1 - Q_2)/2$

(c) $(Q_1 + Q_2)/2$ (d) None of these

R15. The strength of a dielectric is defined as the maximum electric intensity which it can withstand. A parallel-plate capacitor of plate area A on one side is filled with a dielectric of strength E and permittivity ε. The maximum charge which can be given to the capacitor is

(a) εEA (b) EA/ε

(c) $E/\varepsilon A$ (d) $\varepsilon E/A$

R16. A parallel-plate air capacitor has capacity C. A dielectric slab of dielectric constant K, whose thickness is half of the air gap between the plates, is now inserted between the plates. The capacity of the capacitor will now be

(a) $KC/2$ (b) $2KC/(K+1)$

(c) $2KC/(K-1)$ (d) $C(K+1)/(K-1)$

R17. When two uncharged, conducting spheres of radius 0.1 mm each collide, 7 electrons get transferred from one to the other. The potential difference between the spheres will be about

(a) 4×10^{-6} V (b) 2×10^{-6} V

(c) 2×10^{-4} V (d) 10^{-4} V

R18. In a parallel-plate capacitor, the plates are kept vertical. The upper half of the space between the plates is filled with a dielectric with dielectric constant K and the lower half with a dielectric with dielectric constant $2K$. The ratio of the charge density on the upper half of the plates to the charge density on the lower half of the plates will be equal to

(a) 1 (b) 2

(c) 1/2 (d) 3/2

R19. A parallel-plate capacitor, whose plates are kept horizontal, is charged from a cell and then isolated from it. A dielectric slab which can just fit in the gap between the plates is now inserted to fill exactly half of the gap and then left alone. Neglect gravity and friction. The slab will

(a) remain stationary

(b) move further into the gap

(c) move out of the gap

(d) either (b) or (c) depending on whether its dielectric constant is greater or less than 2

R20. 1000 identical drops of mercury are charged to a potential of 1 V each. They join to form a single drop. The potential of this drop will be

(a) 0.01 V (b) 0.1 V

(c) 10 V (d) 100 V

R21. Three charges $+Q$, $+Q$ and $-Q$ are placed at the corners of an equilateral triangle. The ratio of the force on a positive charge to the force on the negative charge will be equal to

(a) 1 (b) 2

(c) $\sqrt{3}$ (d) $1/\sqrt{3}$

R22. A simple pendulum has time period T. Charges are now fixed at the point of suspension of the pendulum and on the bob. If the pendulum continues to oscillate, its time period will now be

(a) greater than T

(b) equal to T

(c) less than T

(d) either (a) or (c) depending on whether the charges attract or repel each other

R23. A parallel-plate capacitor is connected to a cell. A sheet of dielectric and a metal sheet are now introduced between the plates of the capacitor, parallel to these plates. The electric intensity between the positive plate of the capacitor and the dielectric sheet is E_1, between the dielectric sheet and the metal sheet is E_2, and between the metal sheet and the negative plate is E_3. Then

(a) $E_1 = E_2 = E_3$

(b) $E_1 = E_2 \neq E_3$

(c) $E_1 \neq E_2 = E_3$

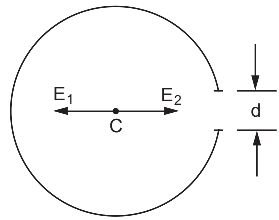
(d) $E_1 \neq E_2 \neq E_3$

Answers to Revision Exercise 3

R1. a	**R2.** b	**R3.** a	**R4.** b	**R5.** b
R6. a	**R7.** d	**R8.** b	**R9.** c	**R10.** d
R11. b	**R12.** b	**R13.** b	**R14.** d	**R15.** a
R16. b	**R17.** c	**R18.** c	**R19.** b	**R20.** d
R21. d	**R22.** b	**R23.** a		

Hints and Solutions to Selected Questions

R1. Let Q = charge on entire ring.



Charge on section removed

$$= dQ = \frac{Qd}{2\pi R}.$$

For the complete ring, field at C is zero. Let E_1 and E_2 be the fields at the centre due to dQ and due to the remaining ring respectively.

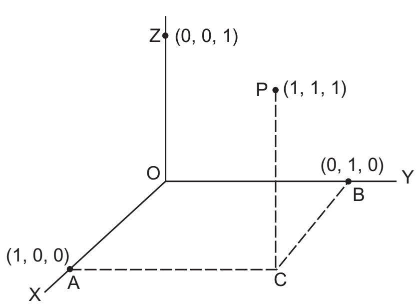
$\therefore$ E_1 and E_2 must be equal and opposite.

E_2 is directed towards gap $= E_1 = k \cdot \frac{dQ}{R^2} = k \cdot \frac{Qd}{2\pi R^3}$.

R2. If Y is fixed, the force acting on Y to keep it fixed is external to the system 'X + Y'. Hence p is not conserved. As this force does not perform any work, as its point of application is fixed, E is conserved.

R3. This is similar to one-dimensional, elastic collisions between bodies of equal masses, and results in exchange of velocities.

R4.



In a uniform electric field, equal displacements cause equal changes in potential.

Each of the displacements *OA*, *AC* and *CP* cause a drop in potential of 2 V. Thus, potential at P is $10 - 3 \times 2 = 4$ V.

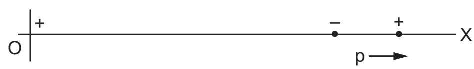
R5. Let initial charge on B be Q. Then, $V = kQ/b$. There is no initial charge on A, as A and B are at the same potential. Let q charge be now given to A. Potential of A now is

$$k\frac{q}{a} + k\frac{Q}{b} = 0 \qquad \text{or} \qquad q = -\frac{Qa}{b}.$$

Potential of B now is

$$k\left(\frac{Q+q}{b}\right) = \frac{k}{b}\left[Q - \frac{Qa}{b}\right] = k\frac{Q}{b}\left(1 - \frac{a}{b}\right) = V\left(1 - \frac{a}{b}\right).$$

R6.



The dipole exerts an attractive force on the charge at the origin. As a reaction, it is attracted by this charge towards the origin.

R7. The potential due to the dipole is positive at $(r, 0)$ and negative at $(-r, 0)$. Also, this potential is inversely proportional to r^2. Work is done by the field in moving the charge.

R8. At the centre, the intensity is effectively due to one charge and the potential is due to $(n - 1)$ charges.

R9. Use Gauss's theorem.

R10. Initially, B is at a higher potential than C. When they are connected, charge will flow from B to C. When they reach the same potential, there will be no field between them. This alters the capacity between A and D, and charge will flow through the cell.

R11. Intensity will depend on the charge distribution but potential will depend only on distance.

R12. The droplets repel each other. Hence, the system does work when the drop breaks up.

R13. The A + B capacitor and the C + B capacitor are in parallel. B is the negative plate for both.

R14. All charge on the inner conductor will flow to the other conductor, as A + B now form a single conductor, and all charge on a conductor must reside on its outer surface.

R15. Let K = dielectric constant, $\varepsilon = \varepsilon_0 K$.

$$\sigma = \frac{Q}{A} \qquad E = \frac{\sigma}{\varepsilon_0 K} \qquad \text{or} \qquad Q = \varepsilon E a.$$

R16. $C = \varepsilon_0 Ad$

Let capacity with dielectric be C'

$$\sigma = \frac{Q}{A} \qquad V = \frac{\sigma}{\varepsilon_0} \cdot \frac{d}{2} + \frac{\sigma}{K\varepsilon_0} \cdot \frac{d}{2} = \frac{Q}{A}\frac{d}{2\varepsilon_0}\left(1 + \frac{1}{K}\right)$$

$$C' = \frac{Q}{V} = \frac{2\varepsilon_0 A}{d\left(1 + \frac{1}{K}\right)} = C \cdot \frac{2K}{K+1} \cdot$$

R17. $Q = 7e \qquad C = 4\pi\varepsilon_0 r \qquad V = \frac{Q}{C} = \frac{7e}{4\pi\epsilon_0 r}$

$$\text{p.d.} = 2V = \frac{2 \times 7 \times 1.6 \times 10^{-19}}{10^{-4}} \times 9 \times 10^9 \simeq 2 \times 10^{-4}\,V.$$

R18. The p.d. between the plates is the same at all the points.

As p.d. = intensity × distance, the intensity E is the same in both dielectrics.

Let σ_1 and σ_2 be the charge densities in the upper and lower halves.

$$\therefore \quad E = \frac{\sigma_1}{\varepsilon_0 K} = \frac{\sigma_2}{\varepsilon_0 \times 2K} \cdot \qquad \therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{1}{2} \cdot$$

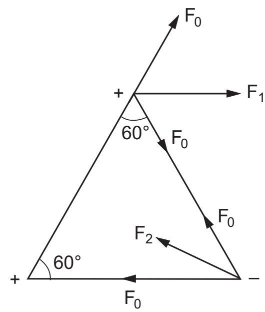
R19. Energy $= \frac{Q^2}{2C} \cdot$ Here, Q is constant.

As the slab moves into the gap, C increases and hence energy decreases. A system tends to move towards lower energy.

R20. Volume $= \frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$ or $r = \frac{R}{10}$

$Q = CV = (4\pi\varepsilon_0 R) \times V = 1000(4\pi\varepsilon_0 r)\,1$ or $V = 10^3 \frac{\tau}{R} = \frac{10^3}{10} = 100$ V

R21.



$F_1 = 2F_0 \cos 60° = F_0$

$F_2 = 2F_0 \cos 30° = \sqrt{3}\, F_0$

$\frac{F_1}{F_2} = \frac{1}{\sqrt{3}}$

R22. The additional electrostatic force acts along the string. This does not have a component in the direction of motion of the bob.

R23. The dielectric sheet and the metal sheet are without charge. Apply Gauss's law.

□

2

Electric Current in Conductors

• *Type 1* •

Choose the correct option (a, b, c or d).

1. The carrier density (number of free electrons per m^3) in metallic conductors is of the order of

(a) 10^{10} (b) 10^{16}

(c) 10^{22} (d) 10^{28}

2. The drift velocity of electrons in a metallic conductor carrying a current is usually of the order of

(a) 1 cm/s (b) 10 m/s

(c) 10^4 m/s (d) 10^8 m/s

3. The resistance of a metallic conductor increases with temperature due to

(a) change in carrier density

(b) change in the dimensions of the conductor

(c) increase in the number of collisions among the carriers

(d) increase in the rate of collisions between the carriers and the vibrating atoms of the conductor

4. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(a) each of them increases (b) each of them decreases

(c) copper increases and that of germanium decreases

(d) copper decreases and that of germanium increases

5. A straight conductor of uniform cross-section carries a current I. Let s = specific charge of an electron. The momentum of all the free electrons per unit length of the conductor, due to their drift velocities only, is

(a) Is (b) I/s

(c) $\sqrt{I/s}$ (d) $(I/s)^2$

6. Current flows through a metallic conductor whose area of cross-section increases in the direction of the current. If we move in this direction,

(a) the current will change

(b) the carrier density will change

(c) the drift velocity will increase

(d) the drift velocity will decrease

7. A nonconducting ring of radius R has charge Q distributed unevenly over it. If it rotates with an angular velocity ω, the equivalent current will be

(a) zero (b) $Q\omega$

(c) $Q\dfrac{\omega}{2\pi}$ (d) $Q\dfrac{\omega}{2\pi R}$

8. All the edges of a block with parallel faces are unequal. Its longest edge is twice its shortest edge. The ratio of the maximum to minimum resistance between parallel faces is

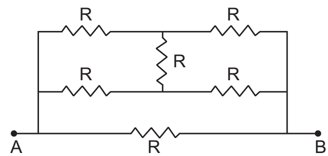
(a) 2

(b) 4

(c) 8

(d) indeterminate unless the length of the third edge is specified

9. In the network shown below, the equivalent resistance between A and B is

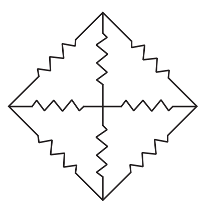


(a) $R/2$ (b) R
(c) $2R$ (d) $4R$

10. A and B are two points on a uniform ring of resistance R. The $\angle ACB = \theta$, where C is the centre of the ring. The equivalent resistance between A and B is

(a) $\frac{R}{4\pi^2}(2\pi - \theta)\theta$ (b) $R\left(1 - \frac{\theta}{2\pi}\right)$
(c) $R\frac{\theta}{2\pi}$ (d) $R\frac{2\pi - \theta}{4\pi}$

11.



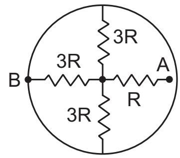
In the network shown, each resistance is equal to R. The equivalent resistance between diagonally opposite corners is

(a) R (b) $R/3$
(c) $2R/3$ (d) $4R/3$

12. In the previous question, the equivalent resistance between adjacent corners is

(a) R (b) $\frac{2}{3}R$
(c) $\frac{3}{7}R$ (d) $\frac{8}{15}R$

13. In the network shown below, the ring has zero resistance. The equivalent resistance between the point A and B is

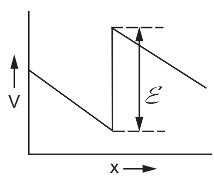


(a) $2R$ (b) $4R$

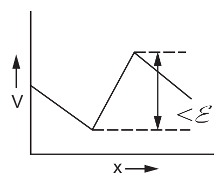
(c) $7R$ (d) $10R$

14. The two ends of a uniform conductor are joined to a cell of emf $\mathcal{E}$ and some internal resistance. Starting from the midpoint P of the conductor, we move in the direction of the current and return to P. The potential V at every point on the path is plotted against the distance covered (x). Which of the following best represents the resulting curve?

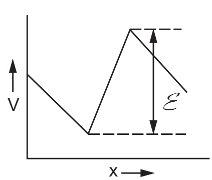
(a)



(b)



(c)



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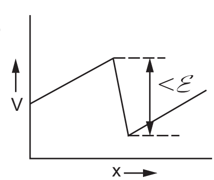


15. The emf of a cell is $\mathcal{E}$ and its internal resistance is r. Its terminals are connected to a resistance R. The potential difference between the terminals is 1.6 V for $R = 4\ \Omega$, and 1.8 V for $R = 9\ \Omega$. Then,

(a) $\mathcal{E} = 1$ V, $r = 1\ \Omega$ (b) $\mathcal{E} = 2$ V, $r = 1\ \Omega$

(c) $\mathcal{E} = 2$ V, $r = 2\ \Omega$ (d) $\mathcal{E} = 2.5$ V, $r = 0.5\ \Omega$

16. N identical cells are connected to form a battery. When the terminals of the battery are joined directly (short-circuited), current I flows in the circuit. To obtain the maximum value of I,

(a) all the cells should be joined in series
(b) all the cells should be joined in parallel
(c) two rows of $N/2$ cells each should be joined in parallel
(d) $\sqrt{N}$ rows of $\sqrt{N}$ cells each should be joined in parallel, given that $\sqrt{N}$ is an integer

17. N identical cells, each of emf $\mathcal{E}$ and internal resistance r, are joined in series. Out of these, n cells are wrongly connected, i.e., their terminals are connected in reverse of that required for series connection. $n < N/2$. Let $\mathcal{E}_0$ be the emf of the resulting battery and r_0 be its internal resistance,

(a) $\mathcal{E}_0 = (N - n)\mathcal{E},\ r_0 = (N - n)r$
(b) $\mathcal{E}_0 = (N - 2n)\mathcal{E},\ r_0 = (N - 2n)r$
(c) $\mathcal{E}_0 = (N - 2n)\mathcal{E},\ r_0 = Nr$
(d) $\mathcal{E}_0 = (N - n)\mathcal{E},\ r_0 = Nr$

18. n identical cells, each of emf $\mathcal{E}$ and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is

(a) zero (b) $\mathcal{E}$
(c) $\dfrac{\mathcal{E}}{n}$ (d) $\dfrac{n-1}{n}\mathcal{E}$

19. n identical cells, each of emf $\mathcal{E}$ and internal resistance r, are joined in series to form a closed circuit. One cell (A) is joined with reversed polarity. The potential difference across each cell, except A, is

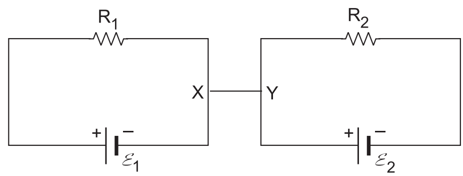
(a) $\dfrac{2\mathcal{E}}{n}$ (b) $\dfrac{n-1}{n}\mathcal{E}$
(c) $\dfrac{n-2}{n}\mathcal{E}$ (d) $\dfrac{2n}{n-2}\mathcal{E}$

20. In the previous question, the potential difference across A is

(a) $\dfrac{2\mathcal{E}}{n}$ (b) $\mathcal{E}\left(1 - \dfrac{1}{n}\right)$

(c) $2\mathcal{E}\left(1-\frac{1}{n}\right)$ (d) $\mathcal{E}\left(\frac{n-2}{n}\right)$

21.



In the circuit shown above, the conductor XY is of negligible resistance. Then

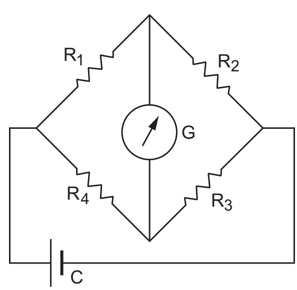
(a) current will flow through XY if $\mathcal{E}_1 \neq \mathcal{E}_2$

(b) current will flow through XY if $\frac{\mathcal{E}_1}{R_1} \neq \frac{\mathcal{E}_2}{R_2}$

(c) current will flow through XY if $\frac{\mathcal{E}_1+\mathcal{E}_2}{R_1+R_2} \neq \frac{|\mathcal{E}_1-\mathcal{E}_2|}{R_1-R_2}$

(d) no current will flow through XY

22.



The Wheatstone bridge shown in the above figure is balanced. If the positions of the cell C and the galvanometer G are now interchanged, G will show zero deflection

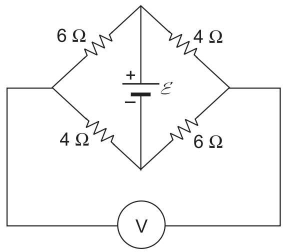
(a) in all cases

(b) only if all the resistances are equal

(c) only if $R_1 = R_3$ and $R_2 = R_4$

(d) only if $R_1/R_3 = R_2/R_4$

23.



In the circuit shown above, the voltmeter is of large resistance. The emf of the cell is $\mathcal{E}$. The reading of the voltmeter is

(a) zero (b) $\frac{\mathcal{E}}{10}$

(c) $\frac{\mathcal{E}}{5}$ (d) $\frac{\mathcal{E}}{2}$

24. A milliammeter of range 10 mA has a coil of resistance 1 Ω. To use it as an ammeter of range 1 A, the required shunt must have a resistance of

(a) $\frac{1}{101}\ \Omega$ (b) $\frac{1}{100}\ \Omega$

(c) $\frac{1}{99}\ \Omega$ (d) $\frac{1}{9}\ \Omega$

25. To use the milliammeter of the previous question as a voltmeter of range 10 V, a resistance R is placed in series with it. The value of R is

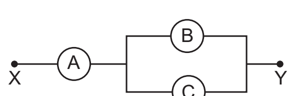
(a) 9 Ω (b) 99 Ω

(c) 999 Ω (d) 1000 Ω

26. A milliammeter of range 10 mA gives full-scale deflection for a current of 100 mA when a shunt of 0.1 Ω is connected in parallel to it. The coil of the milliammeter has a resistance of

(a) 0.9 Ω (b) 1 Ω (c) 1.1 Ω (d) 0.11 Ω

27. A, B and C are voltmeters of resistances R, $1.5R$ and $3R$ respectively. When some potential difference is applied between X and



Y, the voltmeter readings are V_A, V_B and V_C respectively.

(a) $V_A = V_B = V_C$
(b) $V_A \neq V_B = V_C$
(c) $V_A = V_B \neq V_C$
(d) $V_B \neq V_A = V_C$

28. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V respectively. If a resistance is now joined in parallel with the voltmeter,

(a) both A and V will increase
(b) both A and V will decrease
(c) A will decrease, V will increase
(d) A will increase, V will decrease

29. In a moving-coil instrument, the coil is suspended in a radial magnetic field instead of a uniform magnetic field. This is done to

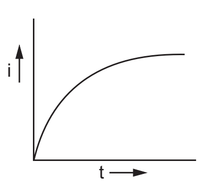
(a) increase the sensitivity of the instrument
(b) increase the accuracy of the instrument
(c) make the instrument compact and portable
(d) make its deflection proportional to the current through it

30. A cell of internal resistance r drives a current through an external resistance R. The power delivered by the cell to the external resistance is maximum when

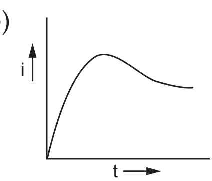
(a) $R = r$
(b) $R >> r$
(c) $R << r$
(d) $R = 2r$

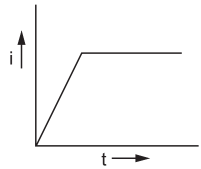
31. When an electric heater is switched on, the current flowing through it (i) is plotted against time (t). Taking into account the variation of resistance with temperature, which of the following best represents the resulting curve?

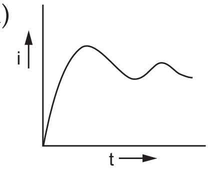
(a)



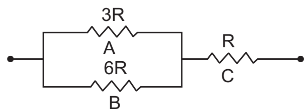
(b)



(c) 

(d) 

32.



The three resistances A, B and C have values $3R$, $6R$ and R respectively. When some potential difference is applied across the network, the thermal powers dissipated by A, B and C are in the ratio

(a) 2 : 3 : 4 (b) 2 : 4 : 3

(c) 4 : 2 : 3 (d) 3 : 2 : 4

33. An electric bulb is designed to draw P_0 power at V_0 voltage. If the voltage is V, it draws P power. Then,

(a) $P = \frac{V_0}{V} P_0$ (b) $P = \frac{V}{V_0} P_0$

(c) $P = \left(\frac{V}{V_0}\right)^2 P_0$ (d) $P = \left(\frac{V_0}{V}\right)^2 P_0$

34. An electric bulb rated for 500 watts at 100 volts is used in a circuit having a 200-volt supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 watts is

(a) 10 Ω (b) 20 Ω

(c) 50 Ω (d) 100 Ω

35. Two electric bulbs A and B are designed for the same voltage. Their power ratings are P_A and P_B respectively, with $P_A > P_B$. If they are joined in series across a V-volt supply,

(a) A will draw more power than B

(b) B will draw more power than A

(c) the ratio of powers drawn by them will depend on V

(d) A and B will draw the same power

36. n identical light bulbs, each designed to draw P power from a certain voltage supply, are joined in series across that supply. The total power which they will draw is

(a) nP (b) P

(c) P/n (d) P/n^2

37. When a 500-W electric bulb and a 500-W heater operate at their rated voltages, the filament of the bulb reaches a much higher temperature than the filament of the heater. The most important reason for this is that

(a) their resistances are not equal

(b) they are made of different materials

(c) their dimensions are very different

(d) they radiate different powers at different temperatures

38. A 100-W bulb and a 25-W bulb are designed for the same voltage. They have filaments of the same length and material. The ratio of the diameter of the 100-W bulb to that of the 25-W bulb is

(a) 4 : 1 (b) 2 : 1

(c) $\sqrt{2} : 1$ (d) 1 : 2

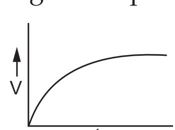
39. If the length of the filament of a heater is reduced by 10%, the power of the heater will

(a) increase by about 9% (b) increase by about 11%

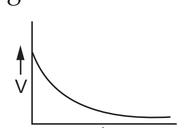
(c) increase by about 19% (d) decrease by about 10%

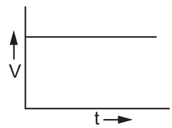
40. An ideal cell is connected to a capacitor through a voltmeter. The reading V of the voltmeter is plotted against time. Which of the following best represents the resulting curve?

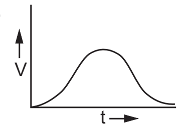
(a)



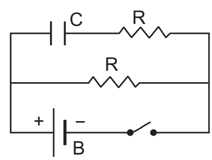
(b)



(c) 

(d) 

41.



In the circuit shown, when the switch is closed, the capacitor charges with a time constant

(a) RC (b) $2RC$

(c) $\frac{1}{2}RC$ (d) $RC \ln 2$

42. In the previous question, if the switch is opened after the capacitor has been charged, it will discharge with a time constant

(a) RC (b) $2RC$

(c) $\frac{1}{2}RC$ (d) $RC \ln 2$

43. A capacitor is charged and then made to discharge through a resistance. The time constant is τ. In what time will the potential difference across the capacitor decrease by 10%?

(a) $\tau \ln (0.1)$ (b) $\tau \ln (0.9)$

(c) $\tau \ln (10/9)$ (d) $\tau \ln (11/10)$

44. In the previous question, after how many time constants will the potential difference across the capacitor fall to 10% of its initial value?

(a) 2 (b) 2.303

(c) $\frac{1}{0.693}$ (d) $\frac{1}{0.37}$

45. A capacitor charges from a cell through a resistance. The time constant is τ. In what time will the capacitor collect 10% of its final charge?

(a) $\tau \ln (0.1)$

(b) $\tau \ln (0.9)$

(c) $\tau \ln (10/9)$

(d) $\tau \ln (11/10)$

46. In the previous question, after how many time constants will the charge on the capacitor be 10% less than its final charge?

(a) 2

(b) 2.303

(c) $\frac{1}{0.693}$

(d) $\frac{1}{0.37}$

47. A capacitor of capacitance C has charge Q. It is connected to an identical capacitor through a resistance. The heat produced in the resistance is

(a) $\frac{Q^2}{2C}$

(b) $\frac{Q^2}{4C}$

(c) $\frac{Q^2}{8C}$

(d) dependent on the value of the resistance

48. The charge on a capacitor decreases η times in time t, when it discharges through a circuit with a time constant τ.

(a) $t = \eta\tau$

(b) $t = \tau \ln \eta$

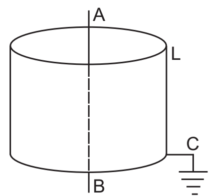
(c) $t = \tau(\ln \eta - 1)$

(d) $t = \tau \ln \left(1 - \frac{1}{\eta}\right)$

• Type 2 •

Choose the correct options. One or more options may be correct.

49.



A straight conductor AB lies along the axis of a hollow metal cylinder L, which is connected to earth through a conductor C. A quantity of charge will flow through C

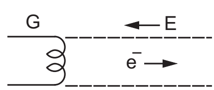
(a) if a current begins to flow through AB

(b) if the current through AB is reversed

(c) if AB is removed, and a beam of electrons flows in its place

(d) if AB is removed, and a beam of protons flows in its place

50.



A beam of electrons emitted from the electron gun G is accelerated by an electric field *E*. The area of cross-section of the beam remains constant. As the beam moves away from G,

(a) the speed of the electrons increases

(b) the current constituted by the beam increases

(c) the number of electrons per unit volume in the beam increases

(d) the number of electrons per unit volume in the beam decreases

51. The charge flowing in a conductor varies with time as $Q = at - bt^2$. Then, the current

(a) decreases linearly with time

(b) reaches a maximum and then decreases

(c) falls to zero after a time period $t = \dfrac{a}{2b}$

(d) changes at a rate $-2b$

52. The charge flowing in a conductor varies with time as

$$Q = at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3,$$

where a, b, c are positive constants. Then, the current

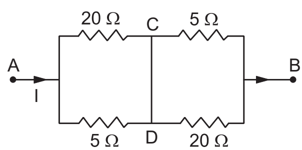
(a) has an initial value $i = a$

(b) reaches a minimum value after a time period $t = b/c$

(c) reaches a maximum value after a time period $t = b/c$

(d) has either a maximum or a minimum value $i = a - \dfrac{b^2}{2c}$

53.



When some potential difference is maintained between A and B, current I enters the network at A and leaves at B.

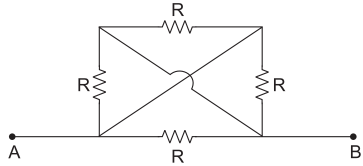
(a) The equivalent resistance between A and B is 8 Ω.

(b) C and D are at the same potential.

(c) No current flows between C and D.

(d) Current $3I/5$ flows from D to C.

54.



In the circuit shown above, each of the four conductors is of resistance R. The potential difference between A and B is V. The current flowing between A and B is

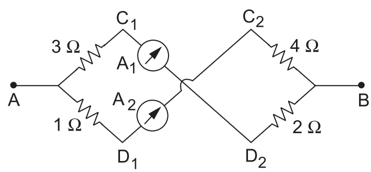
(a) $\dfrac{V}{R}$ (b) $\dfrac{2V}{R}$ (c) $\dfrac{3V}{R}$ (d) $\dfrac{4V}{R}$

55. In the previous question, if the wires forming the diagonals touch at the centre, the equivalent resistance between A and B is

(a) R (b) $\frac{R}{4}$

(c) zero (d) infinite

56.



In the circuit shown above, A_1 and A_2 are ammeters of resistance 5 Ω each. When an ideal cell of emf 10 V is applied between A and B,

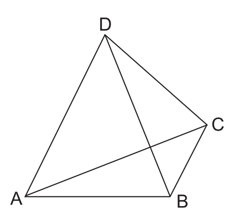
(a) the current drawn from the cell is 1 A

(b) the reading of A_1 is 1 A

(c) the reading of A_2 is 1 A

(d) if C_1 is joined to C_2 and D_1 is joined to D_2, the ammeter readings will become equal

57.



Six identical wires of resistance R each are joined to form a pyramid, as shown in the figure above.

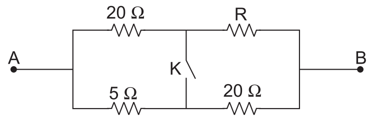
(a) The equivalent resistance between any two corners will depend on the choice of corners.

(b) The equivalent resistance between A and B is $R/2$

(c) The equivalent resistance between D and C is zero.

(d) If an electric current enters at A and flows out at B, no current will pass through DC.

58.



When the switch K is open, the equivalent resistance between A and B is 20 Ω. Then, which is the correct statement?

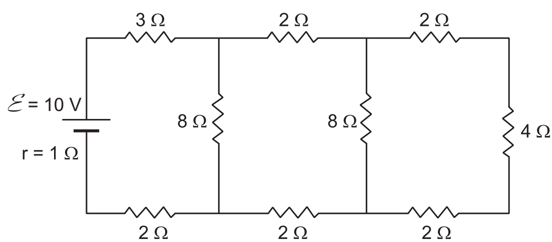
(a) $R = 80\ \Omega$.

(b) No current flows through K when it is closed.

(c) The powers dissipated in R and in the 5-Ω resistor are always equal.

(d) The powers dissipated in the two 20-Ω resistors are unequal.

59.



In the circuit shown above, the cell has an emf of 10 V and an internal resistance of 1 Ω.

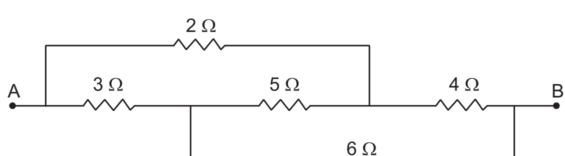
(a) The current through the 3-Ω resistor is 1 A.

(b) The current through the 3-Ω resistor is 0.5 A.

(c) The current through the 4-Ω resistor is 0.5 A.

(d) The current through the 4-Ω resistor is 0.25 A.

60.



In the circuit shown above, some potential difference is applied between A and B. The equivalent resistance between A and B is R.

(a) No current flows through the 5-Ω resistor.

(b) $R = 15\ \Omega$

(c) $R = 12.5\ \Omega$

(d) $R = \frac{18}{5}\ \Omega$

61. A uniform wire of resistance R is shaped into a regular n-sided polygon (n is even). The equivalent resistance between any two corners can have

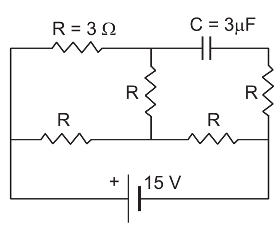
(a) the maximum value $\frac{R}{4}$

(b) the maximum value $\frac{R}{n}$

(c) the minimum value $R\left(\frac{n-1}{n^2}\right)$

(d) the minimum value $\frac{R}{n}$

62.

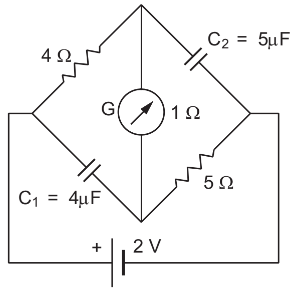


In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance is of 3 Ω. The potential difference across the capacitor is

(a) zero (b) 9 V

(c) 12 V (d) 15 V

63. In the circuit shown below, the cell is ideal, with emf = 2 V. The resistance of the coil of the galvanometer G is 1 Ω.



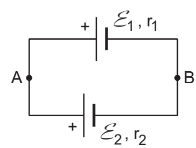
(a) No current flows in G.

(b) 0.2-A current flows in G.

(c) Potential difference across C_1 is 1 V.

(d) Potential difference across C_2 is 1.2 V.

64.



Two cells of unequal emfs, $\mathcal{E}_1$ and $\mathcal{E}_2$, and internal resistances r_1 and r_2 are joined as shown. V_A and V_B are the potentials at A and B respectively.

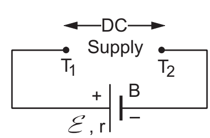
(a) One cell will continuously supply energy to the other.

(b) The potential difference across both the cells will be equal.

(c) The potential difference across one cell will be greater than its emf.

(d) $V_A - V_B = \dfrac{(\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1)}{(r_1 + r_2)}$.

65. An accumulator battery (storage cell) B of emf $\mathcal{E}$ and internal resistance r is being charged from a DC supply whose terminals are T_1 and T_2.



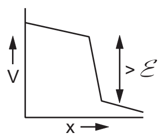
(a) Potential difference between T_1 and T_2 must be > $\mathcal{E}$.

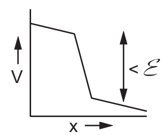
(b) T_1 must be positive with respect to T_2.

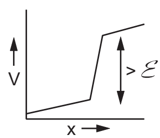
(c) In the battery, current flows from the positive to the negative terminal.

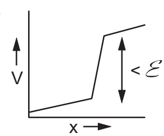
(d) All the above options are incorrect.

66. In the previous question, the connecting wires have uniform resistance. Moving from T_1 to T_2 through B, the potential (V) is plotted against distance (x). The correct curve is

(a)


(b)


(c)


(d)


67. A voltmeter and an ammeter are connected in series to an ideal cell of emf $\mathcal{E}$. The voltmeter reading is V and the ammeter reading is I.

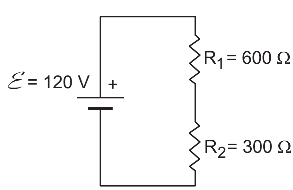
(a) $V < \mathcal{E}$.

(b) The voltmeter resistance is V/I.

(c) The potential difference across the ammeter is $(\mathcal{E} - V)$.

(d) Voltmeter resistance plus ammeter resistance = $\mathcal{E}/I$.

68.



In the circuit, the battery is ideal. A voltmeter of resistance 600 Ω is connected in turn across R_1 and R_2, giving readings of V_1 and V_2 respectively.

(a) $V_1 = 80$ V

(b) $V_1 = 60$ V

(c) $V_2 = 30$ V (d) $V_2 = 40$ V

69. A voltmeter and an ammeter are joined in series to an ideal cell, giving readings V and A respectively. If a resistance equal to the resistance of the ammeter is now joined in parallel to the ammeter,

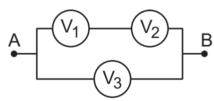
(a) V will not change

(b) V will increase slightly

(c) A will become exactly half of its initial value

(d) A will become slightly more than half of its initial value

70.

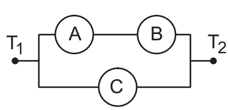


Three voltmeters, all having different resistances, are joined as shown. When some potential difference is applied across A and B, their readings are V_1, V_2, V_3.

(a) $V_1 = V_2$ (b) $V_1 \neq V_2$

(c) $V_1 + V_2 = V_3$ (d) $V_1 + V_2 > V_3$

71.



Three ammeters A, B and C of resistances R_A, R_B and R_C respectively are joined as shown. When some potential difference is applied across the terminals T_1 and T_2, their readings are I_A, I_B and I_C respectively.

(a) $I_A = I_B$ (b) $I_A R_A + I_B R_B = I_C R_C$

(c) $\dfrac{I_A}{I_C} = \dfrac{R_C}{R_A}$ (d) $\dfrac{I_B}{I_C} = \dfrac{R_C}{R_A + R_B}$

72. A microammeter has a resistance of 100 Ω and a full-scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).

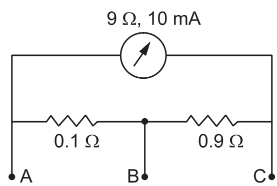
(a) Range 50 V, with a 10-kΩ resistance in series

(b) Range 10 V, with a $(2 \times 10^5 - 100)$-Ω resistance in series

(c) Range 5 mA, with a 1.01-Ω resistance in parallel

(d) Range 10 mA, with a 1-Ω resistance in parallel

73.



A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The metre gives full-scale deflection for current *I* when A and B are used as its terminals, i.e., current enters at A and leaves at B (C is left isolated). The value of *I* is

(a) 100 mA (b) 900 mA

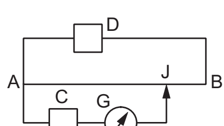
(c) 1 A (d) 1.1 A

74. In the previous question, if A and C are used as terminals, with B isolated, full-scale deflection is obtained for a current of

(a) 90 mA (b) 100 mA

(c) 900 mA (d) 1 A

75.



The figure shows a potentiometer arrangement. D is the driving cell. C is the cell whose emf is to be determined. AB is the potentiometer wire and G is a galvanometer. J is a sliding contact which can touch any point on AB. Which of the following are essential conditions for obtaining balance?

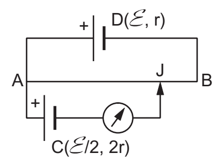
(a) The emf of D must be greater than the emf of C.

(b) Either the positive terminals of both D and C or the negative terminals of both D and C must be joined to A.

(c) The positive terminals of D and C must be joined to A.

(d) The resistance of G must be less than the resistance of AB.

76.



In the potentiometer arrangement shown, the driving cell D has emf $\mathcal{E}$ and internal resistance r. The cell C, whose emf is to be measured, has emf $\mathcal{E}/2$ and internal resistance $2r$. The potentiometer wire is 100-cm long. If balance is obtained, the length AJ $= l$.

(a) $l = 50$ cm.

(b) $l > 50$ cm.

(c) Balance will be obtained only if resistance of AB is $> r$.

(d) Balance cannot be obtained.

77. A cell drives a current through a circuit. The emf of the cell is equal to the work done in moving unit charge

(a) from the positive to the negative plate of the cell

(b) from the positive plate, back to the positive plate

(c) from the negative plate, back to the negative plate

(d) from any point in the circuit back to the same point

78. A cell of emf $\mathcal{E}$ and internal resistance r drives a current i through an external resistance R.

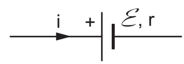
(a) The cell supplies $\mathcal{E}i$ power.

(b) Heat is produced in R at the rate $\mathcal{E}i$.

(c) Heat is produced in R at the rate $\mathcal{E}i\left(\frac{R}{R+r}\right)$.

(d) Heat is produced in the cell at the rate $\mathcal{E}i\left(\frac{r}{R+r}\right)$.

79.



Current i is being driven through a cell of emf $\mathcal{E}$ and internal resistance r, as shown.

(a) The cell absorbs energy at the rate of $\mathcal{E}i$.

(b) The cell stores chemical energy at the rate of $(\mathcal{E}i - i^2r)$.

(c) The potential difference across the cell is $\mathcal{E} + ir$.

(d) Some heat is produced in the cell.

80. Two electric bulbs rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220-V voltage source. The 25-W and 100-W bulbs now draw P_1 and P_2 powers respectively.

(a) $P_1 = 16$ W
(b) $P_1 = 4$ W
(c) $P_2 = 16$ W
(d) $P_2 = 4$ W

81. Two heaters designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in times t_1 and t_2 respectively. When used together across the same source, they produce H amount of heat in time t.

(a) If they are in series, $t = t_1 + t_2$.

(b) If they are in series, $t = 2(t_1 + t_2)$.

(c) If they are in parallel, $t = \frac{t_1 t_2}{(t_1 + t_2)}$.

(d) If they are in parallel, $t = \frac{t_1 t_2}{2(t_1 + t_2)}$.

82. In a household electric circuit,

(a) all electric appliances drawing power are joined in parallel

(b) a switch may be either in series or in parallel with the appliance which it controls

(c) if a switch is in parallel with an appliance, it will draw power when the switch is in the 'off' position (open)

(d) if a switch is in parallel with an appliance, the fuse will blow (burn out) when the switch is put 'on' (closed)

83. Two identical fuses are rated at 10 A. If they are joined

(a) in parallel, the combination acts as a fuse of rating 20 A

(b) in parallel, the combination acts as a fuse of rating 5 A

(c) in series, the combination acts as a fuse of rating 10 A

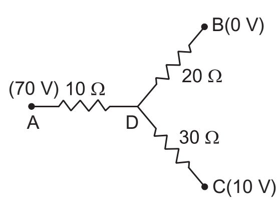
(d) in series, the combination acts as a fuse of rating 20 A

84. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$. The total heat produced in R is

(a) $\dfrac{a^3R}{6b}$ (b) $\dfrac{a^3R}{3b}$

(c) $\dfrac{a^3R}{2b}$ (d) $\dfrac{a^3R}{b}$

85.



In the network shown, points A, B and C are at potentials of 70 V, zero and 10 V respectively.

(a) Point D is at a potential of 40 V.

(b) The currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1.

(c) The currents in the sections AD, DB, DC are in the ratio 1 : 2 : 3.

(d) The network draws a total power of 200 W.

86. Two identical capacitors A and B are charged to the same potential and then made to discharge through resistances R_A and R_B respectively, with $R_A > R_B$.

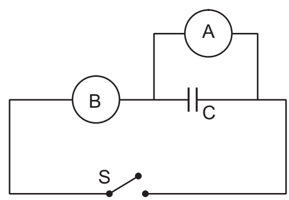
(a) A will require greater time then B to discharge completely.

(b) More heat will be produced in A than in B.

(c) More heat will be produced in B than in A.

(d) All the above options are incorrect.

87.

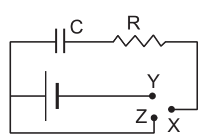


A capacitor of capacitance C is connected to two voltmeters A and B. A is ideal, having infinite resistance, while B has resistance R. The capacitor is charged and then the switch S is closed. The readings of A and B will be equal

(a) at all times (b) after time RC

(c) after time $RC \ln 2$ (d) only after a very long time

88.



The capacitor C is initially without charge. X is now joined to Y for a long time, during which H_1 heat is produced in the resistance R. X is now joined to Z for a long time, during which H_2 heat is produced in R.

(a) $H_1 = H_2$

(b) $H_1 = \frac{1}{2} H_2$

(c) $H_1 = 2H_2$

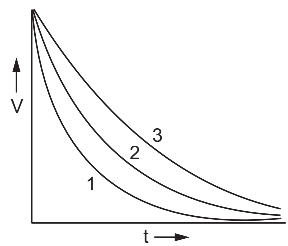
(d) The maximum energy stored in C at any time is H_1.

89. In the previous question, the energy supplied by the cell during charging is equal to

(a) H_1 (b) H_2

(c) $2H_2$ (d) $H_1 + H_2$

90.



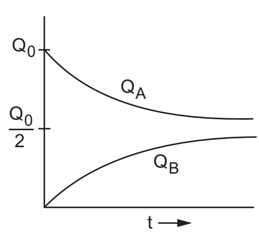
Three identical capacitors A, B and C are charged to the same potential and then made to discharge through three resistances R_A, R_B and R_C, where $R_A > R_B > R_C$. Their potential differences (V) are plotted against time t, giving the curves 1, 2 and 3. Find the correlations between A, B, C and 1, 2, 3.

(a) $1 \to A$ (b) $2 \to B$

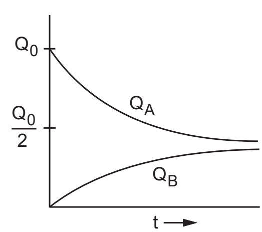
(c) $1 \to C$ (d) $3 \to A$

91. A capacitor A with charge Q_0 is connected through a resistance to another identical capacitor B, which has no charge. The charges on A and B after time t are Q_A and Q_B respectively, and they are plotted against time t. Find the correct curves.

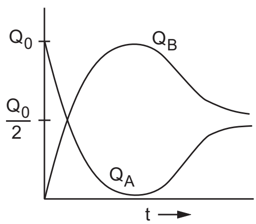
(a)



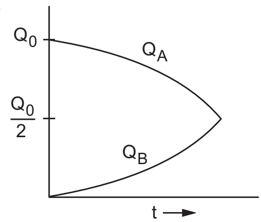
(b)



(c)



(d)



92. A parallel-plate capacitor, filled with a dielectric of dielectric constant k, is charged to a potential V_0. It is now disconnected from the cell and the slab is removed. If it now discharges, with time constant τ, through a resistance, the potential difference across it will be V_0 after time

(a) $k\tau$

(b) $\tau \ln k$

(c) $\tau \ln\left(1 - \frac{1}{k}\right)$

(d) $\tau \ln (k - 1)$

93. When a capacitor discharges through a resistance R, the time constant is τ and the maximum current in the circuit is i_0.

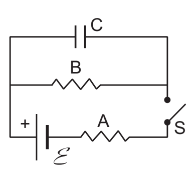
(a) The initial charge on the capacitor was $i_0\tau$.

(b) The initial charge on the capacitor was $\frac{1}{2} i_0\tau$.

(c) The initial energy stored in the capacitor was $i_0^2 R\tau$.

(d) The initial energy stored in the capacitor was $\frac{1}{2} i_0^2 R\tau$.

94.



In the circuit shown, A and B are equal resistances. When S is closed, the capacitor C charges from the cell of emf $\mathcal{E}$ and reaches a steady state.

(a) During charging, more heat is produced in A than in B.

(b) In the steady state, heat is produced at the same rate in A and B.

(c) In the steady state, energy stored in C is $\frac{1}{4}C\mathcal{E}^2$.

(d) In the steady state, energy stored in C is $\frac{1}{8}C\mathcal{E}^2$.

95. Capacitors $C_1 = 1\ \mu F$ and $C_2 = 2\ \mu F$ are separately charged from the same battery. They are then allowed to discharge separately through equal resistors.

(a) The currents in the two discharging circuits at $t = 0$ is zero.

(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.

(c) The currents in the two discharging circuits at $t = 0$ are unequal.

(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

Answers

1. d	**2.** a	**3.** d	**4.** d	**5.** b
6. d	**7.** c	**8.** b	**9.** a	**10.** a
11. c	**12.** d	**13.** a	**14.** b	**15.** b
16. b	**17.** c	**18.** a	**19.** a	**20.** c
21. d	**22.** a	**23.** c	**24.** c	**25.** c
26. a	**27.** a	**28.** d	**29.** d	**30.** a
31. b	**32.** c	**33.** c	**34.** b	**35.** b
36. c	**37.** c	**38.** b	**39.** b	**40.** b
41. a	**42.** b	**43.** c	**44.** b	**45.** c
46. b	**47.** b	**48.** b	**49.** c, d	**50.** a, d
51. a, c, d	**52.** a, b, d	**53.** a, b, d	**54.** d	**55.** c
56. b, c, d	**57.** b, d	**58.** a, b, c, d	**59.** a, d	**60.** a, d
61. a, c	**62.** c	**63.** b, c, d	**64.** a, b, c, d	**65.** a, b, c
66. a	**67.** a, b, c, d	**68.** b, c	**69.** b, d	**70.** b, c
71. a, b, d	**72.** b, c	**73.** c	**74.** b	**75.** a, b
76. b, c	**77.** b, c, d	**78.** a, c, d	**79.** a, b, c, d	**80.** a, d
81. a, c	**82.** a, c, d	**83.** a, c	**84.** a	**85.** a, b, d
86. d	**87.** a	**88.** a, d	**89.** c, d	**90.** b, c, d
91. a	**92.** b	**93.** a, d	**94.** a, b, d	**95.** c, d

Hints and Solutions to Selected Questions

5. $I = Avne$. No. of free electrons per unit length $= 1 \times A \times n$.

Momentum of each free electron $= mv$.

$\therefore$ momentum per unit length $= Anmv = \dfrac{I}{e} m = \dfrac{I}{(e/m)} = \dfrac{I}{s}$.

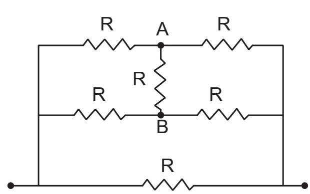
7. With each rotation, charge Q crosses any fixed point P near the ring. Number of rotations per second $= \omega/2\pi$.

$\therefore$ charge crossing P per second = current $= \dfrac{Q\omega}{2\pi}$.

8. Let the edges be $2l$, a and l, in decreasing order.

$R_{max} = \rho \dfrac{2l}{al} = \dfrac{2\rho}{a}$ $R_{min} = \rho \dfrac{l}{2la} = \dfrac{\rho}{2a}$ $\dfrac{R_{max}}{R_{min}} = 4.$

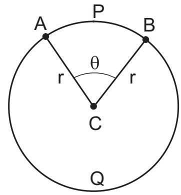
9. TIP: Any circuit element situated between points at the same potential can be removed from the circuit.



By symmetry, points A and B are at the same potential. Thus, removing the resistance R between A and B, the circuit reduces to three resistances of $2R$, $2R$ and R in parallel.

10. Resistance per unit length $= \rho = \dfrac{R}{2\pi r}$.

Lengths of sections APB and AQB are $r\theta$ and $r(2\pi - \theta)$.



Resistances of sections APB and AQB are

$$R_1 = \rho r\theta = \frac{R}{2\pi r} \cdot r\theta = \frac{R\theta}{2\pi} \quad \text{and}$$

$$R_2 = \frac{R}{2\pi r} r(2\pi - \theta) = \frac{\pi(2\pi - \theta)}{2\pi}.$$

As R_1 and R_2 are in parallel between A and B, $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$.

11, 12. TIP Use symmetry to identify points at the same potential. Remove the resistances connected between such points. The circuit is then rearranged and simplified.

13. As the ring has no resistance, the three resistances of $3R$ each are in parallel.

14. When we move in the direction of the current in a uniform conductor, the potential decreases linearly. When we pass through the cell, from its negative to its positive terminal, the potential increases by an amount equal to its potential difference. This is less than its emf, as there is some potential drop across its internal resistance when the cell is driving current.

15. Current in the circuit $= i = \dfrac{\mathcal{E}}{R + r}$.

p.d. across cell = p.d. across $R = iR = \dfrac{\mathcal{E}R}{R + r}$.

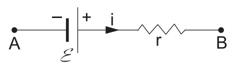
Set up two equations with the given data and solve for $\mathcal{E}$, r.

16. For series connection, $I_{max} = \dfrac{N\mathcal{E}}{Nr} = \dfrac{\mathcal{E}}{r}$.

For parallel connection, $I_{\text{max}} = \dfrac{\mathcal{E}}{r/n} = \dfrac{N\mathcal{E}}{r}$.

17. TIP For every cell that is wrongly connected, the emf decreases by $2\mathcal{E}$. However, internal resistance does not depend on direction and therefore remains the same for all cells.

18.



Current in the circuit is $i = \dfrac{n\mathcal{E}}{nr} = \dfrac{\mathcal{E}}{r}$.

The equivalent circuit of one cell is shown in the figure above. Potential difference across the cell equals

$$V_{\text{A}} - V_{\text{B}} = -\mathcal{E} + ir = -\mathcal{E} + \frac{\mathcal{E}}{r} \cdot r = 0.$$

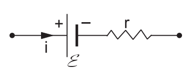
19. See the figure in the previous question.

$$i = \frac{(n-2)\mathcal{E}}{nr}. \qquad \text{(See the hint to Q. \textbf{17})}$$

$$V_{\text{B}} - V_{\text{A}} = -ir + \mathcal{E} = \mathcal{E} - \frac{(n-2)\mathcal{E}}{nr} r$$

$$= \mathcal{E}\left[1 - \frac{n-2}{n}\right] = \frac{2\mathcal{E}}{n}.$$

20.



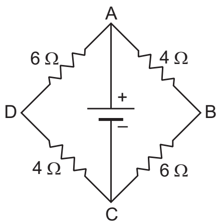
For the cell shown in the figure above, current i flows opposite to the direction of its emf.

Therefore, p.d. $= \mathcal{E} + ir = \mathcal{E} + \dfrac{(n-2)\mathcal{E}}{nr} r = \mathcal{E}\left(1 + \dfrac{n-2}{n}\right) = \mathcal{E}\left(\dfrac{2n-2}{n}\right)$.

21. Current can flow in a circuit or part of a circuit only when it has a 'return path', i.e., it can return to its starting point. Here, current flowing from X to Y cannot return to X.

22. TIP In a Wheatstone bridge, the deflection in the galvanometer does not change if the battery and galvanometer are interchanged.

23.



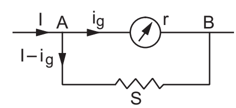
Let $\mathcal{E}$ = emf of the cell.

$$V_A - V_B = \frac{\mathcal{E}}{4\ \Omega + 6\ \Omega} \times 4\ \Omega = \frac{4\mathcal{E}}{10}.$$

$$V_A - V_D = \frac{\mathcal{E}}{6\ \Omega + 4\ \Omega} \times 6\ \Omega = \frac{6\mathcal{E}}{10}.$$

$$V_B - V_D = \frac{\mathcal{E}}{5}.$$

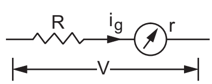
24. $i_g = 10\ \text{mA} = 0.01\ \text{A} \qquad r = 1\ \Omega \qquad I = 1\ \text{A}$



$$V_A - V_B = i_g r = (I - i_g)S.$$

$$S = \frac{i_g r}{I - i_g} = \frac{0.01\ \text{A} \times 1\ \Omega}{1\ \text{A} - 0.01\ \text{A}} = \frac{1}{99}\ \Omega.$$

25.

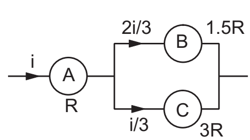


$i_g = 0.01\ \text{A} \qquad r = 1\ \Omega \qquad V = 10\ \text{V}$

$$V = (r + R)i_g$$

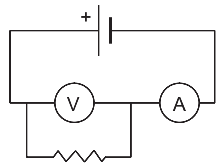
$$R = \frac{V}{i_g} - r = \frac{10\ \text{V}}{0.01\ \text{A}} - 1\ \Omega = 999\ \Omega.$$

27.



$$V_A = iR \qquad V_B = \frac{2i}{3}(1.5R) = iR \qquad V_C = \left(\frac{i}{3}\right)(3R) = iR.$$

28.



When a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases. Current will increase and ammeter reading will increase. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.

30. Let $\mathcal{E}$ = emf of the cell, i = current in the circuit.

$$i = \frac{\mathcal{E}}{R + r}.$$

Power delivered to $R = P = i^2R = \dfrac{\mathcal{E}^2 R}{(R+r)^2} = f(R)$.

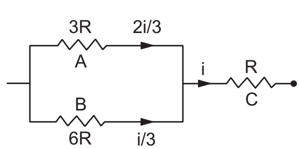
For P to be maximum, $\dfrac{dP}{dR} = 0 = \mathcal{E}^2 \left[\dfrac{1}{(R+r)^2} - \dfrac{2R}{(R+r)^3} \right]$

or $\quad 2R = R + r$

or $\quad R = r.$

31. The filament of the heater reaches its steady resistance when the heater reaches its steady temperature, which is much higher than the room temperature. The resistance at room temperature is thus much lower than the resistance at its steady state. When the heater is switched on, it draws a larger current than its steady-state current. As the filament heats up, its resistance increases and the current falls to its steady-state value.

32.



Thermal power in A $= P_A = \left(\dfrac{2i}{3}\right)^2 3R = \dfrac{4}{3} i^2 R.$

Thermal power in B $= P_B = \left(\frac{i}{3}\right)^2 6R = \frac{2}{3} i^2 R.$

Thermal power in C $= P_C = i^2 R.$

33. Let R = resistance of the bulb.

$$P_0 = \frac{V_0^2}{R} \qquad \text{or} \quad R = \frac{V_0^2}{P_0} \cdot$$

$$P = \frac{V^2}{R} = \frac{V^2}{(V_0^2/P_0)} = \left(\frac{V}{V_0}\right)^2 P_0.$$

35. $R = V^2/R$. The bulb with higher power rating has lower resistance, and vice versa. When the bulbs are joined in series, they draw the same current. As $P = i^2 R$, the bulb with the lower power rating draws more power.

36. Let V = voltage of the source, R = resistance of each bulb.

$\therefore \quad R = V^2/P.$

When n bulbs are joined in series across V, current in each bulb

$$= i = \frac{V}{nR} \cdot$$

Power drawn by each bulb $= i^2 R = \frac{V^2}{n^2 R^2} \cdot R = \frac{V^2}{n^2 R} = \frac{P}{n^2} \cdot$

Total power drawn $= n \times P/n^2 = P/n.$

37. As the bulb and the heater have the same power rating, they must radiate the same power. The power radiated depends on the surface area and the temperature of the filament. The filament of the heater is much longer and thicker than that of the bulb, and hence has greater surface area. It can therefore radiate the same power as the bulb, at a temperature lower than that of the bulb.

40. This is basically an RC circuit, charging from a cell. The resistance (R) of the voltmeter is the resistance in the circuit. The voltage across R = circuit current $\times R$ = reading of the voltmeter

(V). Thus, the nature of the V–t curve is the same as the nature of the I–t curve.

41. The resistance in the middle plays no part in the charging process of C, as it does not alter either the potential difference across the RC combination or the current through it.

42. C discharges through $R + R$ in series.

43. $Q = Q_0 e^{-t/\tau}$ and potential difference across C is proportional to Q. For the p.d. to fall by 10%, Q must fall by 10%.

$Q = 0.9Q_0 = Q_0 e^{-t/\tau}$

or $\quad e^{t/\tau} = \dfrac{10}{9}$ $\qquad$ or $\quad \dfrac{t}{\tau} = \ln\left(\dfrac{10}{9}\right)$.

44. $Q = 0.1Q_0 = Q_0 e^{-t/\tau}$

or $\quad e^{t/\tau} = 10$ $\qquad$ or $\quad t/\tau = \ln 10 = 2.303$.

45. $Q = Q_0(1 - e^{-t/\tau}) = 0.1Q_0$

or $\quad e^{-t/\tau} = 0.9$ $\qquad$ or $\quad e^{t/\tau} = 10/9$.

46. $Q = Q_0(1 - e^{-t/\tau}) = 0.9Q_0$

or $\quad e^{-t/\tau} = 0.1$ $\qquad$ or $\quad e^{t/\tau} = 10$.

47. As the capacitors are identical, they will finally have charge $Q/2$ each.

Initial energy of the system $= E_i = \dfrac{Q^2}{2C}$.

Final energy of the system $= E_f = 2\left[\dfrac{(Q/2)^2}{2C}\right] = \dfrac{Q^2}{4C}$.

Heat produced = loss in energy $= E_i - E_f = \dfrac{Q^2}{4C}$.

48. $Q = Q_0\, e^{-t/\tau} = Q_0/\eta$

or $e^{-t/\tau} = \dfrac{1}{\eta}$ or $e^{t/\tau} = \eta$ or $\dfrac{t}{\tau} = \ln \eta$.

49. When current flows through the conductor AB, it remains electrically neutral. Therefore, no charges are induced by it in the cylinder. A beam of electrons or protons has net negative or positive charge. They will induce bound and free charges on L. The free charges will flow through C to the earth.

50. The speed of the electrons increases as they are accelerated. The current I remains constant, as the number of electrons emitted per second by the gun is constant. Now, $I = Avne$. As I, A and e are constant, while v increases, n must decrease.

51. $i = \dfrac{dQ}{dt} = a - 2bt$

$i = v$ for $t = \dfrac{a}{2b}$. $\dfrac{di}{dt} = -2b$.

52. $Q = at - (½)bt^2 + (⅙)ct^3$

$i = \dfrac{dQ}{dt} = a - bt + \dfrac{1}{2}ct^2$ $i = a$ for $t = 0$

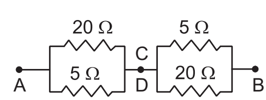
$\dfrac{di}{dt} = -b + ct$

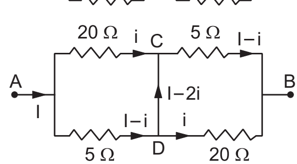
For i to be maximum or minimum, $\dfrac{di}{dt} = 0$ or $t = \dfrac{b}{c}$.

For this value of t, $i = a - \dfrac{b^2}{c} + \dfrac{1}{2}c \cdot \left(\dfrac{b^2}{c^2}\right) = a - \dfrac{b^2}{2c}$.

As this value of i is less than that at $t = 0$, it must be a minimum.

53. As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as 8 Ω. If we distribute current in the network, using symmetry,



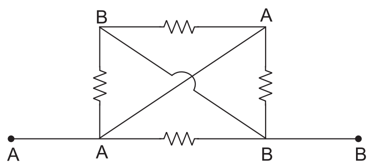


$$V_A - V_D = V_A - V_C$$

or $20i = 5(I - i)$ or $i = I/5$

$$\therefore \quad I - 2i = I - \frac{2I}{5} = \frac{3I}{5} = \text{current flowing from D to C.}$$

54. TIP Treat all points joined by a connecting wire as the same point. Give names (e.g., A, B, etc.) to each such set of points. Rearrange the circuit in terms of these points.



All four resistances are effectively joined between A and B, and are therefore joined in parallel.

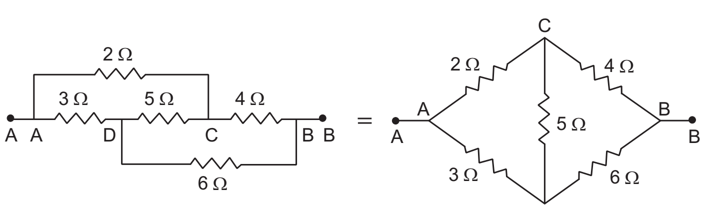
55. Since the diagonals touch at the centre in the figure in the previous question, all four corners are joined directly and hence reduce to the same point.

56. The electrical paths AC_1D_2B and AD_1C_2B are in parallel. The resistance of each of them is 10 Ω, and hence the current is 1 A. When C_1C_2 and D_1D_2 are joined, the ammeters are in parallel combination. As they are of a same resistance, their readings will be equal.

57. Use symmetry arguments. If a p.d. is applied between A and B, the paths ADB and ACB become equivalent, while D and C are at the same potential (and hence can be removed from the circuit).

58. Apply the concepts used in the questions **53**, **54** and **56**.

60.



Rearrangement of the circuit as shown gives a balanced Wheatstone bridge, and no current flows through the 5-Ω resistor. It can thus be removed from the circuit.

61. The resistance of each side = R/n.

For resistance between opposite corners, we have two resistors of resistance $R/2$ in parallel.

For resistance between adjacent corners, we have two resistors of resistances $\frac{R}{n}$ and $\frac{(n-1)R}{n}$ in parallel.

62. TIP A fully charged capacitor draws no current. If the capacitor is removed from the circuit, we can distribute current and find the potential difference across each resistance.

63. Disregard the capacitors and find the current through G. The potential difference across each capacitor is then found from the potential differences across the resistors in parallel with them.

64. Let $\mathcal{E}_1 < \mathcal{E}_2$.

Current in the circuit $= i = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2}$.

$$V_A - V_B = \mathcal{E}_2 + ir_2 = \text{p.d. across each cell.}$$

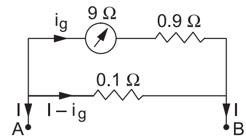
Here, $V_A - V_B > \mathcal{E}_2$.

Current flows in the cell of emf $\mathcal{E}_2$ from the positive plate to the negative plate inside the cell and hence it absorbs energy.

67–71. TIP Treat all voltmeters and ammeters as resistances. Draw the circuit and find the currents and potential differences for each section.

The voltmeter reading is the potential difference across its terminals when it is connected in the circuit. The ammeter reading is the current passing through it.

73. $i_g = 10 \text{ mA} = 0.01 \text{ A}.$

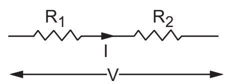


$$V_A - V_B = (I - i_g)(0.1 \text{ A}) = i_g \times 9.9 \text{ A}$$

or $$I \times 0.1 = i_g \times 10$$

$$\therefore \quad I = \frac{0.01 \text{ A} \times 10}{0.1} = 1 \text{ A}.$$

80.



Let $V = 220$ V and R_1 and R_2 be the resistances of the 25-W and 100-W bulbs.

Now, $P_1 = 25 \text{ W} = \dfrac{V^2}{R_1}$ and $P_2 = 100 \text{ W} = \dfrac{V^2}{R_2}\cdot$

$$\therefore \quad R_1 = \frac{V^2}{25 \text{ W}} \text{ and } R_2 = \frac{V^2}{100 \text{ W}}\cdot$$

When the bulbs are joined in series, the current is $I = \dfrac{V}{R_1 + R_2}\cdot$

Power in the 25-W bulb is $R_1 I^2$ and that in the 100-W bulb is $R_2 I^2$.

81. Let R_1 and R_2 be the resistances of the two heaters.

Let H be the heat produced.

$$\therefore \quad H = \left(\frac{V^2}{R_1}\right) t_1 = \left(\frac{V^2}{R_2}\right) t_2.$$

When used in series, $H = \left(\dfrac{V^2}{R_1 + R_2}\right) t.$

When used in parallel, $H = \left(\dfrac{V^2}{R_1} + \dfrac{V^2}{R_2}\right) t.$

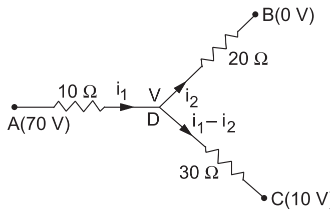
84. $Q = at - bt^2.$

$$i = \frac{dQ}{dt} = a - 2bt.$$

$i = 0$ for $t = t_0 = a/2b$, i.e., current flow from $t = 0$ to $t = t_0$.

The heat produced $= \displaystyle\int_0^{t_0} i^2 R\, dt.$

85.



Let V be the potential at D.

$70\text{ V} - V = i_1 \times 10\ \Omega.$

$V - 0\text{ V} = i_2 \times 20\ \Omega.$

$V - 10\text{ V} = (i_1 - i_2) \times 30\ \Omega.$

Solve for i_1, i_2 and V.

87. A and B are effectively in parallel and hence give the same reading at all times.

88, 89. When X is joined to Y for a long time (charging), the energy stored in the capacitor = heat produced in $R = H_1$.

When X is joined to Z (discharging), the energy stored in C$(= H_1)$ reappears as heat (H_2) in R. Thus, $H_1 = H_2$.

92. When the slab is removed, the potential of the capacitor increases k times, i.e., it becomes kV_0. For the potential to fall to V_0, see the hint to Q. **48**.

□

3

Electromagnetism

• Type 1 •

Choose the correct option (a, b, c or d).

1. If c = the velocity of light, which of the following is correct?

(a) $\mu_0\varepsilon_0 = c$ (b) $\mu_0\varepsilon_0 = c^2$

(c) $\mu_0\varepsilon_0 = \frac{1}{c}$ (d) $\mu_0\varepsilon_0 = \frac{1}{c^2}$

2. If E and B denote electric and magnetic fields respectively, which of the following is dimensionless?

(a) $\sqrt{\mu_0\varepsilon_0}\,\frac{E}{B}$ (b) $\mu_0\varepsilon_0\,\frac{E}{B}$

(c) $\mu_0\varepsilon_0\left(\frac{B}{E}\right)^2$ (d) $\frac{E}{\varepsilon_0} \times \frac{\mu_0}{B}$

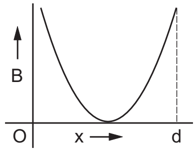
3. A vertical wire carries a current upwards. The magnetic field at a point due north of the wire is directed

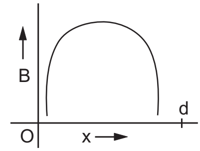
(a) upward (b) due south

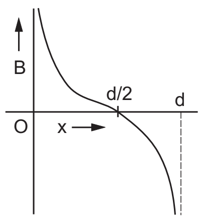
(c) due west (d) due east

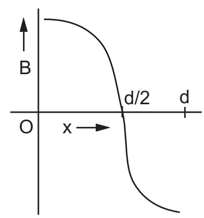
4. Two parallel beams of protons and electrons, carrying equal currents, are fixed at a separation d. The protons and electrons move in opposite directions. P is a point on a line joining the beams, at a distance x from any one beam. The magnetic field at

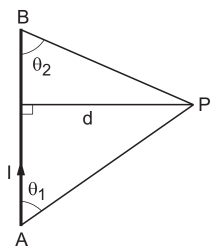
P is B. If B is plotted against x, which of the following best represents the resulting curve?

(a) 

(b) 

(c) 

(d) 

5. 

AB is a section of a straight wire carrying a current I. P is a point at a distance d from AB. The magnetic field at P due to AB has magnitude

(a) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$ (b) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 - \cos\theta_2)$

(c) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$ (d) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$

6. In the previous question, the magnetic field at P is normal to the plane APB

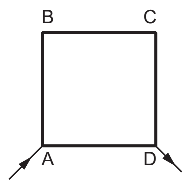
(a) only if $\theta_1 = \theta_2$

(b) only if θ_1 and θ_2 are very small

(c) only if the length of the section AB is small compared to *d*

(d) in all cases

7.



ABCD is a square loop made of a uniform conducting wire. A current enters the loop at A and leaves at D. The magnetic field is

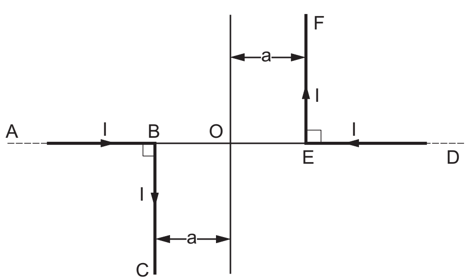
(a) zero only at the centre of the loop

(b) maximum at the centre of the loop

(c) zero at all points outside the loop

(d) zero at all points inside the loop

8.



Two long thin wires ABC and DEF are arranged as shown. They carry currents I as shown. The magnitude of the magnetic field at O is

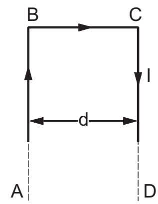
(a) zero

(b) $\frac{\mu_0 I}{4\pi a}$

(c) $\frac{\mu_0 I}{2\pi a}$

(d) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$

9.



AB and CD are long straight conductors, distance d apart, carrying a current I. The magnetic field on BC due to the currents in AB and CD

(a) is zero at all points

(b) is zero only at its midpoint

(c) has different magnitudes at different points

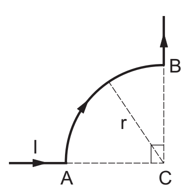
(d) is maximum at its midpoint

10. In the previous question, the magnetic field at the midpoint of BC has magnitude

(a) $\dfrac{\mu_0 I}{8\pi d}$ (b) $\dfrac{\mu_0 I}{4\pi d}$

(c) $\dfrac{\mu_0 I}{2\pi d}$ (d) $\dfrac{\mu_0 I}{\pi d}$

11.



A wire carrying a current I is shaped as shown. Section AB is a quarter circle of radius r. The magnetic field at C is directed

(a) along the bisector of the angle ACB, away from AB

(b) along the bisector of the angle ACB, towards AB

(c) perpendicular to the plane of the paper, directed into the paper

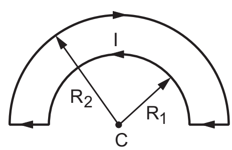
(d) at an angle $\pi/4$ to the plane of the paper

12. In the previous question, the magnitude of the magnetic field at C is

(a) $\frac{\mu_0 I}{2r}$ (b) $\frac{\mu_0 I}{4r}$

(c) $\frac{\mu_0 I}{8r}$ (d) $\frac{\mu_0 I}{4\pi r}$

13.



The wire loop formed by joining two semicircular sections of radii R_1 and R_2, and centre C, carries a current I, as shown. The magnetic field at C has magnitude

(a) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ (b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

(c) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (d) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

14. An electron moving in a circular orbit of radius r makes n rotations per second. The magnetic field produced at the centre has magnitude

(a) zero (b) $\frac{\mu_0 ne}{2\pi r}$

(c) $\frac{\mu_0 ne}{2r}$ (d) $\frac{\mu_0 n^2 e}{2r}$

15. In the previous question, the orbital electron has magnetic moment

(a) zero (b) $\pi r^2 ne$

(c) $\pi r^2 n^2 e$ (d) $\frac{\mu_0}{2\pi} r^2 ne$

16. A flat circular coil of n turns and radius r carries a current i. Its magnetic moment is

(a) $\pi r^2 ni$ (b) $2\pi rni$

(c) $\mu_0\left(\frac{ni}{2\pi r}\right)$ (d) $\mu_0\pi r^2 ni$

17. A point P lies on the axis of a flat coil carrying a current. The magnetic moment of the coil is μ. The distance of P from the coil is d, which is large compared to the radius of the coil. The magnetic field at P has magnitude

(a) $\frac{\mu_0}{2\pi}\left(\frac{\mu}{d^3}\right)$ (b) $\frac{\mu_0}{4\pi}\left(\frac{\mu}{d^3}\right)$

(c) $\frac{\mu_0}{2\pi}\left(\frac{\mu}{d^2}\right)$ (d) $\frac{\mu_0}{4\pi}\left(\frac{\mu}{d^2}\right)$

18. Current flows through a straight, thin-walled tube of radius r. The magnetic field at a distance x from the axis of the tube has magnitude B.

(a) $B \propto x$, for $0 < x < r$ (b) $B \propto 1/x$, for $0 < x < r$

(c) $B = 0$, for $0 \le x < r$ (d) $B = 0$, only for $x = 0$

19. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in opposite directions. Let B_1 and B_2 be the magnetic fields in the regions between the conductors, and outside the conductor, respectively.

(a) $B_1 \ne 0,\ B_2 \ne 0$ (b) $B_1 = B_2 = 0$

(c) $B_1 \ne 0,\ B_2 = 0$ (d) $B_1 = 0,\ B_2 \ne 0$

20. In the previous question, if both conductors carry equal currents in the same direction,

(a) $B_1 \ne 0,\ B_2 \ne 0$ (b) $B_1 = B_2 = 0$

(c) $B_1 \ne 0,\ B_2 = 0$ (d) $B_1 = 0,\ B_2 \ne 0$

21. In Q. No. 20, let B be the magnetic field at a point between the two conductors, at a distance x from the axis. Let B_2 be the magnetic field at a point outside the outer conductor, at a distance $2x$ from the axis.

(a) $B_2 = B_1$ (b) $B_1 = 2B_2$

(c) $B_2 = 2B_1$ (d) $B_2 = 4B_1$

22. A long straight conductor carrying a current lies along the axis of a ring. The conductor will exert a force on the ring if the ring

(a) carries a current

(b) has uniformly distributed charge

(c) has nonuniformly distributed charge

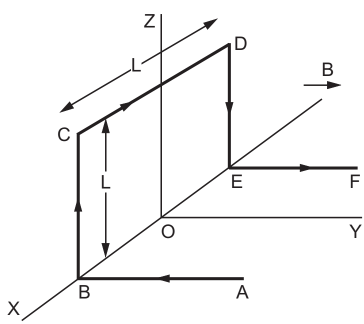
(d) none of the above

23. In the previous question, let the ring have nonuniformly distributed charge, and let it be spinning about its axis. Let F be the net force on the ring and τ be the torque on the ring due to the straight conductor.

(a) $F \neq 0,\ \tau \neq 0$ (b) $F = 0,\ \tau \neq 0$

(c) $F \neq 0,\ \tau = 0$ (d) $F = 0,\ \tau = 0$

24.



A conductor ABCDEF, with each side of length L, is bent as shown. It is carrying a current I in a uniform magnetic induction (field) B, parallel to the positive y-direction. The force experienced by the wire is

(a) BIL in the positive y-direction

(b) BIL in the positive z-direction

(c) $3BIL$

(d) zero

25. A closed loop lying in the xy plane carries a current. If a uniform magnetic field B is present in the region, the force acting on the loop will be zero if B is in

(a) the x-direction (b) the y-direction

(c) the z-direction (d) any of the above directions

26. An irregular closed loop carrying a current has a shape such that the entire loop cannot lie in a single plane. If this is placed in a uniform magnetic field, the force acting on the loop

(a) must be zero

(b) can never be zero

(c) may be zero

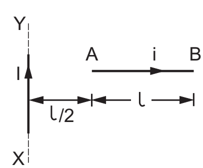
(d) will be zero only for one particular direction of the magnetic field

27. A horizontal straight conductor of mass m and length l is placed in a uniform vertical magnetic field of magnitude B. An amount of charge Q passes through the rod in a very short time such that the conductor begins to move only after all the charge has passed through it. Its initial velocity will be

(a) $BQlm$ (b) $\frac{BQ}{lm}$

(c) $\frac{BQl}{m}$ (d) $\frac{Bl}{mQ}$

28.



A conductor AB of length l, carrying a current i, is placed perpendicular to a long straight conductor XY carrying a current I, as shown. The force on AB will act

(a) upward (b) downward

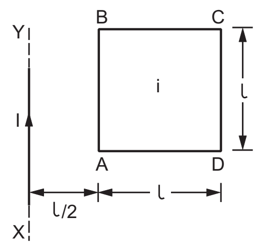
(c) to the right (d) to the left

29. In the previous question, the force on AB has magnitude

(a) $\frac{\mu_0 Ii}{2\pi} \ln 2$ (b) $\frac{\mu_0 Ii}{2\pi} \ln 3$

(c) $\frac{3\mu_0 Ii}{2\pi}$ (d) $\frac{2\mu_0 Ii}{3\pi}$

30.



A square loop ABCD, carrying a current *i*, is placed near and coplanar with a long straight conductor XY carrying a current *I*.

(a) There is no net force on the loop.

(b) The loop will be attracted by the conductor only if the current in the loop flows clockwise.

(c) The loop will be attracted by the conductor only if the current in the loop flows anticlockwise.

(d) The loop will always be attracted by the conductor.

31. In the previous question, the net force on the loop will be

(a) $\frac{2\mu_0 Ii}{3\pi}$ (b) $\frac{\mu_0 Ii}{2\pi}$

(c) $\frac{2\mu_0 Iil}{3\pi}$ (d) $\frac{\mu_0 Iil}{2\pi}$

32. A flat coil of n turns, area A and carrying a current i is placed in a uniform magnetic field of magnitude B. The plane of the coil makes an angle θ with the direction of the field. The torque acting on the coil is

(a) $BinA\sin\theta$ (b) $\frac{nAi}{B}\sin\theta$

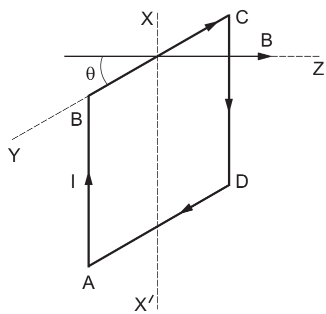
(c) $BinA\cos\theta$ (d) $Bin^2A\cos\theta$

33. A flat coil carrying a current has a magnetic moment μ. It is initially in equilibrium, with its plane perpendicular to a magnetic field of magnitude B. If it is now rotated through an angle θ, the work done is

(a) $\mu B\theta$ (b) $\mu B\cos\theta$

(c) $\mu B(1 - \cos\theta)$ (d) $\mu B\sin\theta$

34.

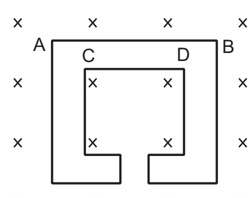


The square loop ABCD, carrying a current I, is placed in a uniform magnetic field B, as shown. The loop can rotate about the axis XX′. The plane of the loop makes an angle θ ($\theta < 90°$) with the direction of B. Through what angle will the loop rotate by itself before the torque on it becomes zero?

(a) θ (b) $90° - \theta$

(c) $90° + \theta$ (d) $180° - \theta$

35.

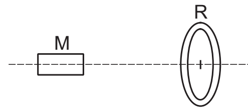


A wire is bent to form the double loop shown in the figure. There is a uniform magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, current will flow from

(a) A to B and C to D (b) B to A and D to C

(c) A to B and D to C (d) B to A and C to D

36.



A conducting ring R is placed on the axis of a bar magnet M. The plane of R is perpendicular to this axis. M can move along this axis.

(a) M will repel R when it is moving towards R.

(b) M will attract R when it is moving towards R.

(c) M will repel R when moving towards as well as away from R.

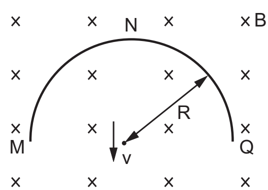
(d) M will attract R when moving towards as well as away from R.

37. A conductor PQ, with $\overrightarrow{PQ} = \vec{r}$, moves with a velocity $\vec{v}$ in a uniform magnetic field of induction $\vec{B}$. The emf induced in the rod is

(a) $(\vec{v} \times \vec{B}) \cdot \vec{r}$ (b) $\vec{v} \cdot (\vec{r} \times \vec{B})$

(c) $\vec{B} \cdot (\vec{r} \times \vec{v})$ (d) $|\vec{r} \times (\vec{v} \times \vec{B})|$

38.



A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ, the speed of the ring is v. The potential difference developed across the ring is

(a) zero

(b) $\frac{1}{2} Bv\pi R^2$, and M is at a higher potential

(c) πRBv, and Q is at a higher potential

(d) $2RBv$, and Q is at a higher potential

39. The magnitude of the earth's magnetic field at a place is B_0 and the angle of dip is δ. A horizontal conductor of length l, lying north-south, moves eastwards with a velocity v. The emf induced across the rod is

(a) zero (b) B_0lv

(c) $B_0lv\sin\delta$ (d) $B_0lv\cos\delta$

40. In the previous question, if the conductor lies east-west and moves vertically up with a speed v, the induced emf is

(a) zero (b) B_0lv

(c) $B_0lv\sin\delta$ (d) $B_0lv\cos\delta$

41. The two ends of a horizontal conducting rod of length l are joined to a voltmeter. The whole arrangement moves with a horizontal velocity v, the direction of motion being perpendicular to the rod. The vertical component of the earth's magnetic field is B. The voltmeter reading is

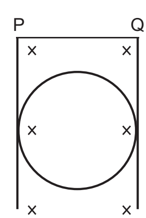
(a) Blv only if the rod moves eastward

(b) Blv only if the rod moves westward

(c) Blv if the rod moves in any direction

(d) zero

42. A vertical ring of radius r and resistance R falls vertically. It is in contact with two vertical rails which are joined at the top. The rails are without friction and resistance. There is a horizontal uniform magnetic field of magnitude B perpendicular to the plane of the ring and the rails. When the speed of the ring is v, the current in the section PQ is



(a) zero (b) $\frac{2Brv}{R}$ (c) $\frac{4Brv}{R}$ (d) $\frac{8Brv}{R}$

43. A horizontal ring of radius r spins about its axis with an angular velocity ω in a uniform vertical magnetic field of magnitude B. The emf induced in the ring is

(a) zero (b) $\pi r^2\omega B$ (c) $\frac{1}{2}Br^2\omega$ (d) $Br^2\omega$

44. A metal rod of resistance R is fixed along a diameter of a conducting ring of radius r. There is a magnetic field of magnitude B perpendicular to the plane of the loop. The ring spins with an angular velocity ω about its axis. The centre of the ring is joined to its rim by an external wire W. The ring and W have no resistance. The current in W is

(a) zero (b) $\frac{Br^2\omega}{2R}$

(c) $\frac{Br^2\omega}{R}$ (d) $\frac{2Br^2\omega}{R}$

45. Three identical rings move with the same speed on a horizontal surface in a uniform horizontal magnetic field normal to the planes of the rings. The first (A) slips without rolling, the second (B) rolls without slipping, and the third rolls with slipping.

(a) The same emf is induced in all three rings.

(b) No emf is induced in any of the rings.

(c) In each ring all points are at the same potential.

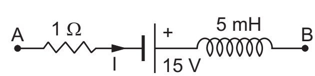
(d) B develops the maximum induced emf, and A, the least.

46. A long solenoid of N turns has a self-inductance L and area of cross-section A. When a current i flows through the solenoid, the magnetic field inside it has magnitude B. The current i is equal to

(a) $\frac{BAN}{L}$ (b) $BANL$

(c) $\frac{BN}{AL}$ (d) $\frac{B}{ANL}$

47. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5 A, and is decreasing at a rate 10^3 A/s then $V_B - V_A$ is



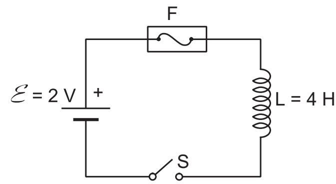
(a) 20 V (b) 15 V (c) 10 V (d) 5 V

48. In the previous question, if the direction of I is reversed, $V_B - V_A$ will be

(a) 20 V (b) 15 V

(c) 10 V (d) 5 V

49.

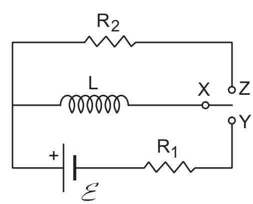


In the circuit shown, the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow

(a) almost at once (b) after 2 s

(c) 5 s (d) after 10 s

50.



In the circuit shown, X is joined to Y for a long time, and then X is joined to Z. The total heat produced in R_2 is

(a) $\dfrac{L\mathcal{E}^2}{2R_1^2}$ (b) $\dfrac{L\mathcal{E}^2}{2R_2^2}$

(c) $\dfrac{L\mathcal{E}^2}{2R_1R_2}$ (d) $\dfrac{L\mathcal{E}^2R_2}{2R_1^3}$

51. Two coils, A and B, are linked such that emf $\mathcal{E}$ is induced in B when the current in A is changing at the rate $\dot{I}$. If i current is now made to flow in B, the flux linked with A will be

(a) $(\mathcal{E}/\dot{I})i$ (b) $\mathcal{E}\, i\, \dot{I}$

(c) $(\mathcal{E}\dot{I})i$ (d) $i\dot{I}/\mathcal{E}$

52. Two coils of inductances L_1 and L_2 are linked such that their mutual inductance is M.

(a) $M = L_1 + L_2$

(b) $M = \frac{1}{2}(L_1 + L_2)$

(c) The maximum value of M is $(L_1 + L_2)$.

(d) The maximum value of M is $\sqrt{L_1 L_2}$.

53. A small coil of radius r is placed at the centre of a large coil of radius R, where $R >> r$. The two coils are coplanar. The mutual induction between the coils is proportional to

(a) r/R (b) r^2/R

(c) r^2/R^2 (d) r/R^2

54. A uniformly wound long solenoid of inductance L and resistance R is broken into two equal parts, which are then joined in parallel. This combination is then joined to a cell of emf $\mathcal{E}$. The time constant of the circuit is

(a) L/R (b) $L/2R$

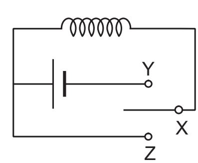
(c) $2L/R$ (d) $L/4R$

55. In the previous question, the steady-state current is

(a) $\mathcal{E}/R$ (b) $2\mathcal{E}/R$

(c) $4\mathcal{E}/R$ (d) $8\mathcal{E}/R$

56. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z.



(a) The total heat produced in the coil is $P\tau$.

(b) The total heat produced in the coil is $\frac{1}{2} P\tau$.

(c) The total heat produced in the coil is $2P\tau$.

(d) The data given is not sufficient to reach a conclusion.

57. When a coil is joined to a cell, the current through the cell grows with a time constant τ. The current will reach 10% of its steady-state value in time

(a) 0.1τ (b) $\tau \ln (0.1)$

(c) $\tau \ln (0.9)$ (d) $\tau \ln (10/9)$

58. In the previous question, the current will be 10% less than its steady-state value after time

(a) 0.9τ (b) $\tau \ln (0.9)$

(c) $\tau \ln (9)$ (d) $\tau \ln (10)$

59. At $t = 0$, an inductor of zero resistance is joined to a cell of emf $\mathcal{E}$ through a resistance. The current increases with a time constant τ. The emf across the coil after time t is

(a) $\mathcal{E} t/\tau$ (b) $\mathcal{E}(1 - e^{-t/\tau})$

(c) $\mathcal{E} e^{-t/\tau}$ (d) $\mathcal{E} e^{-2t/\tau}$

60. In the previous question, after what time will the potential difference across the coil be equal to that across the resistance?

(a) τ (b) $\tau \ln 2$

(c) $\tau(1 - \ln 2)$ (d) $\tau/\ln 2$

61. When a coil carrying a steady current is short-circuited, the current in it decreases η times in time t_0. The time constant of the circuit is

(a) $t_0 \ln \eta$ (b) $\dfrac{t_0}{\ln \eta}$

(c) $\dfrac{t_0}{\eta}$ (d) $\dfrac{t_0}{\eta - 1}$

62. When a coil is joined to a cell, current in it grows with a time constant τ. The coil is disconnected from the cell before the current has reached its steady-state value, and, it is then short-circuited. The current will now decrease with a time constant

(a) τ

(b) $> \tau$

(c) $< \tau$

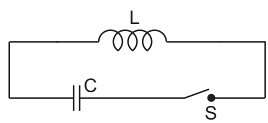
(d) either (b) or (c) depending on the instant at which it was disconnected from the cell

63. In the previous question, after the coil is short-circuited, the energy stored in it will decrease with a time constant

(a) τ (b) $\tau/2$

(c) 2τ (d) $\frac{3}{2}\tau$

64.

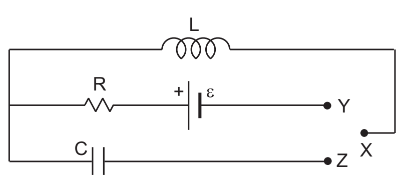


The capacitor of capacity C is given charge Q and then connected to the coil of inductance L by closing the switch S. The maximum current flowing in the circuit at any later time will be

(a) $\frac{Q}{2\sqrt{LC}}$ (b) $\frac{Q}{\sqrt{LC}}$

(c) $\frac{2Q}{\sqrt{LC}}$ (d) $\frac{2}{\pi}\frac{Q}{\sqrt{LC}}$

65.



In the circuit shown, the symbols have their usual meanings. The cell has emf $\mathcal{E}$. X is initially joined to Y for a long time. Then, X is joined to Z. The maximum charge on C at any later time will be

(a) $\frac{\mathcal{E}}{R\sqrt{LC}}$ (b) $\frac{\mathcal{E}R}{2\sqrt{LC}}$

(c) $\frac{\mathcal{E}\sqrt{LC}}{2R}$ (d) $\frac{\mathcal{E}\sqrt{LC}}{2R}$

66. A coil of self-inductance L is placed in an external magnetic field (no current flows in the coil). The total magnetic flux linked with the coil is φ. The magnetic field energy stored in the coil is

(a) zero (b) $\frac{\varphi^2}{2L}$

(c) $\frac{\varphi^2}{L}$ (d) $\frac{2\varphi^2}{L}$

67. When a charged capacitor is made to discharge through an inductance, the charge Q on the capacitor and the current i in the inductor vary sinusoidally. The phase difference between Q and i is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) zero

LR, RC and LC circuits in AC

68. A charged capacitor discharges through a resistance R with time constant τ. The two are now placed in series across an AC source of angular frequency $\omega = \frac{1}{\tau}\cdot$ The impedance of the circuit will be

(a) $\frac{R}{\sqrt{2}}$ (b) R

(c) $\sqrt{2R}$ (d) $2R$

69. An electric lamp designed for operation on 110 V AC is connected to a 220 V AC supply, through a choke coil of inductance 2H, for proper operation. The angular frequency of the AC is $100\sqrt{10}$ rad/s. If a capacitor is to be used in place of the choke coil, its capacitance must be

(a) 1 μF (b) 2 μF

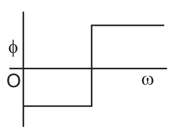
(c) 5 μF (d) 10 μF

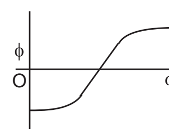
70. A resistance R draws P power when connected to an AC source. If an inductance is now placed in series with the resistance, such that the impedance of the circuit becomes Z, the power drawn will be

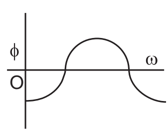
(a) $P\left(\frac{R}{Z}\right)^2$ (b) $P\left(\frac{R}{Z}\right)$

(c) $P\sqrt{\frac{R}{Z}}$ (d) P

71. An inductor and a capacitor are joined in series to an AC source. The frequency of the AC is gradually increased. The phase difference φ between the emf and the current is plotted against the angular frequency ω. Which of the following best represents the resulting curve?

(a)


(b)


(c)


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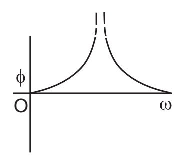
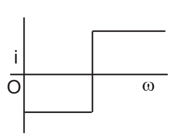
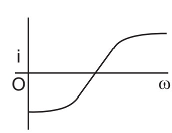
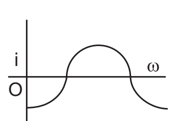
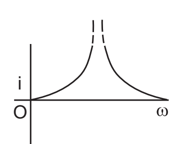


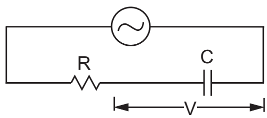
72. In the previous question, if the current i in the circuit is plotted against ω, which of the following best represents the resulting curve?

(a)


(b)


(c)


(d)


73. 

A resistance R and a capacitor C are joined to a source of AC of constant emf and variable frequency. The potential difference across C is V. If the frequency of AC is gradually increased, V will

(a) increase

(b) decrease

(c) remain constant

(d) first increase and then decrease

74. An inductance L, a capacitance C and a resistance R may be connected to an AC source of angular frequency ω, in three different combinations of RC, RL and LC in series. Assume that $\omega L = \frac{1}{\omega C}$. The power drawn by the three combinations are P_1, P_2, P_3 respectively. Then,

(a) $P_1 > P_2 > P_3$ (b) $P_1 = P_2 < P_1$

(c) $P_1 = P_2 > P_1$ (d) $P_1 = P_2 = P_3$

75. An electrical heater and a capacitor are joined in series across a 220 V, 50 Hz AC supply. The potential difference across the heater is 90 V. The potential difference across the capacitor will be about

(a) 200 V (b) 130 V

(c) 110 V (d) 90 V

76. In an AC circuit, the reactance is equal to the resistance. The power factor of the circuit will be

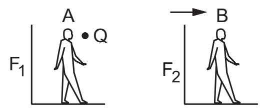
(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) zero

• Type 2 •

Choose the correct options. One or more options may be correct.

77.



An observer A and a charge Q are fixed in a stationary frame F_1. An observer B is fixed in a frame F_2, which is moving with respect to F_1.

(a) Both A and B will observe electric fields.

(b) Both A and B will observe magnetic fields.

(c) Neither A nor B will observe magnetic fields.

(d) B will observe a magnetic field, but A will not.

78. A long straight wire carries a current along the x-axis. Consider the points A(0, 1, 0), B(0, 1, 1), C(1, 0, 1) and D(1, 1, 1). Which of the following pairs of points will have magnetic fields of the same magnitude?

(a) A and B (b) A and C

(c) B and C (d) B and D

79. In the previous question, if the current is i and the magnetic field at D has magnitude B,

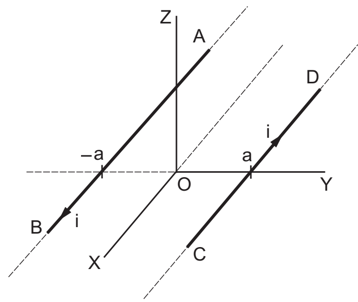
(a) $B = \dfrac{\mu_0 i}{2\sqrt{2}\pi}$

(b) $B = \dfrac{\mu_0 i}{2\sqrt{3}\pi}$

(c) B is parallel to the z-axis

(d) B makes an angle of 45° with the xy plane

80.



Two long parallel wires, AB and CD, carry equal currents in opposite directions. They lie in the xy plane, parallel to the x-axis, and pass through the points $(0, -a, 0)$ and $(0, a, 0)$ respectively. The resultant magnetic field is

(a) zero on the x-axis

(b) maximum on the x-axis

(c) directed along the z-axis at the origin, but not at other points on the z-axis

(d) directed along the z-axis at all points on the z-axis

81. A straight conductor carries a current. Assume that all free electrons in the conductor move with the same drift velocity v. A and B are two observers on a straight line XY parallel to the conductor. A is stationary. B moves along XY with a velocity v in the direction of the free electrons.

(a) A and B observe the same magnetic field.

(b) A observes a magnetic field, B does not.

(c) A and B observe magnetic fields of the same magnitude but opposite directions.

(d) A and B do not observe any electric field.

82. A straight conductor carries a current along the z-axis. Consider the points $A(a, 0, 0)$, $B(0, -a, 0)$, $C(-a, 0, 0)$ and $D(0, a, 0)$.

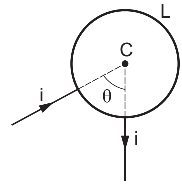
(a) All four points have magnetic fields of the same magnitude.

(b) All four points have magnetic fields in different directions.

(c) The magnetic fields at A and C are in opposite directions.

(d) The magnetic fields at A and B are mutually perpendicular.

83. L is a circular ring made of a uniform wire. Current enters and leaves the ring through straight conductors which, if produced, would have passed through the centre C of the ring. The magnetic field at C



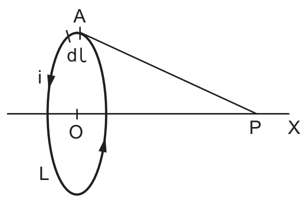
(a) due to the straight conductors is zero

(b) due to the loop is zero

(c) due to the loop is proportional to θ

(d) due to the loop is proportional to (π – θ)

84.



L is a circular loop carrying a current. P is a point on its axis OX. *dl* is an element of length on the loop at a point A on it. The magnetic field at P

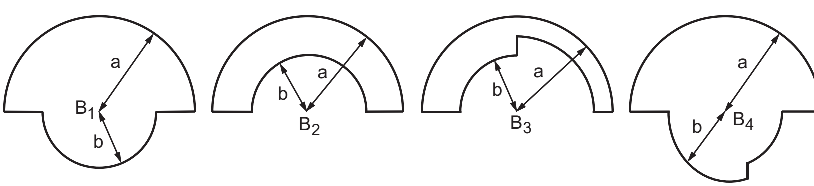
(a) due to L is directed along OX

(b) due to *dl* is directed along OX

(c) due to *dl* is perpendicular to OX

(d) due to *dl* is perpendicular to AP

85.



In the loops shown, all curved sections are either semicircles or quarter circles. All the loops carry the same current. The magnetic fields at the centres have magnitudes B_1, B_2, B_3 and B_4.

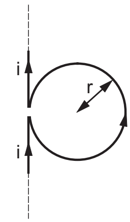
(a) B_4 is maximum.

(b) B_3 is minimum.

(c) $B_4 > B_1 > B_2 > B_3$

(d) $B_1 > B_4 > B_3 > B_2$

86. A long straight conductor, carrying a current i, is bent to form an almost complete circular loop of radius r on it. The magnetic field at the centre of the loop



(a) is directed into the paper

(b) is directed out of the paper

(c) has magnitude $\frac{\mu_0 i}{2r}\left(1-\frac{1}{\pi}\right)$

(d) has magnitude $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$

87. A flat circular coil, carrying a current, has a magnetic moment μ.

(a) μ has only magnitude; it does not have direction.

(b) The direction of μ is along the normal to the plane of the coil.

(c) The direction of μ depends on the direction of the current flow.

(d) The direction of μ does not change if the current in the coil is reversed.

88. Current flows through a straight cylindrical conductor of radius r. The current is distributed uniformly over its cross-section. The magnetic field at a distance x from the axis of the conductor has magnitude B.

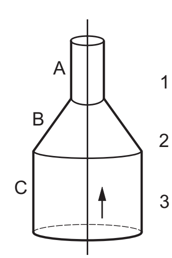
(a) $B = 0$ at the axis.

(b) $B \propto x$ for $0 \le x \le r$.

(c) $B \propto \frac{1}{x}$ for $x > r$.

(d) B is maximum for $x = r$.

89. A long, straight, hollow conductor (tube) carrying a current has two sections A and C of unequal cross-sections joined by a conical section B. 1, 2 and 3 are points on a line parallel to the axis of the conductor. The magnetic fields at 1, 2 and 3 have magnitudes B_1, B_2 and B_3.



(a) $B_1 = B_2 = B_3$

(b) $B_1 = B_2 \ne B_3$

(c) $B_1 < B_2 < B_3$

(d) B_2 cannot be found unless the dimensions of the section B are known.

90. A conductor AB carries a current i in a magnetic field $\vec{B}$. If $\vec{AB} = \vec{r}$ and the force on the conductor is $\vec{F}$,

(a) $\vec{F}$ does not depend on the shape of AB

(b) $\vec{F} = i(\vec{r} \times \vec{B})$

(c) $\vec{F} = i(\vec{B} \times \vec{r})$

(d) $|\vec{F}| = i(\vec{r} \cdot \vec{B})$

91. A semicircular wire of radius r, carrying a current i, is placed in a magnetic field of magnitude B. The force acting on it

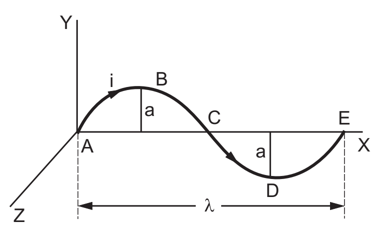
(a) can never be zero

(b) can have the maximum magnitude $2Bir$

(c) can have the maximum magnitude $Bi\pi r$

(d) can have the maximum magnitude Bir

92.



A conductor ABCDEF, shaped as shown, carries a current i. It is placed in the xy plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be

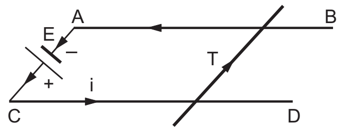
(a) zero, if B is in the x-direction

(b) λBi in the z-direction, if B is in the y-direction

(c) λBi in the negative y-direction, if B is in the z-direction

(d) $2aBi$, if B is in the x-direction

93.



AB and CD are smooth, parallel, horizontal rails on which a conductor T can slide. A cell, E, drives current i through the rails and T.

(a) The current in the rails will set up a magnetic field over T.

(b) T will experience a force to the right.

(c) T will experience a force to the left.

(d) T will not experience any force.

94. In the previous question,

(a) the force on T is proportional to i

(b) the force on T is proportional to i^2

(c) if the direction of i is reversed in the circuit by reversing E, the force on T will reverse in direction

(d) if the direction of i is reversed in the circuit by reversing E, the force on T will remain in the same direction

95. Two long, thin, parallel conductors, separated by a distance d, carry currents i_1 and i_2. The force acting on unit length of any one conductor is F.

(a) F is attractive, if i_1 and i_2 flow in the same direction.

(b) F is attractive, if i_1 and i_2 flow in opposite directions.

(c) F is the same for both conductors.

(d) F is different for the two conductors.

96. In the previous question,

(a) $F \propto (i_1 i_2)$ (b) $F \propto (i_1 i_2)^2$

(c) $F \propto \frac{1}{d^2}$ (d) $F \propto \frac{1}{d}$

97. A conducting gas is in the form of a long cylinder. Current flows through the gas along the length of the cylinder. The current is distributed uniformly across the cross-section of the gas. Disregard thermal and electrostatic forces among the gas molecules. Due to the magnetic fields set up inside the gas and the forces which they exert on the moving ions, the gas will tend to

(a) expand

(b) contract

(c) expand and contract alternately

(d) none of the above

98. A current-carrying ring is placed in a magnetic field. The direction of the field is perpendicular to the plane of the ring.

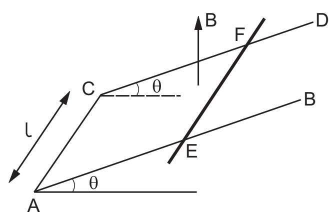
(a) There is no net force on the ring.

(b) The ring will tend to expand.

(c) The ring will tend to contract.

(d) Either (b) or (c) depending on the directions of the current in the ring and the magnetic field.

99.



AB and CD are smooth parallel rails, separated by a distance l, and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current i. For EF to be in equilibrium,

(a) i must flow from E to F (b) $Bil = mg\tan\theta$

(c) $Bil = mg\sin\theta$ (d) $Bil = mg$

100. In the previous question, if B is normal to the plane of the rails,

(a) $Bil = mg\tan\theta$

(b) $Bil = mg\sin\theta$

(c) $Bil = mg\cos\theta$

(d) equilibrium cannot be reached

101. A flat coil carrying a current has a magnetic moment $\vec{\mu}$. It is placed in a magnetic field $\vec{B}$. The torque on the coil is $\vec{\tau}$.

(a) $\vec{\tau} = \vec{\mu} \times \vec{B}$

(b) $\vec{\tau} = \vec{B} \times \vec{\mu}$

(c) $|\vec{\tau}| = \vec{\mu} \cdot \vec{B}$

(d) $\vec{\tau}$ is perpendicular to both $\vec{\mu}$ and $\vec{B}$.

102. A flat coil carrying a current has a magnetic moment $\vec{\mu}$. It is placed in a magnetic field $\vec{B}$ such that $\vec{\mu}$ is antiparallel to $\vec{B}$. The coil is

(a) not in equilibrium (b) in stable equilibrium

(c) in unstable equilibrium (d) in neutral equilibrium

103. The magnetic flux (ϕ) linked with a coil depends on time t as $\phi = at^n$, where a and n are constants. The emf induced in the coil is e.

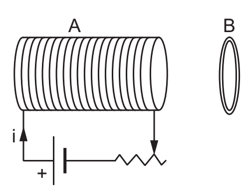
(a) If $0 < n < 1$, $e = 0$.

(b) If $0 < n < 1$, $e \neq 0$ and $|e|$ decreases with time.

(c) If $n = 1$, e is constant.

(d) If $n > 1$, $|e|$ increases with time.

104.



An aluminium ring B faces an electromagnet A. The current i through A can be altered.

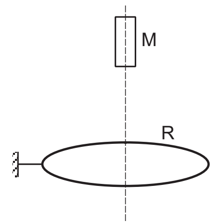
(a) If i increases, A will repel B.

(b) If i increases, A will attract B.

(c) If i decreases, A will attract B.

(d) If i decreases, A will repel B.

105.



A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be

(a) $< g$ when it is above R and moving towards R

(b) $> g$ when it is above R and moving towards R

(c) $< g$ when it is below R and moving away from R

(d) $> g$ when it is below R and moving away from R

106. In the previous question, the directions of the current flow in the ring, when M is above R and below R will be

(a) the same in all cases

(b) opposite in all cases

(c) the same only if the N-pole of M moves towards R when M is above R

(d) the same only if the S-pole of M moves towards R when M is above R

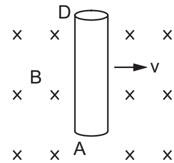
107. The magnetic flux linked with a coil is ϕ and the emf induced in it is e.

(a) If $\phi = 0$, e must be 0.

(b) If $\phi \neq 0$, e cannot be 0.

(c) If e is not 0, ϕ may or may not be 0.

(d) None of the above is correct.

108. 

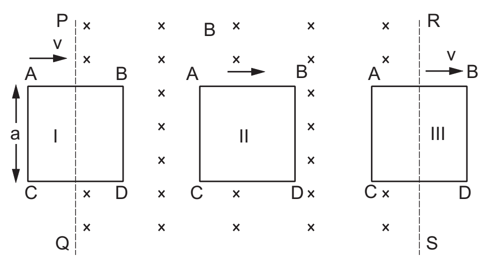
The conductor AD moves to the right in a uniform magnetic field directed into the paper.

(a) The free electrons in AD will move towards A.

(b) D will acquire a positive potential with respect to A.

(c) If D and A are joined by a conductor externally, a current will flow from A to D in AD.

(d) The current in AD flows from lower to higher potential.

109. 

A square loop ABCD of edge *a* moves to the right with a velocity *v*, parallel to AB. There is a uniform magnetic field of magnitude *B*, directed into the paper, in the region between PQ and RS only. I, II and III are three positions of the loop.

(a) The emf induced in the loop has magnitude *Bav* in all three positions.

(b) The induced emf is zero in position II.

(c) The induced emf is anticlockwise in position I.

(d) The induced emf is clockwise in position III.

110. In the previous question, in position I of the loop,

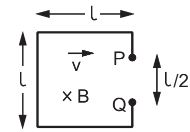
(a) the induced emf will increase linearly as the loop enters the field

(b) the induced emf will increase from 0 to *Bav* sharply as the edge BC crosses PQ

(c) the induced emf will have a constant value Ba^2v

(d) the loop will experience a force to the left after entering the field partially

111. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between P and Q is e.



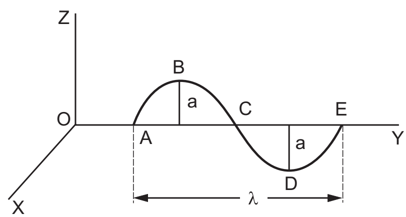
(a) $e = \frac{1}{2} Blv$

(b) $e = Blv$

(c) P is positive with respect to Q.

(d) Q is positive with respect to P.

112.



The conductor ABCDE has the shape shown. It lies in the yz plane, with A and E on the y-axis. When it moves with a velocity v in a magnetic field B, an emf e is induced between A and E.

(a) $e = 0$, if v is in the y-direction and B is in the x-direction.

(b) $e = 2Bav$, if v is in the y-direction and B is in the x-direction.

(c) $e = B\lambda v$, if v is in the z-direction and B is in the x-direction.

(d) $e = B\lambda v$, if v is in the x-direction and B is in the z-direction.

113. The magnitude of the earth's magnetic field at the North Pole is B_0. A horizontal conductor of length l moves with a velocity v. The direction of v is perpendicular to the conductor. The induced emf is

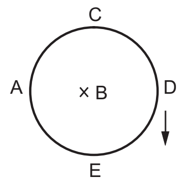
(a) zero, if v is vertical

(b) $B_0 lv$, if v is vertical

(c) zero, if v is horizontal

(d) $B_0 lv$, if v is horizontal

114.



A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B. The direction of B is perpendicular to the plane of the ring. When the speed of the ring is v,

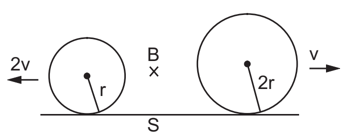
(a) no current flows in the ring

(b) A and D are at the same potential

(c) C and E are at the same potential

(d) the potential difference between A and D is $2BRv$, with D at a higher potential

115.



Two conducting rings of radii r and $2r$ move in opposite directions with velocities $2v$ and v respectively on a conducting surface S. There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is

(a) zero (b) $2rvB$

(c) $4rvB$ (d) $8rvB$

116. The magnetic field perpendicular to the plane of a conducting ring of radius r changes at the rate $\frac{dB}{dt}$.

(a) The emf induced in the ring is $\pi r^2 \frac{dB}{dt}$.

(b) The emf induced in the ring is $2\pi r \frac{dB}{dt}$.

(c) The potential difference between diametrically opposite points on the ring is half of the induced emf.

(d) All points on the ring are at the same potential.

117. In the previous question, let E be the electric intensity at any point on the ring.

(a) E is tangential to the ring everywhere.

(b) $E = 0$ everywhere on the ring.

(c) $E = r\dfrac{dB}{dt}\cdot$

(d) $E = \dfrac{1}{2}r\dfrac{dB}{dt}\cdot$

118. A nonconducting ring of radius r has charge Q. A magnetic field perpendicular to the plane of the ring changes at the rate $\frac{dB}{dt}\cdot$ The torque experienced by the ring is

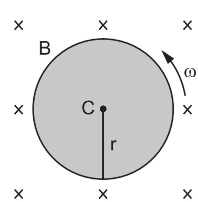
(a) zero

(b) $Qr^2\dfrac{dB}{dt}$

(c) $\dfrac{1}{2}Qr^2\dfrac{dB}{dt}$

(d) $\pi r^2 Q\dfrac{dB}{dt}$

119.



A conducting disc of radius r spins about its axis with an angular velocity ω. There is a uniform magnetic field of magnitude B perpendicular to the plane of the disc. C is the centre of the ring.

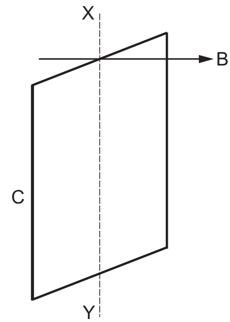
(a) No emf is induced in the disc.

(b) The potential difference between C and the rim is $\frac{1}{2}Br^2\omega$.

(c) C is at a higher potential than the rim.

(d) Current flows between C and the rim.

120.



A flat coil, C, of n turns, area A and resistance R is placed in a uniform magnetic field of magnitude B. The plane of the coil is initially perpendicular to B. If the coil is rotated by an angle θ about the axis XY, charge of amount Q flows through it.

(a) If $\theta = 90°$, $Q = \frac{BAn}{R}$.
(b) If $\theta = 180°$, $Q = \frac{BAn}{R}$.
(c) If $\theta = 180°$, $Q = 0$.
(d) If $\theta = 360°$, $Q = 0$.

121. In the previous question, the plane of the coil is initially kept parallel to B. All other details remain the same.

(a) If $\theta = 90°$, $Q = \frac{BAn}{R}$.
(b) If $\theta = 180°$, $Q = \frac{2BAn}{R}$.
(c) If $\theta = 180°$, $Q = 0$.
(d) If $\theta = 360°$, $Q = 0$.

122. In the previous question, if the coil rotates about XY with a constant angular velocity ω, the emf induced in it

(a) is zero
(b) changes nonlinearly with time
(c) has a constant value = $BAn\omega$
(d) has a maximum value = $BAn\omega$

123. The SI unit of inductance, the henry, can be written as

(a) weber/ampere
(b) volt second/ampere
(c) joule/ampere^2
(c) ohm second

Answers

1. d	**2.** a	**3.** c	**4.** c	**5.** a
6. d	**7.** a	**8.** c	**9.** c	**10.** d
11. c	**12.** c	**13.** d	**14.** c	**15.** b
16. a	**17.** a	**18.** c	**19.** c	**20.** a
21. a	**22.** d	**23.** d	**24.** a	**25.** d
26. a	**27.** c	**28.** a	**29.** b	**30.** b
31. a	**32.** c	**33.** c	**34.** c	**35.** c
36. a	**37.** a	**38.** d	**39.** c	**40.** d
41. d	**42.** d	**43.** a	**44.** d	**45.** a
46. a	**47.** b	**48.** b	**49.** d	**50.** a
51. a	**52.** d	**53.** b	**54.** a	**55.** c
56. b	**57.** d	**58.** d	**59.** c	**60.** b
61. b	**62.** a	**63.** b	**64.** b	**65.** d
66. b	**67.** b	**68.** c	**69.** c	**70.** a
71. a	**72.** d	**73.** b	**74.** b	**75.** a
76. c	**77.** a, d	**78.** b, d	**79.** a, d	**80.** b, d
81. a, d	**82.** a, b, c, d	**83.** a, b	**84.** a, d	**85.** a, b, c
86. b, c	**87.** b, c	**88.** a, b, c, d	**89.** a	**90.** a, b
91. b	**92.** a, b, c	**93.** a, b	**94.** b, d	**95.** a, c
96. a, d	**97.** b	**98.** a, d	**99.** a, b	**100.** b
101. a, d	**102.** c	**103.** b, c, d	**104.** a, c	**105.** a, c
106. b	**107.** c	**108.** a, b, c, d	**109.** b, c, d	**110.** b, d
111. a, c	**112.** a, c, d	**113.** a, d	**114.** c, d	**115.** d
116. a, d	**117.** a, d	**118.** c	**119.** b, c	**120.** a, b, d
121. a, c, d	**122.** a, d	**123.** a, b, c, d		

Hints and Solutions to Selected Questions

7. The resistance of AD is one-third of the resistance along the parallel path AB + BC + CD. Hence if current i enters at A, $3i/4$ will flow along AD and $i/4$, along AB. The magnetic field at the centre due to AD is equal and opposite to the combined effects of AB, BC and CD.

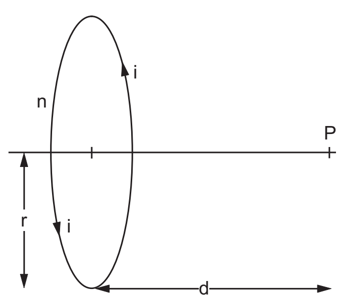
8. Sections AB and DE produce no field at O. Sections BC and EF produce equal fields at O, each = $\mu_0 I/(4\pi a)$.

9, 10. The sections AB and CD produce fields in the same direction at all points on BC. The magnitude of the field is variable, being a function of the distances from AB and CD.

At the midpoint of BC, $B = 2 \times \dfrac{\mu_0 I}{4\pi(d/2)}$.

14. The charge flowing past any point on the orbital path per second is ne. This is the equivalent orbital current.

17.

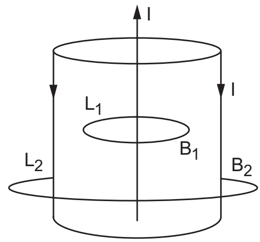


The magnetic field at P due to the flat coil of n turns, radius r, carrying current i is

$$B = \frac{\mu_0}{2} \cdot \frac{nir^2}{(d^2 + r^2)^{3/2}} \cong \frac{\mu_0}{2} \cdot \frac{nir^2}{d^3} \qquad (d >> r)$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{n(\pi r^2)i}{d^3} = \frac{\mu_0}{2\pi} \cdot \frac{\mu}{d^3}.$$

19.



Apply Ampere's law to the coaxial circular loops L_1 and L_2. The magnetic field is B_1 at all points on L_1 and B_2 at all points on L_2. $\Sigma i \neq 0$ for L_1 and 0 for L_2.

20. See the figure of Q. No. 19. If I is in the same direction for both conductors, $\Sigma i \neq 0$ for both L_1 and L_2.

21. See the figure of Q. No. 19. Let x and $2x$ be the radii of L_1 and L_2.

For L_1, $\oint_{L_1} \vec{B} \cdot \vec{dl} = 2\pi x B_1 = \mu_0 \Sigma i = \mu_0 I$ or $B_1 = \dfrac{\mu_0 I}{2\pi x}$.

For L_2, $\oint_{L_2} \vec{B} \cdot \vec{dl} = 2\pi(2x)B_2 = \mu_0 \Sigma i = \mu_0(2I)$ or $B_2 = \dfrac{\mu_0 I}{2\pi x}$.

24. The force on BC and DE are equal and opposite, and cancel out. The force on CD is BIL.

25, 26. **TIP** A closed, current-carrying loop of any shape placed in any uniform magnetic field, experiences no force.

27. The Ampere force on the conductor is $F = Bil$. The impulse given by this force to the conductor is $J = \int F\,dt = Bl \int i\,dt = BlQ$.

If u = initial velocity, $J = mu = BlQ$.

29. Consider an element of length dx on AB, at a distance x from XY.

Force on the element $= dF = \left(\dfrac{\mu_0 I}{2\pi x}\right) i\,dx$.

Total force on AB $= \int_{1/2}^{3/2} \frac{\mu_0 \, Ii}{2\pi x} dx = \frac{\mu_0 \, Ii}{2\pi} \ln 3.$

31. Force on AB due to XY $= F_1 = \left[\frac{\mu_0 \, I}{2\pi(1/2)}\right] il = \frac{\mu_0 \, Ii}{\pi}.$

Force on CD due to XY $= F_2 = \left[\frac{\mu_0 \, I}{2\pi(3/2)}\right] il = \frac{\mu_0 \, Ii}{3\pi}.$

F_1 and F_2 act in opposite directions, as AB and CD carry currents in opposite directions.

33. $\vec{\tau} = \vec{\mu} \times \vec{B}$ or $|\vec{\tau}| = \tau = \mu B \sin\theta.$

Work done$= \int \tau \, d\theta = \int_0^{\theta} \mu B \sin\theta = \mu B(1 - \cos\theta).$

34. In the position shown, AB is outside and CD is inside the plane of the paper. The Ampere force on AB acts into the paper. The torque on the loop will be clockwise, as seen from above. The loop must rotate through an angle $(90° + \theta)$ before the plane of the loop becomes normal to the direction of B and the torque becomes zero.

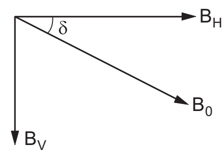
35. Applying Lenz's law, clockwise emf is induced in both the loops. As the outer loop has greater area, larger emf is induced in it.

36. Let the N-pole of M face R. As M moves towards R, applying Lenz's law, clockwise current flows in R, as seen from the right of R. Hence, the face of R towards M will behave as a N-pole and repel M. A similar argument will apply to the S-pole of M facing R. Also, M will attract R when moving away from R.

37. Consider a unit charge inside the conductor. As the conductor moves with a velocity $\vec{v}$, the charge inside it will experience the force $\vec{F} = \vec{v} \times \vec{B}$. If this unit charge is now moved from one end of the conductor to the other, its displacement is $\vec{r}$. Hence, the work done on it is $\vec{F} \cdot \vec{r} = (\vec{v} \times \vec{B}) \cdot \vec{r}$. This, by definition, is the emf across the conductor.

38. TIP The emf induced in a conductor does not depend on its shape but only on its end points. As a simple application of this principle (which follows from Q. No. 37), replace the actual conductor by an imaginary straight conductor joining its two ends.

39.



The vertical component of the earth's magnetic field is $B_V = B_0 \sin\delta$. A conductor moving horizontally is normal to B_V.

40. See the hint to Q. No. 39. The horizontal component of the earth's magnetic field is $B_H = B_0 \cos\delta$. A conductor lying east-west and moving vertically is normal to B_H.

41. TIP A closed loop of any shape, moving in a uniform magnetic field, induces no emf along the loop.

The voltmeter joined to the two ends of a rod forms a closed loop.

42. TIP When a ring moves in a magnetic field perpendicular to its plane, replace the ring by a diameter perpendicular to the direction of motion. The emf is induced across this diameter.

In the question, current flow in the ring will be through the two semicircular portions, in parallel.

Induced emf, $e = B(2r)v$.

Resistance of each half of ring $= R/2$.

As these are in parallel, the equivalent resistance $= R/4$.

Current in the circuit $= \dfrac{B(2r)v}{(R/4)} = \dfrac{8Brv}{R}$.

43. TIP A ring spinning about its axis in a magnetic field induces no emf.

44. The emf induced across a radius $= e = \frac{1}{2}Br^2\omega =$ potential difference between the centre and the rim. The rod fixed along a diameter is equivalent to two radii joined in parallel between the centre and the rim. As each radius has a resistance $R/2$, the equivalent resistance between the centre and the rim is $R/4$.

$$\text{Current} = \frac{e}{(r/4)} = \frac{4}{R}\cdot\frac{1}{2}Br^2\omega = \frac{2Br^2\omega}{R}.$$

45. The emf is induced in a ring due to its linear motion and does not depend on the spin.

46. $\phi = Li = BAN.$

47. Moving from A to B,

$$V_A - V_B = [1\times 5 - 15 + 5\times 10^{-3}(-10^3)]\ \text{V} = -15\ \text{V}$$

or $\quad V_B - V_A = 15\ \text{V} \quad [\frac{di}{dt}$ is negative as i is decreasing].

48. Moving from B to A, $V_B - V_A = [5\times 10^{-3}(-10^3) + 15 + 1\times 5]\ \text{V}$

$= 15\ \text{V}.$

49. $\mathcal{E} = L\frac{di}{dt} \qquad$ or $\quad di = \left(\frac{\mathcal{E}}{L}\right)dt \qquad$ or $\quad i = \left(\frac{2}{4}\right)t = 0.5t$

For $\ i = 5$ A, $\ t = 10$ s.

50. Steady-state current in $L = i_0 = \frac{\mathcal{E}}{R_1}.$

Energy stored in $L = \frac{1}{2}Li_0^2 = \frac{1}{2}L\left(\frac{\mathcal{E}^2}{R_1^2}\right) =$ heat produced in R_2 during discharge.

51. Let $M =$ mutual inductance between A and B.

$$e_B = M\frac{di_A}{dt}$$

or $\quad \mathcal{E} = M\dot{I} \qquad$ or $\quad M = \mathcal{E}/\dot{I}$

$$\phi_A = Mi_B = (\mathcal{E}/\dot{I})\, i.$$

53. Let I current flow in the larger coil.

Magnetic field at the centre $= B = \dfrac{\mu_0 I}{2R}\cdot$

Flux linked with the smaller coil $= \phi = \pi r^2 B = (\pi r^2)\dfrac{\mu_0 I}{2R}\cdot$

But, $\phi = MI$ or $M = \dfrac{\phi}{I} = \left(\dfrac{\mu_0 \pi}{2}\right)\dfrac{r^2}{R}\cdot$

56. Let L and R be the inductance and the resistance of the coil respectively. Let $\mathcal{E}$ = emf of the cell.

$\tau = L/R$.

Steady-state current $= i_0 = \mathcal{E}/R$.

$P = i_0^2 R = \dfrac{\mathcal{E}^2}{R^2}\cdot R = \dfrac{\mathcal{E}^2}{R}\cdot$

Energy stored in the coil $= \dfrac{1}{2}Li_0^2 = \dfrac{1}{2}L\cdot\dfrac{\mathcal{E}^2}{R^2} = \dfrac{1}{2}\left(\dfrac{L}{R}\right)\left(\dfrac{\mathcal{E}^2}{R}\right) = \dfrac{1}{2}\tau P$

= total heat produced in the coil.

59. $i = i_0(1 - e^{-t/\tau})$

$\dfrac{di}{dt} = -i_0\left(-\dfrac{1}{\tau}\right)e^{-t/\tau} = \dfrac{\mathcal{E}}{R}\cdot\dfrac{R}{L}\cdot e^{-t/\tau}$

or $L\dfrac{di}{dt} = \mathcal{E}e^{-t/\tau}$ = emf across the coil.

60. p.d. across the coil $= \mathcal{E}\cdot e^{-t/\tau}$ = p.d. across $R = Ri = R\cdot \mathcal{E}/R\,(1 - e^{-t/\tau})$

or $2e^{-t/\tau} = 1$ or $e^{t/\tau} = 2$ or $t = \tau \ln 2$.

61. $i = i_0/\eta = i_0 e^{-t_0/\tau}$

or $e^{t_0/\tau} = \eta$ or $t_0 = \tau \ln \eta$.

63. $i = i_0 e^{-t/\tau}$

Energy stored in the coil $= E = \frac{1}{2} L i^2$.

$$\text{The rate of decrease of energy} = -\frac{dE}{dt}$$

$$= -\frac{L}{2} \cdot 2i \cdot \frac{di}{dt} = -L(i_0 e^{-t/\tau})\left(-\frac{i_0}{\tau} e^{-t/\tau}\right)$$

$$= \frac{L i_0^2}{\tau} e^{-2t/\tau} = \frac{L i_0^2}{\tau} e^{-t/(\tau/2)}.$$

64. From energy conservation, $\frac{Q^2}{2C} = \frac{1}{2} L i_{max}^2$

65. When X is joined to Y, in the steady state, the current is $i_{max} = \frac{\mathcal{E}}{R}$ · The maximum charge can be obtained from the relation $Q_{max} = i_{max} \sqrt{LC}$ (see solution to Q. No. 23)

66. Let l and A be the length and area of cross-section of the coil. Then, $\varphi = BAln$, where n is the number of turns per unit length. Also, if current i flows through the coil, the magnetic induction in the coil is $B = \mu_0 n i$. Now, $\varphi = Li$ and the energy stored in the coil is $\frac{1}{2} L i^2$.

67. Q and i are of the form $Q = Q_0 \sin \omega t$ and, $i = i_0 \cos \omega t$, or $Q = Q_0 \cos \omega t$ and $i = i_0 \sin \omega t$

68. $\tau = RC \quad \therefore \quad \omega = \frac{1}{RC}$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = \sqrt{2}R$$

69. To obtain the same reactance,

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad C = \frac{1}{\omega^2 L} = \frac{1}{(100\sqrt{10})^2 \times 2} = 5 \times 10^{-6}\text{F}.$$

70. For resistance R only, $P = e_{\text{rms}} \times i_{\text{rms}} \times \cos\varphi = e_{\text{rms}} \times \frac{e_{\text{rms}}}{R} \times 1$

$$= e_{\text{rms}}^2 / R.$$

For R and L, $P = e_{\text{rms}} \times i_{\text{rms}} \times \cos\varphi = e_{\text{rms}} \times \frac{e_{\text{rms}}}{Z} \times \frac{R}{Z}$

$$= (PR)\frac{R}{Z^2} = P\left(\frac{R}{Z}\right)^2.$$

71. $\tan\varphi = \dfrac{\omega L - \frac{1}{\omega C}}{R} \cdot$ For $R = 0$, $\varphi = \pm\frac{\pi}{2} \cdot$

At low frequencies, $\frac{1}{\omega C} > \omega L$, $\varphi = -\frac{\pi}{2} \cdot$

At high frequencies, $\omega L > \frac{1}{\omega C}$, $\varphi = \frac{\pi}{2} \cdot$

72. $i = \frac{e}{Z}$, $Z = \omega L - \frac{1}{\omega C}$ for $\omega = \frac{1}{\sqrt{LC}}$, $Z = 0$, $i \to \infty$.

73. Current $i = \frac{e}{Z} = \frac{eX}{\sqrt{R^2 + X^2}}$ where $X = \frac{1}{\omega C} \cdot$

Potential difference across $C = V = iX = \frac{eX}{\sqrt{R^2 + X^2}}$

$$\text{or} \quad V = \frac{e}{\sqrt{1 + \left(\frac{R}{X}\right)^2}} = \frac{e}{\sqrt{1 + (\omega CR)^2}} \cdot$$

As ω increases, V will decrease

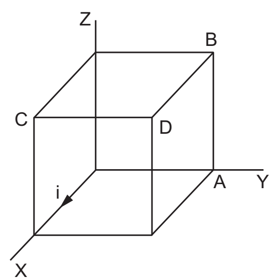
74. The LC circuit draws no power. When $\omega L = \frac{1}{\omega C}$, the impedances of the RC and LR circuits are equal, and they draw the same power.

75. The supply voltage E, the potential difference (V_R) across the resistance, and the potential difference (V_C) across the capacitor are related as

$E^2 = V_R^2 + V_C^2$. $\quad\therefore\quad V_C = \sqrt{220^2 - (90)^2} = 10^2\sqrt{4.03}$

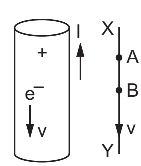
76. $\tan\varphi = \frac{X}{R} = 1$. Power factor $= \cos\varphi = \frac{1}{\sqrt{2}}$.

78, 79.



The magnitude of the magnetic field depends only on the distance from the x-axis. Points A and C are at distances of 1 unit each from the x-axis. Points B and D are at distances of $\sqrt{2}$ unit each from the x-axis.

81. A is stationary and observes the current I. B observes the free electrons to be at rest, but the unbalanced positive charges in the conductor will appear to move in the direction opposite to that of v. Thus, A and B observe the same current and hence the same magnetic field.



83. See the hint to Q. No. 7.

85. For B_1 and B_4, the contributions due to the different sections add up. For B_2 and B_3, the contributions due to the outer sections oppose the contributions due to the inner sections. Thus, B_1 and B_4 are greater than B_2 and B_3.

For B_4, there is a section with radius $< b$ and hence it contributes more than the semicircular section of radius b does for B_1. Thus $B_4 > B_1$.

For B_3, there is a section with radius $> b$ and hence it contributes less than the semicircular section of radius b does for B_2. Thus $B_3 < B_2$.

86. The field at the centre of the loop due to the straight part is $\mu_0 i/(2\pi r)$, directed into the paper, and the field due to the loop is $\mu_0 i/(2r)$, directed out of the paper.

89. To find the magnetic field outside a thick conductor, the current may be assumed to flow along the axis. As points 1, 2, 3 are equidistant from the axis, $B_1 = B_2 = B_3$.

91, 92. TIP To find the Ampere force on a conductor of any shape, replace the conductor by an imaginary straight conductor joining the two ends of the given conductor.

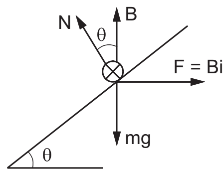
93. Current in AB and CD causes magnetic fields in the same direction on T, upward in this case. Hence there is a net magnetic field over T.

94. The magnetic field over T is proportional to i. Hence, the Ampere force is proportional to i^2. If i is reversed, the direction of B will also reverse. The direction of the Ampere force remains the same.

95, 96. Apply the definition of 1 ampere.

97. Treat the gas as a thick conductor carrying a uniform current. Apply Ampere's law to find the magnetic field. Then apply the left-hand rule to find the direction of the Ampere force.

99.



$N\cos\theta = mg.$ [⊗ indicates *i* current flowing into the paper].

$N\sin\theta = Bil.$

$\tan\theta = \frac{Bil}{mg}\cdot$

100. See the hint to Q. No. 86. If *B* is along *N*, *Bil* force is along the incline.

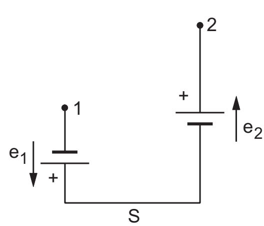
104. See the hint to Q. No. 36.

111, 112. See the hint to Q. No. 38.

113. At the poles, the earth's magnetic field is vertical.

114. Replace the ring by a diameter perpendicular to its direction of motion. The spin of a ring about its axis causes no emf.

115.



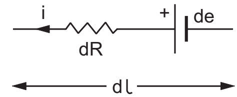
Replace the induced emfs in the rings by cells

$e_1 = B2r(2v) = 4Brv.$

$e_2 = B(4r)v = 4Brv.$

$V_2 - V_1 = e_2 + e_1 = 8Brv.$

116. $\phi = \pi r^2 B \qquad e = \dot{\phi} = \pi r^2 \dfrac{dB}{dt}\cdot$



Let R = resistance of the ring

$\therefore$ the current in the ring = $i = e/R$.

Consider a small element dl on the ring.

Emf induced in the element = $de = \left(\dfrac{e}{2\pi r}\right) dl.$

Resistance of the element = $dR = \left(\dfrac{R}{2\pi r}\right) dl.$

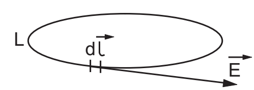
p.d. across the element = $-idR + de = -\left(\dfrac{e}{R}\right)\left(\dfrac{R}{2\pi r}\right) dl + \left(\dfrac{e}{2\pi r}\right) dl = 0.$

$\therefore$ all points on the ring are at the same potential.

117. $\oint_L \vec{E}\cdot \vec{dl}$ = emf in L.

By symmetry in this case, $\vec{E}$ is parallel to $\vec{dl}$ everywhere.

$$\oint_L \vec{E}\cdot\vec{dl} = E\oint \vec{dl} = 2\pi r E = e = \pi r^2 \frac{dB}{dt}$$



or $\quad E = \dfrac{1}{2} r \dfrac{dB}{dt}\cdot$

118. The same emf is induced in a conducting and a nonconducting ring, and it is equal to $\pi r^2 \dfrac{dB}{dt}\cdot$ (See Q. No. 103)

At any point on ring, $E = \dfrac{1}{2} r \dfrac{dB}{dt}\cdot$ (See Q. No. 104)

For any charge dQ on the ring, force = $dF = EdQ$, tangential to the ring

Torque about the centre due to $dF = d\tau = rdF = rEdQ$.

Total torque = $\Sigma rEdQ = r\left(\dfrac{1}{2} r \dfrac{dB}{dt}\right) Q.$

120, 121. TIP Charge flowing in a circuit = $\dfrac{\Delta\phi}{R}$,

where $\Delta\phi$ = change in flux = $\phi_{final} - \phi_{initial}$

and $\quad R$ = resistance in circuit.

122. If the normal to the plane of the coil makes an angle θ with the direction of B, the flux linked with the coil is

$$\phi = BAn\cos\theta = BAn\cos(\omega t)$$

($\because$ the coil rotates with an angular velocity ω.)

$$\text{emf} = e = -\frac{d\phi}{dt} = BAn\omega\sin(\omega t).$$

□

4

Motion of Charged Particles

• *Type 1* •

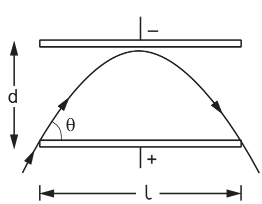
Choose the correct option (a, b, c or d).

1. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to

(a) 1 (b) $(m_p/m_e)^{1/2}$

(c) $(m_e/m_p)^{1/2}$ (d) 1836

2.



An electron enters the region between the plates of a parallel-plate capacitor at an angle θ to the plates. The plate width is l and the plate separation is d. The electron follows the path shown, just missing the upper plate. Neglect gravity.

(a) $\tan\theta = 2d/l$

(b) $\tan\theta = 4d/l$

(c) $\tan\theta = 8d/l$

(d) The data given is insufficient to find a relation between d, l, θ.

3. Which of the following statements is correct?

(a) A charged particle can be accelerated by a magnetic field.

(b) A charged particle cannot be accelerated by a magnetic field.

(c) The speed of a charged particle can be increased by a uniform magnetic field.

(d) The speed of a charged particle can be increased by a nonuniform magnetic field.

4. A charged particle begins to move in a magnetic field, initially parallel to the field. The direction of the field now begins to change, with its magnitude remaining constant.

(a) The magnitude of the force acting on the particle will remain constant.

(b) The magnitude of the force acting on the particle will change.

(c) The particle will always move parallel to the field.

(d) The speed of the particle will change.

5. A proton moves horizontally towards a vertical conductor carrying a current upwards. It will be deflected

(a) to the left (b) to the right

(c) upwards (d) downwards

6. A charged particle moves with velocity $\vec{v} = a\hat{i} + d\hat{j}$ in a magnetic field $\vec{B} = A\hat{i} + D\hat{j}$. The force acting on the particle has magnitude F.

(a) $F = 0$, if $aD = dA$.

(b) $F = 0$, if $aD = -dA$.

(c) $F = 0$, if $aA = -dD$.

(d) $F \propto (a^2 + b^2)^{½} \times (A^2 + D^2)^{½}$.

7. A charged particle moves horizontally without deflection near the earth's surface. In this region,

(a) only electric field is present

(b) only vertical magnetic field is present

(c) only horizontal magnetic field is present

(d) mutually perpendicular electric and magnetic fields are present

8. Two very long, straight, parallel wires carry steady currents I and $-I$. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\dfrac{\mu_0 Iqv}{2\pi d}$ (b) $\dfrac{\mu_0 Iqv}{\pi d}$

(c) $\dfrac{2\mu_0 Iqv}{\pi d}$ (d) zero

9. An electron is ejected from the surface of a long, thick, straight conductor carrying a current, initially in a direction perpendicular to the conductor. The electron will

(a) ultimately return to the conductor

(b) move in a circular path around the conductor

(c) gradually move away from the conductor along a spiral

(d) move in a helical path, with the conductor as the axis

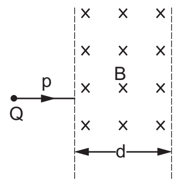
10. A particle with a specific charge s is fired with a speed v towards a wall at a distance d, perpendicular to the wall. What minimum magnetic field must exist in this region for the particle not to hit the wall?

(a) v/sd (b) $2v/sd$

(c) $v/2sd$ (d) $v/4sd$

11. A particle with charge Q, moving with a momentum p, enters a uniform magnetic field normally. The magnetic field has magnitude B and is confined to a region of width d, where

$d < \frac{p}{BQ}$. The particle is deflected by an angle θ in crossing the field.



(a) $\sin\theta = \frac{BQd}{p}$ (b) $\sin\theta = \frac{p}{BQd}$

(c) $\sin\theta = \frac{Bp}{Qd}$ (d) $\sin\theta = \frac{pd}{BQ}$

12. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths

(a) of the same radius

(b) with larger radii for the faster electrons

(c) with smaller radii for the faster electrons

(d) either (b) or (c) depending on the magnitude of the magnetic field

13. In the previous question, time periods of rotation will be

(a) the same for all the electrons

(b) greater for the faster electrons

(c) smaller for the faster electrons

(d) either (b) or (c) depending on the magnitude of the magnetic field

14. A charged particle entering a magnetic field from outside in a direction perpendicular to the field

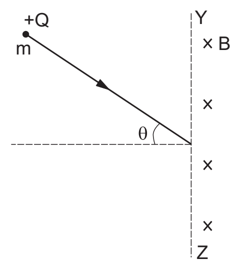
(a) can never complete one rotation inside the field

(b) may or may not complete one rotation in the field depending on its angle of entry into the field

(c) will always complete exactly half of a rotation before leaving the field

(d) may follow a helical path depending on its angle of entry into the field

15.



A particle with charge +*Q* and mass *m* enters a magnetic field of magnitude *B*, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of *B*. Let $T = 2\pi\frac{m}{QB}$. The time spent by the particle in the field will be

(a) $T\theta$ (b) $2T\theta$

(c) $T\left(\frac{\pi + 2\theta}{2\pi}\right)$ (d) $T\left(\frac{\pi - 2\theta}{2\pi}\right)$

16. In the previous question, if the particle has −*Q* charge, the time spend by the particle in the field will be

(a) $T\theta$ (b) $2T\theta$

(c) $T\left(\frac{\pi + 2\theta}{2\pi}\right)$ (d) $T\left(\frac{\pi - 2\theta}{2\pi}\right)$

17. A charged particle moves undeflected in a region of crossed electric and magnetic fields. If the electric field is switched off, the particle has an initial acceleration *a*. If the magnetic field is switched off, instead of the electric field, the particle will have an initial acceleration

(a) equal to 0 (b) $> a$

(c) equal to *a* (d) $< a$

18. A charged particle begins to move from the origin in a region which has a uniform magnetic field in the x-direction and a uniform electric field in the y-direction. Its speed is v when it reaches the point (x, y, z). v will depend

(a) only on x (b) only on y

(c) on both x and y, but not z (d) on x, y and z

• *Type 2* •

Choose the correct options. One or more options may be correct.

19. A charged particle moving in an electric field

(a) must undergo change in velocity

(b) must undergo change in speed

(c) may not undergo change in velocity

(d) may not undergo change in speed

20. A particle with a specific charge s starts from rest in a region where the electric field has a constant direction, but whose magnitude increases linearly with time. The particle acquires a velocity v in time t.

(a) $v \propto s$ (b) $v \propto \sqrt{s}$

(c) $v \propto t$ (d) $v \propto t^2$

21. In the previous question, the particle covers a distance x in time t. x is proportional to

(a) t (b) $t^{3/2}$

(c) t^2 (d) t^3

22. In a parallel-plate capacitor, the potential difference between the plates is V. A particle of mass m and charge $-Q$ leaves the negative plate and reaches the positive plate with a momentum p. p is proportional to

(a) $m^{1/2}$ (b) $Q^{1/2}$

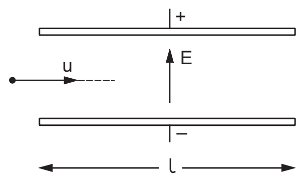
(c) $V^{1/2}$ (d) V

23. In the previous question, if d = plate separation and t = time of travel from the negative plate to the positive plate then t is proportional to

(a) $d^{1/2}$ (b) d

(c) Q/m (d) m/Q

24.



There is a uniform electric field E in the region between two parallel plates. A charged particle with a specific charge s enters the region with a speed u, in a direction parallel to the plates. The length of each plate is l. The deflection of the particle as it crosses the plates is proportional to

(a) E (b) s

(c) l^2 (d) u^{-2}

25. In the previous question, as the particle crosses the plates, its direction of motion changes by an angle θ. Then tan θ is proportional to

(a) E (b) s

(c) l (d) u^{-2}

26. In which of the following situations will a charge experience no force?

(a) It is stationary in an electric field.

(b) It moves parallel to an electric field.

(c) It is stationary in a magnetic field.

(d) It moves parallel to a magnetic field.

27. When a charged particle moves in an electric or a magnetic field, its speed is v and acceleration is a.

(a) In a magnetic field, v is constant if the particle moves in a circular path, and variable if it moves in a helical path.

(b) In a magnetic field, v is always constant; a may or may not be zero.

(c) In an electric field, v can never remain constant.

(d) In a uniform electric field, a must be constant in magnitude and direction.

28. A proton moves horizontally towards a vertical conductor with a uniformly distributed positive charge. It will undergo

(a) horizontal deflection (b) vertical deflection

(c) no deflection (d) retardation

29. Two parallel conductors carrying current in the same direction attract each other, while two parallel beams of electrons moving in the same direction repel each other. Which of the following statements provide part or all of the reason for this?

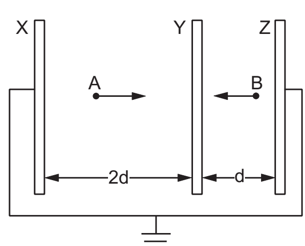
(a) The conductors are electrically neutral.

(b) The conductors produce magnetic fields on each other.

(c) The electron beams do not produce magnetic fields on each other.

(d) The magnetic forces caused by the electron beams on each other are weaker than the electrostatic forces between them.

30.



X, Y and Z are parallel plates. Y is given some positive charge. Two electrons A and B start from X and Z respectively and reach Y in times t_A and t_B respectively.

(a) $t_A = t_B$ (b) $t_A = \sqrt{2}t_B$

(c) $2t_A = t_B$ (d) $t_A = 2t_B$

31. In the set-up of the previous question, an electron E moves from X to Y and a proton P moves from Y to Z. Both particles start from rest.

(a) E reaches Y with greater energy than P.

(b) P reaches Y with greater energy than E.

(c) P and E reach Y with equal energies.

(d) P reaches Y with greater momentum than E.

32. Two particles X and Y with equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

(a) $(R_1/R_2)^{1/2}$ (b) R_2/R_1

(c) $(R_1/R_2)^2$ (d) R_1/R_2

33. A proton, a deuteron and an α-particle with the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_a denote respectively the radii of the trajectories of these particles,

(a) $r_a = r_p < r_d$

(b) $r_a > r_d > r_p$

(c) $r_a = r_d > r_p$

(d) $r_p = r_d = r_a$

34. A neutral atom which is stationary at the origin in gravity-free space emits an α-particle (A) in the *z*-direction. The product atom is P. A uniform magnetic field exists in the *x*-direction. Disregard the electrostatic forces between A and P.

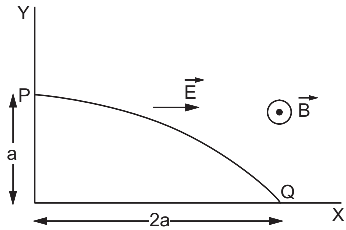
(a) A and P will move along circular paths of equal radii.

(b) A has greater time period of rotation than P.

(c) A has greater kinetic energy than P.

(d) A and P will meet again somewhere in the *yz* plane.

35.



A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$.

(a) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(b) The rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$.

(c) The rate of work done by the electric field at P is 0.

(d) The rate of work done by both the fields at Q is 0.

36. Two long, thin, parallel conductors are kept very close to each other, without touching. One carries a current i, and the other has charge λ per unit length. An electron moving parallel to the conductors is undeflected. Let c = velocity of light.

(a) $v = \frac{\lambda c^2}{i}$

(b) $v = \frac{i}{\lambda}$

(c) $c = \frac{i}{\lambda}$

(d) The electron may be at any distance from the conductor.

37. A region has uniform electric and magnetic fields along the positive x-direction. An electron is fired from the origin at an angle θ ($< 90°$) with the x-axis. It will

(a) move along a helical path of increasing pitch

(b) move along a helical path of decreasing pitch initially

(c) return to the yz plane at some time
(d) come to rest momentarily at some position

38. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. P will also strike T if

(a) $B = 2B_0,\ v = 2v_0$
(b) $B = 2B_0,\ v = v_0$
(c) $B = B_0,\ v = 2v_0$
(d) $B = B_0/2,\ v = 2v_0$

39. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will

(a) never move parallel to the x-axis
(b) move parallel to the x-axis once during every rotation for all values of θ
(c) move parallel to the x-axis at least once during every rotation if $\theta = 45^\circ$
(d) never move perpendicular to the x-direction

40. In the previous question, if the pitch of the helical path is equal to the maximum distance of the particle from the x-axis,

(a) $\cos\theta = \frac{1}{\pi}$
(b) $\sin\theta = \frac{1}{\pi}$
(c) $\tan\theta = \frac{1}{\pi}$
(d) $\tan\theta = \pi$

Answers

1. b	**2.** b	**3.** a	**4.** b	**5.** d
6. a	**7.** a	**8.** d	**9.** a	**10.** a
11. a	**12.** b	**13.** a	**14.** a	**15.** c
16. d	**17.** c	**18.** b	**19.** a, b	**20.** a, d
21. d	**22.** a, b, c	**23.** b	**24.** a, b, c, d	**25.** a, b, c, d
26. c, d	**27.** b, c, d	**28.** c, d	**29.** a, b, d	**30.** d
31. c, d	**32.** c	**33.** a	**34.** a, c, d	**35.** a, b, d
36. a, d	**37.** b, c	**38.** a, b	**39.** a, d	**40.** d

Hints and Solutions to Selected Questions

1. Equal forces act on the two as they have equal charges.

If F = force, d = distance travelled,

$$a_e = \frac{F}{m_e}.$$

$$d = \frac{1}{2} a_e t_1^2 \quad \text{or} \quad t_1^2 = \frac{2d}{a_e} = 2d\frac{me}{F}.$$

$$a_p = \frac{F}{m_p}.$$

$$d = \frac{1}{2} a_p t_2^2 \quad \text{or} \quad t_2^2 = 2d\frac{m_p}{F}.$$

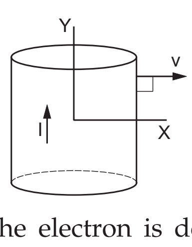
$$\therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_p}{m_e} \quad \text{or} \quad \frac{t_2}{t_1} = \left(\frac{m_p}{m_e}\right)^{1/2}.$$

2. The motion is identical to that of a projectile, with the constant electrical force acting as the gravitational force. Treat l as the horizontal range and, d as the maximum height.

6. $\vec{F} \propto (\vec{v} \times \vec{B}) = \hat{k}\,[aD - dA]$.

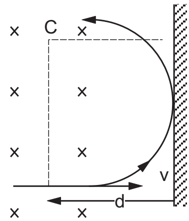
8. $\vec{v}$ is parallel to $\vec{B}$, both perpendicular to the plane of the wires.

9.



The initial force on the electron is downward. As the electron changes direction, the force on it remains in the xy plane, with a component directed towards the conductor.

10.

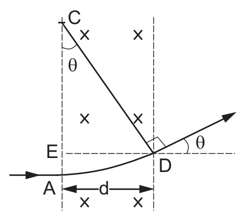


The particle moves in a circular path with radius d if it is to just miss the wall.

$$mv = Bqr$$

or $$B = \frac{v}{(q/m)d} = \frac{v}{sd}.$$

11.



A to D is part of a circle with centre C.

$$CD = r.$$

$$mv = p = BQr.$$

or $$r = p/BQ.$$

$$\sin\theta = \frac{ED}{CD} = \frac{d}{r} = \frac{BQd}{p}.$$

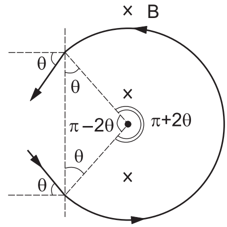
12. $mv = BQr$

or $$r = \left(\frac{m}{BQ}\right)v$$

As m, Q and B are the same for all the electrons, $r \propto v$.

13. $T = 2\pi\dfrac{m}{QB}$ = the same for all electrons, as it is independent of v

15.



TIP

A charged particle enters and leaves a uniform magnetic field *symmetrically*.

The time spent in the field is proportional to the angle $(\pi + 2\theta)$.

16. See the hint to Q. No. 15. Rotation is clockwise.

18. TIP

For a particle moving in any combination of electric and magnetic fields, work is done only by the electric field.

Energy of the particle = work done by the electric field

= electric field × displacement in the direction of the electric field

20, 21. $E = \alpha t$ $(\alpha = \text{constant})$ $F = QE$

$$a = F/m = QE/m = Es = \alpha st.$$

$$\therefore \quad a = \frac{dv}{dt} = \alpha st$$

or $v = \frac{1}{2}\alpha st^2$ $\therefore$ $v \propto s$ and t^2.

$$v = \frac{dx}{dt} = \frac{1}{2}\alpha st^2$$

or $x = \frac{1}{6}\alpha st^3$ or $x \propto t^3$.

22. Energy $= QV = \frac{p^2}{2m}$ or $p = (2mQV)^{1/2}$.

23. $E = \frac{V}{d}$, $F = QE = Q\frac{V}{d}$, $a = \frac{F}{m} = \left(\frac{Q}{m}\right)\frac{V}{d}$.

$$d = \frac{1}{2}at^2$$

or $$t^2 = \frac{2d}{a} = 2d\frac{md}{QV} = d^2\left[2\left(\frac{m}{Q}\right)\frac{1}{V}\right].$$

24. As there is no horizontal force, the horizontal component of the velocity remains constant $= u$.

Time of transit between the plates $= t = l/u$.

In the vertical direction, $F = QE$, $a = F/m = QE/m$.

$\therefore$ displacement $= \frac{1}{2}at^2 = \frac{1}{2}\cdot\frac{QE}{m}\left(\frac{l^2}{u^2}\right) = \frac{1}{2}Es\left(\frac{l^2}{u^2}\right)$.

25. See the hint to Q. No. 24. The vertical component of the velocity at the point of exit from the plates $= v = at = \left(\frac{QE}{m}\right)\frac{l}{u} = \frac{sEl}{u}$.

$$\tan\theta = \frac{v}{u} = \frac{sEl}{u^2}.$$

30. As X and Z are connected, they have the same potential.

$\therefore$ p.d. between X and Y = p.d. between Z and Y = V (say).

For XY, $E_A = \frac{V}{2d}$, $F_A = QE_A$, $a_A = \frac{QE_A}{m} = \frac{Q}{m}\cdot\frac{V}{2d}$.

$$2d = \frac{1}{2}(a_A)\,t_A^2$$

or $$t_A^2 = \frac{4d}{a_A} = 4d\frac{2md}{QV} = \frac{8md^2}{QV}.$$

For ZY, $E_B = \frac{V}{d}$, $F_B = QE_B$, $a_B = \frac{QE_B}{m} = \frac{Q}{m}\cdot\frac{V}{d}$.

$$d = \frac{1}{2}(a_B)\,t_B^2$$

or $$t_B^2 = \frac{2d}{a_B} = 2d\frac{md}{QV} = \frac{2md^2}{QV}.$$

$t_A^2/t_B^2 = 4$ or $t_A/t_B = 2$.

31. The two particles have equal charges and move through the same p.d. They will, therefore, acquire the same energy (E).

Also, $E = p^2/2m$ or $p = \sqrt{2mE}$, where p = momentum.

For the same E, the particle with greater m has greater p.

32. $mv = p = BQR$.

Also, energy $E = p^2/2m$ or $p = \sqrt{2mE}$

$\therefore \quad \sqrt{2mE} = BQR$.

Here, the two particles have the same E, B and Q.

$\therefore \quad r \propto m^{1/2}$

or $\quad m \propto R^2 \qquad \therefore \quad \dfrac{m_X}{m_Y} = \left(\dfrac{R_1}{R_2}\right)^2$.

33. See the hint to Q. No. 32. $\sqrt{2mE} = BQR$, where E = energy.

Here, B and E are the same for all three particles.

$\therefore \quad r \propto \dfrac{\sqrt{m}}{Q}$.

Let $m_p = m, \quad m_d = 2m, \quad m_a = 4m$

$Q_p = e, \quad Q_d = e, \quad Q_a = 2e, \quad k = \text{constant}.$

$r_p = k\dfrac{\sqrt{m}}{e}, \quad r_d = k\dfrac{\sqrt{2m}}{e}, \quad r_a = k\dfrac{\sqrt{4m}}{2e} = k\dfrac{\sqrt{m}}{e} = r_p$

$\therefore \quad r_a = r_p < r_d$.

34. As momentum must be conserved, the two move in opposite directions with equal momenta (p).

$mv = p = BQr$.

Here, B and Q are also the same for the two.

$\therefore \quad r$ is the same.

$T = 2\pi \dfrac{m}{QB}$

The two have the same Q and B, but $m_P > m_A$.

$\therefore \quad T_P > T_A$.

Kinetic energy $= p^2/2m$.

The two have the same p, but $m_P > m_A$.

$\therefore \quad E_A > E_P$.

35. In going from P to Q, increase in kinetic energy

$$= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(3v^2) = \text{work done by electric field}$$

or $\quad \frac{3}{2}mv^2 = Eq \times 2a \qquad$ [See Tip, Q. No. 18.]

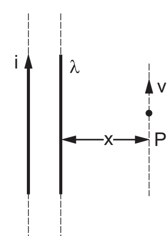
or $\quad E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$.

The rate of work done by E at P = force due to $E \times$ velocity

$$= (qE)v = qv\left[\frac{3}{4}\left(\frac{mv^2}{qa}\right)\right] = \frac{3}{4}\left(\frac{mv^3}{a}\right).$$

At Q, $\vec{v}$ is perpendicular to $\vec{E}$ and $\vec{B}$, and no work is done by either field.

36.



At P, electric field = $E = \frac{\lambda}{2\pi\varepsilon_0 x}$ (to the right),

and magnetic field = $B = \frac{\mu_0 i}{2\pi x}$ (into the paper).

For no deflection, $E = vB \qquad$ or $\quad v = \frac{E}{B}$

or $\quad v = \frac{\lambda}{2\pi\varepsilon_0 x} \times \frac{2\pi x}{\mu_0 i} = \frac{\lambda}{i} \cdot \frac{1}{\varepsilon_0\mu_0} = \frac{\lambda c^2}{i}$.

37. The electric field exerts a force opposite to the component of motion along the x-axis.

38. TIP Let d = distance of the target T from the point of projection. P will strike T if d is an *integral* multiple of the pitch.

$$\text{Pitch} = \left(2\pi \frac{m}{QB}\right) v\cos\theta.$$

Here, m, Q and θ are constant.

$\therefore$ pitch $= k\left(\frac{v}{B}\right)$, where k = constant

Initially, $d = k\left(\frac{v_0}{B_0}\right)$.

40. $\text{Pitch} = \left(2\pi \frac{m}{QB}\right) v\cos\theta.$

Also, $mv\sin\theta = QBr$ for motion perpendicular to the magnetic field

or $\quad r = \frac{mv}{QB}\sin\theta.$

Maximum distance of the particle from the x-axis $= 2r$.

$$\therefore \quad \left(2\pi \frac{m}{QB}\right) v\cos\theta = 2 \cdot \frac{mv}{QB}\sin\theta$$

or $\quad \tan\theta = \pi.$

□

• Revision Exercise 4 •

Choose the correct option in each of the following questions. Only one option is correct in each question.

R1. A rectangular, horizontal, conducting frame carries current, flowing clockwise as seen from above. P is a point vertically above the centre of the frame. The direction of the magnetic field at P due to the current is

(a) vertically upwards

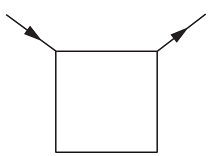
(b) horizontal, parallel to the longer sides of the frame

(c) vertically downwards

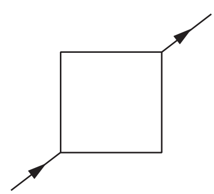
(d) horizontal, parallel to the diagonal of the frame

R2. Current flows through uniform, square frames as shown. In which case is the magnetic field at the centre of the frame not zero?

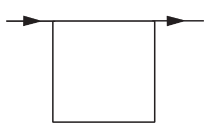
(a)



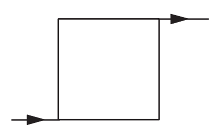
(b)



(c)



(d)



R3. A long, narrow beam of electrons, of uniform cross-section, consists of electrons moving with velocity v (c = velocity of light). The ratio of the electric field to the magnetic field at any point near the beam is

(a) v (b) c

(c) c^2/v (d) v^2/c

R4. A long, straight conductor lies along the axis of a ring. Both carry current I. The force on the ring is proportional to

(a) I (b) $I^{3/2}$ (c) I^2 (d) zero

R5. Two flat, closed, conducting loops A and B face each other. Current I flows in A. A will attract B

(a) in all cases
(b) only if I is increasing
(c) only if I is decreasing
(d) if I is changing

R6. A pair of long, smooth, parallel, horizontal, conducting rails are joined to a cell at one end. There are no external electric or magnetic fields. A metal rod is placed on the rails. The rod will

(a) remain stationary
(b) move towards the cell
(c) move away from the cell
(d) oscillate

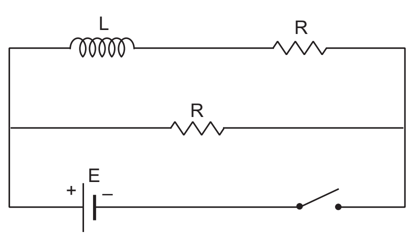
R7. A flat, rectangular coil, carrying current, is placed beside a long, straight conductor carrying current. The two are coplanar. The net force and net torque experienced by the coil are F and τ.

(a) $F = 0, \tau = 0$
(b) $F \neq 0, \tau = 0$
(c) $F \neq 0, \tau \neq 0$
(d) $F = 0, \tau \neq 0$

R8. A small bar magnet moves along the axis of a flat, closed coil. The magnet will attract the coil

(a) only when it moves towards the coil
(b) only when it moves away from the coil
(c) both (a) and (b)
(d) only if its south pole is facing the coil

R9.



A coil of self-inductance L and resistance R is connected to a resistance R and a cell of emf E, as shown. The switch is kept closed for a long time and then opened. The heat produced in the coil, after opening the switch, is

(a) $LE^2/2R^2$
(b) $LE^2/4R^2$
(c) $LE^2/8R^2$
(d) $2LE^2/3R^2$

R10. If L and C denote inductance and capacitance then the quantity L/C has the same dimension as

(a) time (b) $(\text{time})^{-1}$

(c) resistance × time (d) $(\text{resistance})^2$

R11. A metal ring is placed in a magnetic field, with its plane ⊥ to the field. If the magnitude of the field begins to change, the ring will experience

(a) a net force

(b) a torque about its axis

(c) a torque about a diameter

(d) a tension along its length

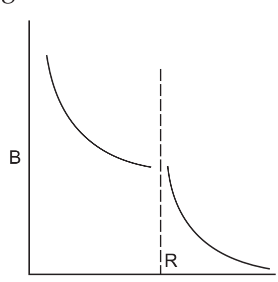
R12. A metal rod of length l is pivoted at its upper end. It is released from a horizontal position. There is a uniform magnetic field ⊥ to its plane of rotation. When it becomes vertical, the p.d. across its ends is proportional to

(a) $l^{1/2}$ (b) l

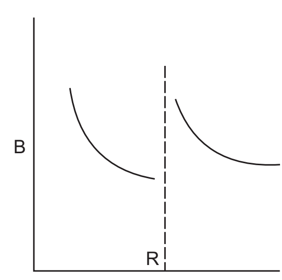
(c) $l^{3/2}$ (d) l^2

R13. A thin, straight conductor lies along the axis of a hollow conductor of radius R. The two carry equal currents in the same direction. The magnetic field B is plotted against the distance r from the axis. Which of the following best represents the resulting curve?

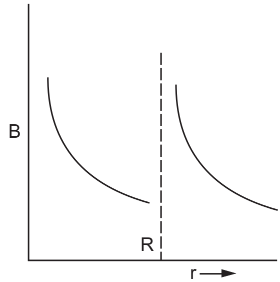
(a)



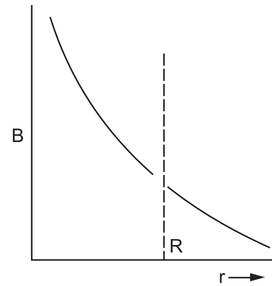
(b)



(c)



(d)



R14. A solenoid is connected to a cell. There is no resistance in the circuit. If the current I flowing in the circuit is plotted against time t, the slope of the curve (dI/dt) will

(a) increase with time
(b) decrease with time
(c) remain constant
(d) be almost infinite

R15. In a region of space with mutually perpendicular electric and magnetic fields, a charged particle moves without deflection, with a speed very much smaller than the speed of light in vacuum (c). The energy densities due to the electric and magnetic fields are u_E and u_B respectively.

(a) $u_E = u_B$

(b) $u_E < u_B$

(c) $u_E > u_B$

(d) The data is not sufficient to reach a conclusion.

R16. A charged particle passes through a uniform electric field existing between the parallel plates of a capacitor. The length of each plate is l. Its initial velocity is parallel to the plates. When it emerges from the field, its deflection from its initial path is δ, and its angular deflection from its initial direction is θ. Then, $\tan\theta$ is equal to

(a) δ/l
(b) $2\delta/l$
(c) $\delta/2l$
(d) none of these

R17. A particle of charge Q and mass m passes through a region of width d, which has a uniform magnetic field B. Its initial velocity

v is perpendicular to B. If d is slightly greater than (mv/BQ), the particle will get deflected through an angle

(a) $\pi/4$ (b) $\pi/3$

(c) $\pi/2$ (d) π

R18. A ring of area A and resistance R is placed on the axis of a solenoid. The mutual inductance between them is M. When the current in the solenoid changes at the rate of I, the magnetic moment of the ring is

(a) $ARMI$ (b) AI/RM

(c) MRI/A (d) AMI/R

R19. A long, straight conductor carries current I. Assume that every charge carrier in it moves with the same drift velocity v. An observer moves parallel to the conductor, in the direction of the current, with a constant velocity $v/2$. He will observe that the current flowing in the conductor is

(a) $I/2$ (b) I

(c) $3I/2$ (d) zero

R20. The flux linked with a coil is 0.8 Wb when 2A current flows through it. If this current begins to increase at the rate of 0.4 A/s, the emf induced in the coil will be

(a) 0.02 V (b) 0.04 V

(c) 0.08 V (d) 0.16 V

R21. A small, flat coil carrying current has magnetic moment μ. It is placed in an external magnetic field B. The maximum potential energy of the system can be

(a) $\mu B/2$ (b) μB (c) $\mu B \ln 2$ (d) $2\mu B$

R22. An electron moves in a circular orbit of radius r, with angular velocity ω. The magnetic field at the centre of the orbit has magnitude (e = electronic charge)

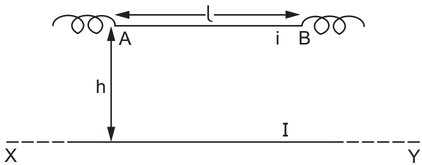
(a) $\mu_0 \omega e/2r$ (b) $\mu_0 \omega e/2\pi r$

(c) $\mu_0 \omega e/4\pi r$ (d) $\mu_0 \omega^2 e/4\pi r$

R23. A small, flat coil of resistance r is placed at the centre of a large, closed coil of resistance R. The coils are coplanar. Their mutual inductance is M. Initially, a constant current i was flowing in the inner coil. If this current is suddenly switched off, what charge will circulate in the outer coil?

(a) Mir/R^2 (b) MiR/r^2 (c) Mi/R (d) Mi/r

R24.



A conductor AB, carrying current i, is placed vertically above and parallel to a long horizontal conductor XY, carrying current I. Assume that AB is free to move and that the wires through which currents enter and leave it do not exert any forces on it. If AB is in equilibrium,

(a) $i = I$

(b) i and I must flow in the same direction

(c) the equilibrium of AB is unstable

(d) if AB is given a small vertical displacement, it will undergo oscillations

R25. In the previous question, let m = mass of AB, l = length of AB, and h = height of AB above XY. For AB to be in equilibrium, which of the following quantities must be constant?

(a) Iil/mh (b) Iih/ml

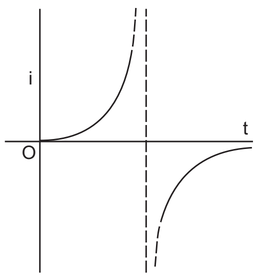
(c) I^2il/mh (d) Ii^2l/mh

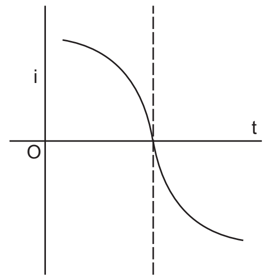
R26. There is a uniform magnetic field B normal to the xy plane. A conductor ABC has length AB $= l_1$, parallel to the x-axis, and length BC $= l_2$, parallel to the y-axis. ABC moves in the xy plane with velocity $v_x \hat{i} + v_y \hat{j}$. The potential difference between A and C is proportional to

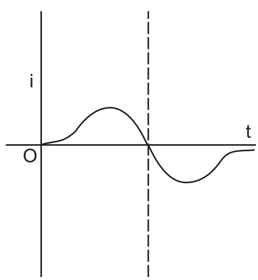
(a) $v_x l_1 + v_y l_2$ (b) $v_x l_2 + v_y l_1$

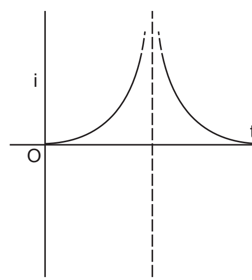
(c) $v_x l_2 - v_y l_1$ (d) $v_x l_1 - v_y l_2$

R27. A long bar magnet moves with constant velocity along the axis of a fixed metal ring. It starts from a large distance from the ring, passes through the ring and then moves away far from the ring. The current *i* flowing in the ring is plotted against time *t*. Which of the following best represents the resulting curve?

(a)


(b)


(c)


(d)


R28. In a cell, or accumulator battery, current flows inside the cell from the negative plate to the positive plate when

(a) it drives current through an external resistance

(b) it is being charged from an external source

(c) its emf is being measured by a potentiometer and the balance position has been reached

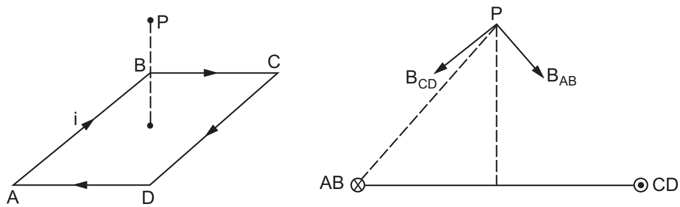
(d) when it is connected to a charged capacitor whose potential difference is greater than its emf, and its positive and negative plates are connected to the plates of similar polarities of the capacitor

Answers to Revision Exercise 4

R1. c	**R2.** c	**R3.** c	**R4.** d	**R5.** c
R6. c	**R7.** b	**R8.** b	**R9.** b	**R10.** d
R11. d	**R12.** c	**R13.** b	**R14.** c	**R15.** b
R16. b	**R17.** d	**R18.** d	**R19.** b	**R20.** d
R21. d	**R22.** c	**R23.** c	**R24.** d	**R25.** a
R26. c	**R27.** a	**R28.** a		

Hints and Solutions to Selected Questions

R1.



Consider the magnetic field at P due to one pair of parallel conductors AB and CD. These are shown as B_{AB} and $B_{CD.}$ By symmetry, their resultant is directed downwards. The same will hold for BC and AD.

R2. In all the four arrangements, the field at the centre due only to the currents flowing in the square frames is zero. The field here due to the currents flowing in and out of the frames is nonzero only in (c).

R3. Let A = area of cross section of the beam, e = electronic charge

n = number of electrons per unit volume in the beam

Current carried by beam = $i = Avne$.

Magnetic field at distance r from the beam = $B = \dfrac{\mu_0 i}{2\pi r}\cdot$

Charge per unit length of beam = $\lambda = Ane$.

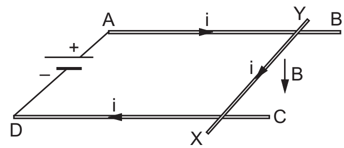
Electric field at distance r from beam = $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}\cdot$

$$\frac{E}{B} = \frac{\lambda}{2\pi\varepsilon_0 r} \times \frac{2\pi r}{\mu_0(Avne)} = \frac{c^2}{v}\cdot$$

R4. The magnetic field due to the ring is along its axis. Therefore, there is no force of interaction between the ring and the straight conductor lying along its axis.

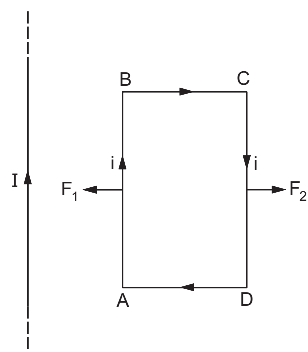
R5. When I decreases, the flux through B decreases. This induces current in B in the same direction as I.

R6.



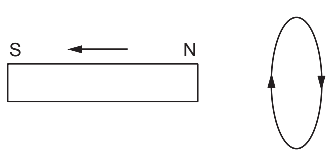
The currents flowing in the rails AB and CD produce a magnetic field directed downwards over the rail XY. This produces an ampere force on XY, directed away from the cell.

R7.



The magnetic field due to I is larger on AB than on CD. Hence, the ampere force F_1 on AB is larger than that on CD, F_2. Also, F_1 and F_2 are coplanar and hence do not produce a torque.

R8.



Let the north pole of the magnet face the coil. When the magnet moves away from the coil, the flux through the coil decreases. This induces current in the coil, flowing clockwise, as seen from the magnet. The face of the coil facing the magnet now behaves as a south pole. The magnet attracts the coil. The opposite will happen when the magnet approaches the coil.

R9. In the steady state, the current in the coil is $I = E/R$. The energy stored in it is $E = \frac{1}{2}LI^2 = \frac{LE^2}{2R^2}\cdot$ When the switch is opened, this energy is shared equally between the two resistances.

R10. $\frac{L}{C} = \frac{L}{R} \times \frac{1}{RC} \times R^2$

$\frac{L}{R}$ and RC each have the dimension of time.

R11. When the field changes, a current flows along the ring. Each section of the ring now experiences an ampere force acting either outward or inward. The ring will tend to expand or contract, and hence experience a tension along its length.

R12. $mg\frac{l}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\left(\frac{ml^2}{3}\right)\omega^2$ or $\omega = \sqrt{\frac{3g}{l}}\cdot$

Induced emf $= \frac{1}{2}B\omega l^2 = \frac{1}{2}B\sqrt{\frac{3g}{l}}\,l^2 \propto l^{3/2}$.

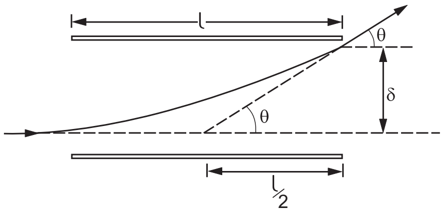
R13. Let i be the current in each conductor. Applying Ampere's law, the magnetic field inside the hollow conductor is due to current i only, while the field outside it is due to current $2i$. Hence, the field just outside it is twice the field just inside it.

R14. $\varepsilon = L\cdot\frac{dI}{dt}$ or $\frac{dI}{dt} = \frac{\varepsilon}{L} = \text{constant}$.

R15. $QE = QvB$ or $v = \frac{E}{B}\cdot$

$u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = \frac{B^2}{2\mu_0}\cdot$ $\quad\therefore\quad \frac{u_E}{u_B} = \varepsilon_0\mu_0\frac{E^2}{B^2} = \frac{v^2}{c^2} < 1.$

R16.

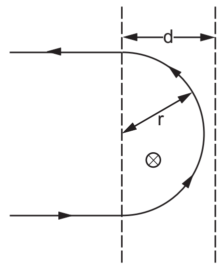


$$\tan\theta = \frac{\delta}{l/2} = \frac{2\delta}{l}$$

R17. $mv = BQr$ or $r = \dfrac{mv}{BQ}\cdot$

Here, $d \gtrsim \dfrac{mv}{BQ}$ or $d \gtrsim r$.

The particle will just complete half of one rotation inside the field.



R18. Let ϕ = flux linked with the ring.

$\dot{\phi} = MI$. Current in ring $= i = \dfrac{\dot{\phi}}{R} = \dfrac{MI}{R}\cdot$

Magnetic moment of ring $= Ai = \dfrac{AMI}{R}\cdot$

R19. He will observe the negative charge carriers moving with velocity $\dfrac{v}{2}$, and an equal number of positive charges moving in the opposite direction with velocity $\dfrac{v}{2}\cdot$

R20. $\phi = Li$ or $L = \dfrac{\phi}{i}\cdot$ Emf $= L\dfrac{di}{dt} = \left(\dfrac{\phi}{i}\right)\dfrac{di}{dt} = \dfrac{0.8\text{ Wb}}{2\text{A}} \times 0.4\text{ A/s} = 0.16\text{ V}.$

R21. Let θ = angle between μ and B.

PE of the system $= \mu B(1 - \cos\theta)$.

For $\theta = \pi$, $(\text{PE})_{\max} = 2\mu B$.

R22. $n = \dfrac{\omega}{2\pi}\cdot$ $i = ne.$ $B = \dfrac{\mu_0 i}{2r} = \dfrac{\mu_0 ne}{2r} = \dfrac{\mu_0 \omega e}{4\pi r}\cdot$

R23. Initial flux linked with outer coil = $\phi = Mi$.

When current is switched off, $\Delta\phi = Mi$.

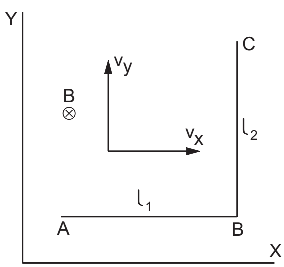
$$Q = \frac{\Delta\phi}{R} = \frac{Mi}{R}$$

R24. The weight of AB is balanced by the ampere force on it. If it is pushed down a little, the ampere force increases. Hence, its acceleration is opposite to its displacement and it will undergo oscillation.

R25. Magnetic field on AB, due to XY = $B = \dfrac{\mu_0 I}{2\pi h}$.

$$\therefore \quad mg = Bil = \left(\frac{\mu_0 I}{2\pi h}\right) il \quad \text{or} \quad \frac{Iil}{mh} = \frac{2\pi g}{\mu_0} = \text{constant.}$$

R26.



Let the magnetic field B be directed into the paper.

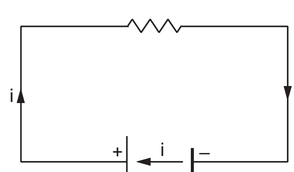
Emf induced in AB = Bl_1v_y directed from B to A.

Emf induced in BC = Bl_2v_x directed from B to C.

$V_A - V_B = v_xl_2 - v_yl_1$

R27. As the bar magnet passes through the coil, the flux linked with the ring first increases and then decreases. The current in the ring will therefore reverse in direction as the magnet passes through it. Also, the rate of change of flux is maximum when the magnet is close to the coil.

R28.



□

Part 6

Modern Physics

1

Modern Physics

• Type 1 •

Choose the correct option (a, b, c or d).

1. The penetrating powers of α, β and γ radiations, in decreasing order, are

(a) γ, α, β (b) γ, β, α

(c) α, β, γ (d) β, γ, α

2. When α, β and γ radiations pass through a gas, their ionizing powers, in decreasing order, are

(a) γ, α, β (b) γ, β, α

(c) α, β, γ (d) β, γ, α

3. If a beam consisting of α, β and γ radiations is passed through an electric field perpendicular to the beam, the deflections suffered by the components, in decreasing order, are

(a) α, β, γ (b) α, γ, β

(c) β, α, γ (d) β, γ, α

4. In a radioactive series, ${}^{238}_{92}U$ changes to ${}^{206}_{82}Pb$ through n_1 α-decay processes and n_2 β-decay processes.

(a) $n_1 = 8,\ n_2 = 8$ (b) $n_1 = 6,\ n_2 = 6$

(c) $n_1 = 8,\ n_2 = 6$ (d) $n_1 = 6,\ n_2 = 8$

5. Let u denote one atomic mass unit. One atom of an element of mass number A has mass exactly equal to A u

(a) for any value of A

(b) only for $A = 1$

(c) only for $A = 12$

(d) for any value of A provided the atom is stable

6. Two nucleons are at a separation of 1 fm. The net force between them is F_1 if both are neutrons, F_2 if both are protons, and F_3 if one is a proton and the other is a neutron.

(a) $F_1 > F_2 > F_3$ (b) $F_2 > F_1 > F_3$

(c) $F_1 = F_3 > F_2$ (d) $F_1 = F_2 > F_3$

7. A sample of radioactive material has mass m, decay constant λ, and molecular weight M. Avogadro constant $= N_A$. The initial activity of the sample is

(a) λm (b) $\dfrac{\lambda m}{M}$

(c) $\dfrac{\lambda m N_A}{M}$ (d) $m N_A e^{\lambda}$

8. In the previous question, the activity of the sample after time t will be

(a) $\left(\dfrac{m N_A}{M}\right) e^{-\lambda t}$ (b) $\left(\dfrac{m N_A \lambda}{M}\right) e^{-\lambda t}$

(c) $\left(\dfrac{m N_A}{M \lambda}\right) e^{-\lambda t}$ (d) $\dfrac{m}{\lambda}\left(1 - e^{-\lambda t}\right)$

9. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 $(t_2 > t_1)$. Its mean life is T.

(a) $A_1 t_1 = A_2 t_2$ (b) $\dfrac{A_1 - A_2}{t_2 - t_1} =$ constant

(c) $A_2 = A_1 e^{(t_1 - t_2/T)}$ (d) $A_2 = A_1 e^{(t_1/T t_2)}$

10. Let T be the mean life of a radioactive sample. 75% of the active nuclei present in the sample initially will decay in time

(a) $2T$ (b) $\frac{1}{2}(\ln 2)T$

(c) $4T$ (d) $2(\ln 2)T$

11. In a sample of radioactive material, what percentage of the initial number of active nuclei will decay during one mean life?

(a) 37 % (b) 50 %

(c) 63 % (d) 69.3 %

12. In a sample of radioactive material, what fraction of the initial number of active nuclei will remain undisintegrated after half of a half-life of the sample?

(a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}-1$

13. Three-fourths of the active nuclei present in a radioactive sample decay in $\frac{3}{4}$ s. The half-life of the sample is

(a) 1 s (b) $\frac{1}{2}$ s

(c) $\frac{3}{4}$ s (d) $\frac{3}{8}$ s

14. 90 % of the active nuclei present in a radioactive sample are found to remain undecayed after 1 day. The percentage of undecayed nuclei left after two days will be

(a) 85 % (b) 81 %

(c) 80 % (d) 79 %

15. A fraction f_1 of a radioactive sample decays in one mean life, and a fraction f_2 decays in one half-life.

(a) $f_1 > f_2$ (b) $f_1 < f_2$

(c) $f_1 = f_2$

(d) May be (a), (b) or (c) depending on the values of the mean life and half-life.

16. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ.

(a) $\lambda = \lambda_1 + \lambda_2$

(b) $\lambda = \frac{1}{2}(\lambda_1 + \lambda_2)$

(c) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(d) $\lambda = \sqrt{\lambda_1 \lambda_2}$

17. A sample of radioactive material is used to provide desired doses of radiation for medical purposes. The total time for which the sample can be used will depend

(a) only on the number of times radiation is drawn from it

(b) only on the intensity of doses drawn from it

(c) on both (a) and (b)

(d) neither on (a) nor on (b)

18. An orbital electron in the ground state of hydrogen has an angular momentum L_1, and an orbital electron in the first orbit in the ground state of lithium has an angular momentum L_2.

(a) $L_1 = L_2$

(b) $L_1 = 3L_2$

(c) $L_2 = 3L_1$

(d) $L_2 = 9L_1$

19. When white light (violet to red) is passed through hydrogen gas at room temperature, absorption lines will be observed in the

(a) Lyman series

(b) Balmer series

(c) both (a) and (b)

(d) neither (a) nor (b)

20. If radiation of all wavelengths from ultraviolet to infrared is passed through hydrogen gas at room temperature, absorption lines will be observed in the

(a) Lyman series

(b) Balmer series

(c) both (a) and (b)

(d) neither (a) nor (b)

21. A photon of energy 10.2 eV corresponds to light of wavelength λ_0. Due to an electron transition from $n = 2$ to $n = 1$ in a hydrogen atom, light of wavelength λ is emitted. If we take into account the recoil of the atom when the photon is emitted,

(a) $\lambda = \lambda_0$

(b) $\lambda < \lambda_0$

(c) $\lambda > \lambda_0$

(d) the data is not sufficient to reach a conclusion

22. White X-rays are called 'white' because

(a) they are produced most abundantly in X-ray tubes

(b) they are electromagnetic waves and hence have a nature similar to white light

(c) they can be converted to visible light using coated screens, and they affect photographic plates, just like light

(d) they have a continuous range of wavelengths

23. The minimum wavelength of X-ray that can be produced in a Coolidge tube depends on

(a) the metal used as the target

(b) the intensity of the electron beam striking the target

(c) the current flowing through the filament

(d) the potential difference between the cathode and the anode

24. If a potential difference of 20,000 volts is applied across an X-ray tube, the cut-off wavelength will be

(a) 6.21×10^{-10} m (b) 6.21×10^{-11} m

(c) 6.21×10^{-12} m (d) 3.1×10^{-11} m

25. If the potential difference applied across a Coolidge tube is increased,

(a) the wavelength of the K_α line will increase

(b) the wavelength of the K_β line will decrease

(c) the difference in wavelength between the K_α and K_β lines will decrease

(d) none of the above

26. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between K and L levels in this atom is about

(a) 0.51 MeV
(b) 1.2 MeV
(c) 59 keV
(d) 13.6 eV

• *Type 2* •

27. When a nucleus with atomic number Z and mass number A undergoes a radioactive decay process,

(a) both Z and A will decrease, if the process is α decay

(b) Z will decrease but A will not change, if the process is β^+ decay

(c) Z will increase but A will not change, if the process is β^- decay

(d) Z and A will remain unchanged, if the process is γ decay

28. When the nucleus of an electrically neutral atom undergoes a radioactive decay process, it will remain neutral after the decay if the process is

(a) an α decay
(b) a β^- decay
(c) a γ decay
(d) a K-capture process

29. Which of the following assertions are correct?

(a) A neutron can decay to a proton only inside a nucleus.

(b) A proton can change to a neutron only inside a nucleus.

(c) An isolated neutron can change into a proton.

(d) An isolated proton can change into a neutron.

30. Two identical nuclei A and B of the same radioactive element undergo β decay. A emits a β-particle and changes to A′. B emits a β-particle and then a γ-ray photon immediately afterwards, and changes to B′.

(a) A′ and B′ have the same atomic number and mass number.

(b) A′ and B′ have the same atomic number but different mass numbers

(c) A′ and B′ have different atomic numbers but the same mass number.

(d) A′ and B′ are isotopes.

31. A and B are isotopes. B and C are isobars. All three are radioactive.

(a) A, B and C must belong to the same element.

(b) A, B and C may belong to the same element.

(c) It is possible that A will change to B through a radioactive-decay process.

(d) It is possible that B will change to C through a radioactive-decay process.

32. A nuclide A undergoes α decay and another nuclide B undergoes β^- decay.

(a) All the α-particles emitted by A will have almost the same speed.

(b) The α-particles emitted by A may have widely different speeds.

(c) All the β-particles emitted by B will have almost the same speed.

(d) The β-particles emitted by B may have widely different speeds.

33. The decay constant of a radioactive sample is λ. Its half-life is $T_{1/2}$ and mean life is T.

(a) $T_{1/2} = \dfrac{1}{\lambda}, \quad T = \dfrac{\ln 2}{\lambda}$ (b) $T_{1/2} = \dfrac{\ln 2}{\lambda}, \quad T = \dfrac{1}{\lambda}$

(c) $T_{1/2} = \lambda \ln 2, \quad T = \dfrac{1}{\lambda}$

(d) $T_{1/2} = \dfrac{\lambda}{\ln 2}, \quad T = \dfrac{\ln 2}{\lambda}$

34. The count rate from 100 cm^3 of a radioactive liquid is *c*. Some of this liquid is now discarded. The count rate of the remaining liquid is found to be *c*/10 after three half-lives. The volume of the remaining liquid, in cm^3, is

(a) 20
(b) 40
(c) 60
(d) 80

35. In the Bohr model of the hydrogen atom, let *R*, *V* and *E* represent the radius of the orbit, speed of the electron and the total energy of the electron respectively. Which of the following quantities are proportional to the quantum number *n*?

(a) VR
(b) RE
(c) $\dfrac{V}{E}$
(d) $\dfrac{R}{E}$

36. Let ν_1 be the frequency of the series limit of the Lyman series, ν_2 be the frequency of the first line of the Lyman series, and ν_3 be the frequency of the series limit of the Balmer series.

(a) $\nu_1 - \nu_2 = \nu_3$
(b) $\nu_2 - \nu_1 = \nu_3$
(c) $\nu_3 = \dfrac{1}{2}(\nu_1 + \nu_2)$
(d) $\nu_1 + \nu_2 = \nu_3$

37. In an electron transition inside a hydrogen atom, orbital angular momentum may change by (*h* = Planck constant)

(a) h
(b) $\dfrac{h}{\pi}$
(c) $\dfrac{h}{2\pi}$
(d) $\dfrac{h}{4\pi}$

38. When a hydrogen atom emits a photon of energy 12.1 eV, its orbital angular momentum changes by

(a) 1.05×10^{-34} J s
(b) 2.11×10^{-34} J s
(c) 3.16×10^{-34} J s
(d) 4.22×10^{-34} J s

39. An electron with kinetic energy = E eV collides with a hydrogen atom in the ground state. The collision will be elastic

(a) for all values of E (b) for $E < 10.2$ eV

(c) for $E < 13.6$ eV (d) only for $E < 3.4$ eV

40. An electron is in an excited state in a hydrogen-like atom. It has a total energy of –3.4 eV. The kinetic energy of the electron is E and its de Broglie wavelength is λ.

(a) $E = 6.8$ eV, $\lambda \sim 6.6 \times 10^{-10}$ m

(b) $E = 3.4$ eV, $\lambda \sim 6.6 \times 10^{-10}$ m

(c) $E = 3.4$ eV, $\lambda \sim 6.6 \times 10^{-11}$ m

(d) $E = 6.8$ eV, $\lambda \sim 6.6 \times 10^{-11}$ m

41. Let the potential energy of a hydrogen atom in the ground state be zero. Then its energy in the first excited state will be

(a) 10.2 eV (b) 13.6 eV

(c) 23.8 eV (d) 27.2 eV

42. When a hydrogen atom emits a photon in going from $n = 5$ to $n = 1$, its recoil speed is almost

(a) 10^{-4} m/s (b) 2×10^{-2} m/s

(c) 4 m/s (d) 8×10^{2} m/s

43. An electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are

(a) $n_1 = 4$, $n_2 = 2$ (b) $n_1 = 8$, $n_2 = 2$

(c) $n_1 = 8$, $n_2 = 1$ (d) $n_1 = 6$, $n_2 = 3$

44. A beam of ultraviolet light of all wavelengths passes through hydrogen gas at room temperature, in the x-direction. Assume that all photons emitted due to electron transitions inside the gas emerge in the y-direction. Let A and B denote the lights emerging from the gas in the x- and y-directions respectively.

(a) Some of the incident wavelengths will be absent in A.
(b) Only those wavelengths will be present in B which are absent in A.
(c) B will contain some visible light.
(d) B will contain some infrared light.

45. Whenever a hydrogen atom emits a photon in the Balmer series,
(a) it may emit another photon in the Balmer series
(b) it must emit another photon in the Lyman series
(c) the second photon, if emitted, will have a wavelength of about 122 nm
(d) it may emit a second photon, but the wavelength of this photon cannot be predicted

46. Which of the following pairs constitute very similar radiations?
(a) Hard ultraviolet rays and soft X-rays
(b) Soft ultraviolet rays and hard X-rays
(c) Very hard X-rays and low-frequency γ-rays
(d) Soft X-rays and γ-rays

47. Let λ_α, λ_β and λ'_α denote the wavelengths of the X-rays of the K_α, K_β and L_α lines in the characteristic X-rays for a metal.
(a) $\lambda'_\alpha > \lambda_\alpha > \lambda_\beta$
(b) $\lambda'_\alpha > \lambda_\beta > \lambda_\alpha$
(c) $\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$
(d) $\frac{1}{\lambda_\alpha} + \frac{1}{\lambda_\beta} = \frac{1}{\lambda'_\alpha}$

48. In a Coolidge tube, the potential difference across the tube is 20 kV, and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The X-ray beam carries a power of
(a) 0.1 W
(b) 1 W
(c) 2 W
(d) 10 W

49. When an electron moving at a high speed strikes a metal surface, which of the following are possible?

(a) The entire energy of the electron may be converted into an X-ray photon.

(b) Any fraction of the energy of the electron may be converted into an X-ray photon.

(c) The entire energy of the electron may get converted to heat.

(d) The electron may undergo elastic collision with the metal surface.

50. When a metal of atomic number Z is used as the target in a Coolidge tube, let ν be the frequency of the K_α line. Corresponding values of Z and ν are known for a number of metals. Which of the following plots will give a straight line?

(a) ν against Z (b) $\frac{1}{\nu}$ against Z

(c) $\sqrt{\nu}$ against Z (d) ν against $\sqrt{Z}$

Answers

1. b	**2.** c	**3.** c	**4.** c	**5.** c
6. c	**7.** c	**8.** b	**9.** c	**10.** d
11. c	**12.** c	**13.** d	**14.** b	**15.** a
16. a	**17.** d	**18.** a	**19.** d	**20.** a
21. c	**22.** d	**23.** d	**24.** b	**25.** d
26. c	**27.** a, b, c, d	**28.** c, d	**29.** b, c	**30.** a
31. d	**32.** a, d	**33.** b	**34.** d	**35.** a, c
36. a	**37.** b, c	**38.** b	**39.** b	**40.** b
41. c	**42.** c	**43.** a, d	**44.** a, c, d	**45.** b, c
46. a, c	**47.** a, c	**48.** b	**49.** a, b, c	**50.** c

Hints and Solutions to Selected Questions

4. $n_2 = \dfrac{238 - 206}{4} = 8.$ $\qquad n_1 = n_2 \times 2 - (92 - 82) = 6.$

6. The attractive nuclear force is the same for any pair of nucleons. Thus, $F_1 = F_3$ when there are no electrostatic forces.
F_2 = attractive nuclear force – repulsive electrostatic force.

7. Activity = number of disintegrations per unit time

$$= \left| \frac{dN}{dt} \right| = \lambda \cdot N, \quad \text{where } N = \text{the total number of nuclei.}$$

Also, $N = \text{number of moles} \times N_A = \left(\dfrac{m}{M} \right) N_A.$

8. Activity after time t = initial activity $\times e^{-\lambda t}$.

9. Let A_0 = initial activity.

Then, $A_1 = A_0 e^{-\lambda t_1}$ and $A_2 = A_0 e^{-\lambda t_2}$. Also, $\lambda = \dfrac{1}{T}$

$$\therefore \quad \frac{A_2}{A_1} = \frac{e^{-\lambda t_2}}{e^{-\lambda t_1}} = e^{(-\lambda t_2 + \lambda t_1)}$$

or $\quad A_2 = A_1 e^{\lambda(t_1 - t_2)} = A_1 e^{(t_1 - t_2)/T}.$

10. When 75% decays, 25% is left undecayed. This requires a time $t = 2T_{1/2}$, where $T_{1/2}$ = half-life $= \dfrac{\ln 2}{\lambda} \cdot$ Also, $T = \dfrac{1}{\lambda} \cdot$

$$\therefore \quad t = 2\left(\frac{\ln 2}{\lambda} \right) = 2(\ln 2)T.$$

11. $N = N_0 e^{-\lambda t}$

$\therefore \quad$ number of nuclei decayed $= N_0 - N.$

∴ percentage of initial nuclei decayed

$$= \frac{N_0 - N}{N_0} \times 100 = \left(1 - \frac{N}{N_0}\right)100$$

$$= (1 - e^{-\lambda t})100 = (1 - e^{-\lambda T})100$$

$$= (1 - e^{-1})100 = (1 - 0.37)100.$$

12. $N = \dfrac{N_0}{2^{(t/T)}}$.

Here, $t = T/2$. ∴ $2^{(t/T)} = 2^{1/2} = \sqrt{2}$.

14. Equal fractions decay in equal times.
∴ if a fraction 0.9 remains undecayed after 1 day, a fraction $(0.9)^2 = 0.81$ will remain undecayed after 2 days.

15. Mean life $T = \dfrac{1}{\lambda}$ and half-life $T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{0.693}{\lambda}$.

∴ $T > T_{1/2}$. Greater fraction will decay in longer time.

19. At room temperature, all the atoms are in the ground state. The minimum photon energy required for absorption is 10.2 eV. White light has photon energies less than this and hence is not absorbed.

21. The total energy available from the transition = 10.2 eV = energy of emitted photon + kinetic energy of recoiling atom.

∴ energy of emitted photon < 10.2 eV.
∴ $\lambda > \lambda_0$.

24. Photon energy at cut-off wavelength = 20,000 eV.

∴ $\lambda_c = \dfrac{1242 \text{ eV nm}}{20 \times 10^3 \text{ eV}} = 6.21 \times 10^{-11}$ m.

26. $E_K - E_L = \dfrac{1242 \text{ eV nm}}{0.021 \text{ nm}} = \dfrac{124.2}{2.1} \times 10^3 \text{ eV} \approx 59$ keV.

32. In α decay, the entire energy is carried away by the α-particle as its kinetic energy.

In β^- decay, the energy is shared between the β-particle and the antineutrino. Hence, the speed of the β-particle will vary, depending on the energy of the antineutrino.

34. Initial count rate (CR) for 1 cm^3 of liquid $= \dfrac{c}{100}\cdot$

After 3 half-lives, CR for 1 cm^3 of liquid $= \dfrac{1}{8} \times \dfrac{c}{100}\cdot$

Let the volume of the remaining liquid $= V$ cm^3.

$\therefore$ CR of this liquid $= V \times \dfrac{c}{800} = \dfrac{c}{10}$

or $V = 80$.

35. $R \propto n^2, \quad V \propto n^{-1}, \quad E \propto n^2$

36. Series limit means the shortest possible wavelength (maximum photon energy), and first line means the longest possible wavelength (minimum photon energy) in the series.

$$\nu = c\left[\frac{1}{n^2} - \frac{1}{m^2}\right], \qquad \text{where } c = \text{constant.}$$

For series limit of the Lyman series, $n = 1,\ m = \infty,\ \nu_1 = c.$

For first line of the Lyman series, $n = 1,\ m = 2,\ \nu_2 = \dfrac{3c}{4}\cdot$

For series limit of the Balmer series, $n = 2,\ m = \infty,\ \nu_3 = \dfrac{c}{4}\cdot$

$\therefore \quad \nu_1 - \nu_2 = \nu_3.$

38. Emission of a photon of 12.1 eV requires a transition from $n = 3$ to $n = 1$.

Change in orbital angular momentum $= \dfrac{h}{2\pi}(3 - 1) = \dfrac{h}{\pi}\cdot$

39. Hydrogen atom in the ground state will only absorb energy greater than 10.2 eV. If this occurs, the collision will be inelastic. If there is no absorption of energy, the collision is elastic.

40. The potential energy $= -2 \times$ kinetic energy $= -2E$.

$\therefore$ total energy $= -2E + E = -E = -3.4$ eV

or $E = 3.4$ eV.

Let p = momentum and m = mass of the electron.

$$\therefore \quad E = \frac{p^2}{2m} \qquad \text{or} \quad p = \sqrt{2mE}.$$

de Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} \sim 6.6 \times 10^{-10}$ m.

42. Photon energy $= h\nu = 13.6\left(1 - \dfrac{1}{25}\right)$ eV $\cong 13$ eV.

Photon momentum = momentum of hydrogen atom $= p = \dfrac{h\nu}{c}$.

$$\therefore \quad mv = \frac{h\nu}{c}$$

$$\text{or} \quad v = \frac{h\nu}{mc} = \frac{13 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^{8}} \cong 4 \text{ m/s}.$$

43. In the nth orbit, let r_n = radius and v_n = speed of electron.

Time period $T_n = \dfrac{2\pi r_n}{v_n} \propto \dfrac{r_n}{v_n}$.

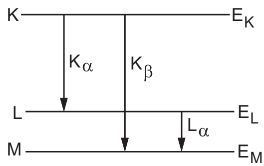
Now $r_n \propto n^2$ and $v_n \propto \dfrac{1}{n}$.

$$\therefore \quad \frac{r_n}{v_n} \propto n^3 \qquad \text{or} \quad T_n \propto n^3.$$

Here, $8 = \left(\dfrac{n_1}{n_2}\right)^3$

or $\dfrac{n_1}{n_2} = 2$ or $n_1 = 2n_2$.

45. Any transition causing a photon to be emitted in the Balmer series must end at $n = 2$. This must be followed by the transition from $n = 2$ to $n = 1$, emitting a photon of energy 10.2 eV, which corresponds to a wavelength of about 122 nm. This belongs to the Lyman series.

47. 

$$E_K - E_L = \frac{hc}{\lambda_\alpha}$$

$$E_K - E_M = \frac{hc}{\lambda_\beta}$$

$$E_L - E_M = \frac{hc}{\lambda'_\alpha} = \frac{hc}{\lambda_\beta} - \frac{hc}{\lambda_\alpha}$$

or $$\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}.$$

Also, $(E_K - E_M) > (E_K - E_L) > (E_L - E_M)$

or $$\frac{hc}{\lambda_\beta} > \frac{hc}{\lambda_\alpha} > \frac{hc}{\lambda'_\alpha}.$$

48. Power drawn by the Coolidge tube $= (20 \times 10^3 \text{ V})(10 \times 10^{-3} \text{ A})$
$= 200 \text{ W}.$

Power of X-ray beam $= \dfrac{0.5}{100} \times 200 \text{ W} = 1 \text{ W}.$

50. From Moseley's law, $\sqrt{\nu} = a(Z - b)$, where a and b are constants.

□

Part 7

Experimental Physics

1

Experimental Physics

• *Type 1* •

Choose the correct option (a, b, c or d).

1. In which of the following instruments is zero error not taken into account for measurements?

(a) Slide callipers

(b) Screw gauge

(c) Spherometer

(d) Correction for zero error has to made in all of the above.

2. A small, hollow metal cylinder is closed at one end. Its mass is known. Which of the following instruments are required to find the density of the metal?

(a) Slide callipers only

(b) Slide callipers and screw gauge

(c) Screw gauge and spherometer

(d) Slide callipers and spherometer

3. Backlash error may occur in which of the following instruments?

(a) Slide callipers and screw gauge

(b) Spherometer and screw gauge

(c) Slide callipers and spherometer

(d) Slide callipers, screw gauge and spherometer

4. A screw gauge is provided with a "ratchet" arrangement, in the form of a knob at the right end of the spindle. The screw should be rotated by this knob only. The purpose of this device is

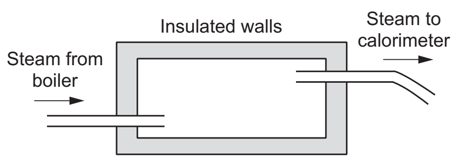
(a) to reduce zero error

(b) to prevent backlash error

(c) to control the rate of rotation of the screw

(d) to prevent damage to the pitch of the screw

5.



In the laboratory method for measuring the latent heat of steam, the steam is passed through the device shown above. The function of the device is

(a) to prevent condensed steam reaching the calorimeter

(b) to reduce the pressure of the steam

(c) to ensure that the pressure of the steam is equal to the atmospheric pressure

(d) to control the rate of flow of steam

6. A plane mirror, a metre scale, a plumb line and a vertical pin are required to measure the focal length of which of the following?

(a) Convex lens (b) Concave lens

(c) Concave mirror (d) Convex mirror

7. In an experiment to measure the focal length of a convex lens, the data for image distances (v) for different object distances (u) are plotted to obtain the three graphs of (1) v against u, (2) $\frac{1}{v}$ against $\frac{1}{u}$, and (3) $u + v$ against u. It is possible to find the focal length directly, without any further calculations, from which of these graphs?

(a) All (b) 1 and 2

(c) 2 and 3 (d) 1 and 3

8. Which of the following quantities can be measured using only a travelling microscope?

(a) Refractive index of a glass slab

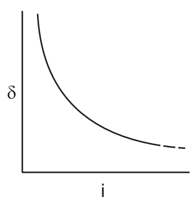
(b) Refractive index of a prism

(c) Refracting angle of a prism

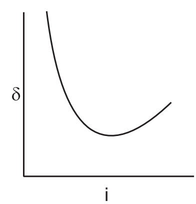
(d) Refractive index of a small drop of water

9. In the experiment to find the minimum deviation for a glass prism, by ray tracing, the deviation (δ) is measured for different values of the angle of incidence (i). Which of the following plots of δ against i is closest to the experimental result?

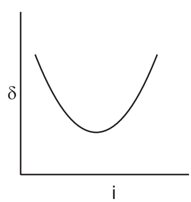
(a)



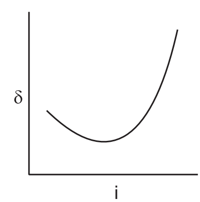
(b)



(c)



(d)



10. To measure the refracting angle (A) of a prism, the paths of rays reflected from the prism surface are traced using vertical pins placed on a sheet of paper. If the angle of minimum deviation for the prism is δ_m then the angle between the rays reflected from two surfaces of the prism will be

(a) $A + \delta_m$ (b) $2(A + \delta_m)$

(c) 2A (d) A

11. The laws of vibration of a string can be verified using a sonometer. If only one string is available on the sonometer, along with tuning forks of several different frequencies, which of the following laws can be verified?

(a) Law of length only (b) Law of tension only

(c) Law of length and law of tension

(d) Law of length, law of tension and law of mass

12. In a resonance-column experiment to measure the velocity of sound, the first resonance is obtained at a length l_1 and the second resonance at a length l_2. Then,

(a) $l_2 > 3l_1$

(b) $l_2 = 3l_1$

(c) $l_2 < 3l_1$

(d) may be any of the above, depending on the frequency of the tuning fork used

13.

Position (1) Position (2)

In a resonance-column experiment, the tuning fork is held above the open end of the glass tube. This can be done in two ways: position (1) one prong above the other, and position (2) the two prongs at the same horizontal level. The correct position is

(a) (1) in all cases

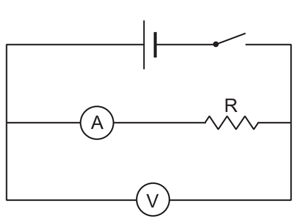
(b) (2) in all cases

(c) (1) for first resonance and (2) for second resonance

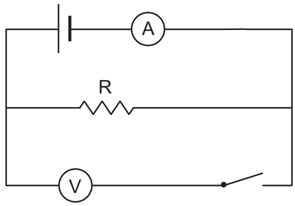
(d) (2) for first resonance and (1) for second resonance

14. Which of the circuits shown below is best suited to measure the resistance of a coil, R? The symbols have their usual meanings. The ammeter has a finite resistance. The voltmeter is ideal.

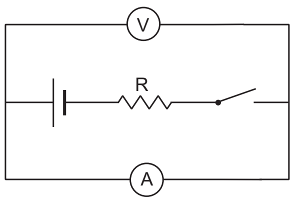
(a)



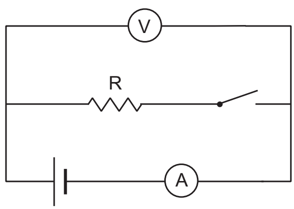
(b)



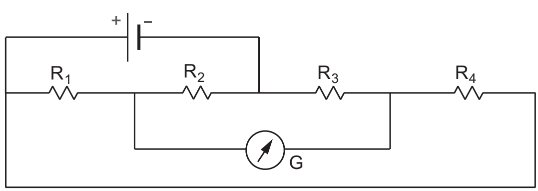
(c)



(d)



15.

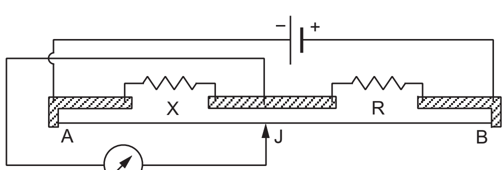


In the given circuit, the galvanometer G will show zero deflection if

(a) $R_1R_2 = R_3R_4$ (b) $R_1R_3 = R_2R_4$

(c) $R_1R_4 = R_2R_3$ (d) none of the above

16.



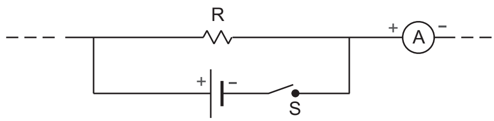
The figure shows a metre-bridge circuit, with $AB = 100$ cm, $X = 12\ \Omega$ and $R = 18\ \Omega$, and the jockey J in the position of balance.

If R is now made 8 Ω, through what distance will J have to be moved to obtain balance?

(a) 10 cm (b) 20 cm

(c) 30 cm (d) 40 cm

17.

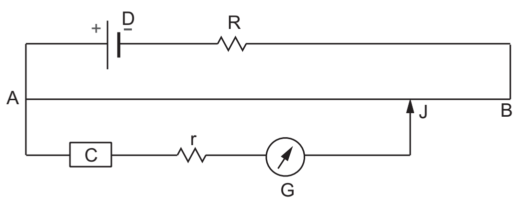


The circuit shown is part of a larger circuit, shown by dotted lines. The switch S is initially open. The potential difference across R is equal to the emf of the cell. The ammeter reading is I. If S is now closed, the ammeter reading will be

(a) zero (b) $2I$

(c) $\frac{I}{2}$ (d) I

18.



In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0. C is a cell of internal resistance r. The galvanometer G does not give zero deflection for any position of the jockey J. Which of the following cannot be a reason for this?

(a) $r > R_0$

(b) $R >> R_0$

(c) emf of C > emf of D

(d) The negative terminal of C is connected to A.

19. In experiments using the metre bridge, post-office box, and potentiometer, a galvanometer is used. Which property of the galvanometer makes it suitable for these experiments?

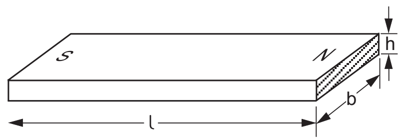
(a) It has a relatively high coil resistance.

(b) It indicates rather then measures the magnitude of the current.

(c) It can indicate currents flowing through it in either direction.

(d) It can be made very sensitive to small currents.

20.



An oscillation magnetometer is used to measure the magnetic moment of a bar magnet. Assume that the horizontal component of the earth's magnetic field and the time period of oscillation are known. In addition, which of the following quantities need to be known: m = mass of magnet, l = length, b = width, h = height.

(a) m only
(b) m and l
(c) m, l and b
(d) m, l, b and h

Answers

1. c	**2.** a	**3.** b	**4.** d	**5.** a
6. d	**7.** a	**8.** a	**9.** b	**10.** c
11. c	**12.** c	**13.** a	**14.** d	**15.** b
16. b	**17.** d	**18.** a	**19.** c	**20.** c

Hints and Solutions to Selected Questions

1. In a spherometer, the zero position is of little consequence. Readings are taken by the number of rotations and the difference between the initial and final readings on the circular scale.

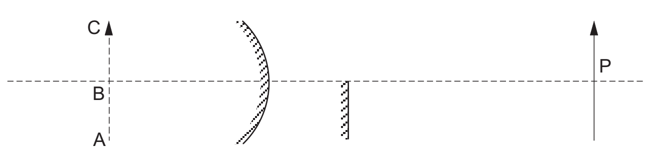
2. Slide callipers can measure the external and internal diameters as well as the depth of a hollow cylinder. This is sufficient to find its volume, and hence density, since mass is known.

3. Backlash error occurs only in instruments using screws.

4. The ratchet slips when extra pressure is applied on the screw head at the zero position. This prevents damage to the pitch of the screw.

5. The arrangement is commonly called a "steam trap". The steam which condenses into water in the rubber pipe from the boiler remains inside this device and does not reach the calorimeter. This removes a major source of error in the experiment.

6.



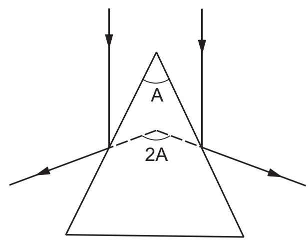
The image AB of the pin P formed by the plane mirror, and BC formed by the concave mirror, are made to coincide. The metre scale and plumb line are necessary to measure distances.

7. The focal length can be obtained from the point where $u = v$ in the $u \sim v$ graph, from the x and y intercepts on the $\frac{1}{v} \sim \frac{1}{u}$ graph, and from the minimum value of $u + v$ in the $u + v \sim u$ graph.

8. When the slab is placed on a flat surface, the real depth and apparent depths can be measured directly by the travelling microscope.

9. Sharp images are obtained for small angles of incidence. For large angles of incidence, the images become indistinct and proper ray tracing becomes difficult.

10.



11. Verification of the law of mass requires several strings with different masses per unit length.

12. λ = wavelength of sound, δ = end correction

$$l_1 = \lambda/4 + \delta \qquad l_2 = 3\lambda/4 + \delta$$

or $\quad l_2 = 3(l_1 - \delta) + \delta = 3l_1 - 2d. \qquad \therefore \quad l_2 < 3l_1.$

13. Most of the sound emitted by a tuning fork is propagated in the plane in which the prongs lie. Here, the sound has to be directed into the tube.

14. The voltmeter must measure the potential difference across the coil only. This is satisfied in (b) and (d). However, in (b) the key is not correctly placed.

15. The circuit can be rearranged as a Wheatstone bridge, in which R_1 and R_3, and R_2 and R_4 are diagonally opposite each other.

16. $\frac{X}{R} = \frac{AJ}{JB}$ for balance

Initially, $\frac{12}{18} = \frac{AJ}{100 - AJ}$, finally $\frac{12}{8} = \frac{AJ'}{100 - AJ'}$ or $AJ' - AJ = 20$ cm.

17. As the emf of the cell is equal to the potential difference across *R*, closing S does not affect the rest of the circuit.

18. In the position of balance, no current flows through the galvanometer, and hence through *r*. Therefore, the magnitude of *r* does not affect the condition for balance. Under the other three conditions, balance cannot be obtained.

19. In the three experiments, the jockey is moved according to the direction of the deflection of the galvanometer.

20. The moment of inertia of the magnet has to be found.

□

Part 8

Miscellaneous Questions

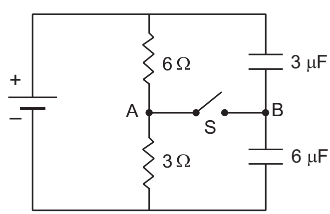
1

Miscellaneous Questions—1

• *Straight-Objective Type 1* •

Choose the correct option (a, b, c or d). Only one option is correct.

1.



In the circuit shown above, when the switch S is closed,

(a) no charge flows through S

(b) charge flows from A to B

(c) charge flows from B to A

(d) charge flows initially from A to B and later from B to A

2. The horizontal range of a projectile is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of the motion of the projectile, giving it a constant horizontal acceleration $= g/2$. Under the same conditions of projection, the horizontal range of the projectile will now be

(a) $R + \frac{H}{2}$ (b) $R + H$

(c) $R + \frac{3H}{2}$ (d) $R + 2H$

3. A small object O is placed in front of a convex mirror, forming a virtual image, I. A narrow beam of light is now made incident on the mirror, aimed at I. After reflection at the mirror, the beam will reach O

(a) in all cases

(b) only if the beam of light moves very close to the axis OI

(c) only if the distance of O from the mirror is small compared to the radius of curvature of the mirror

(d) only if both (b) and (c) are satisfied

4. A particle of mass m and charge Q is placed in an electric field E which varies with time t as $E = E_0 \sin \omega t$. It will undergo simple harmonic motion of amplitude

(a) $\dfrac{QE_0^2}{m\omega^2}$ (b) $\dfrac{QE_0}{m\omega^2}$

(c) $\sqrt{\dfrac{QE_0}{m\omega^2}}$ (d) $\dfrac{QE_0}{m\omega}$

5. In a Young's double-slit experiment using identical slits, when only one slit is used, the total energy reaching the screen is E_0 and the intensity of light at any point on the screen is I_0. When both slits are used, and fringes are formed on the screen, the total energy reaching the screen is E and the maximum intensity on the screen is I. Then,

(a) $E = 2E_0, I = 2I_0$ (b) $E = 4E_0, I = 4I_0$

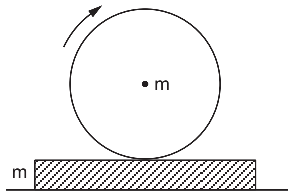
(c) $E = 2E_0, I = 4I_0$ (d) $E = 4E_0, I = 2I_0$

6. In a Young's double-slit experiment using slits of unequal widths, the intensities on the screen due to the slits are in the ratio 4 : 9 when the slits are used separately. When they are used together, the ratio of the intensity at a dark fringe to the intensity at a bright fringe on the screen will be

(a) 4 : 9 (b) 1 : 9

(c) 9 : 16 (d) 1 : 25

7.



A sphere of mass m is given some angular velocity about a horizontal axis through its centre, and gently placed on a plank of mass m. The coefficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the sphere relative to the plank will be

(a) zero (b) μg

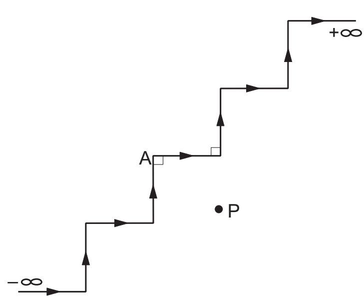
(c) $\frac{7}{5}\mu g$ (d) $2\mu g$

8. When beats are formed between sound waves of slightly different frequencies, the intensity of the sound heard changes from maximum to minimum in 0.2 s. The difference in frequencies of the two sound waves is

(a) 5 Hz (b) 4 Hz

(c) 2.5 Hz (d) 2 Hz

9.

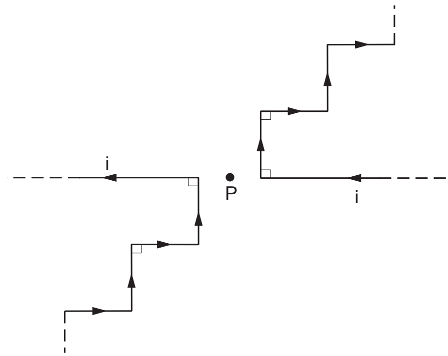


An infinitely long thin conductor, shaped as shown, carries current. Each section is of the same length. The magnetic field at the point P due to the section from $-\infty$ to A is B. The field at P due to the entire conductor is

(a) zero (b) B

(c) $\sqrt{2}B$ (d) $2B$

10.



Two infinitely long conductors carrying equal currents are shaped as shown. The short sections are all of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at P due to any one conductor is B. The total field at P is

(a) zero
(b) B
(c) $\sqrt{2}B$
(d) $2B$

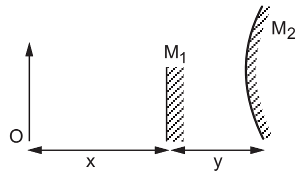
11. A satellite can be in a geostationary orbit around a planet at a distance r from the centre of the planet. If the angular velocity of the planet about its axis doubles, a satellite can now be in a geostationary orbit around the planet if its distance from the centre of the planet is

(a) $\frac{r}{2}$
(b) $\frac{r}{2\sqrt{2}}$
(c) $\frac{r}{4^{1/3}}$
(d) $\frac{r}{2^{1/3}}$

12. A bullet moving with a velocity u passes through a plank which is free to move. The two are of equal mass. After passing through the plank, the velocity of the bullet becomes fu. Its velocity relative to the plank now is

(a) fu
(b) $(1-f)u$
(c) $(2f-1)u$
(d) $(2-f)u$

13.

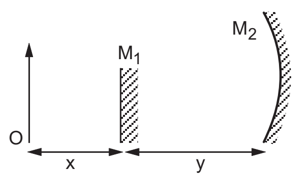


An object O is placed in front of a small plane mirror M_1 and a large convex mirror M_2 of focal length f. The distance between O and M_1 is x, and the distance between M_1 and M_2 is y. The images of O formed by M_1 and M_2 coincide. The magnitude of f is

(a) $\frac{x^2 - y^2}{2y}$ (b) $\frac{x^2 + y^2}{2y}$

(c) $x - y$ (d) $\frac{x^2 + y^2}{x - y}$

14.



An object O is placed in front of a partially reflecting plane mirror M_1 and a concave mirror M_2 of focal length f. The distance between O and M_1 is x and the distance between M_1 and M_2 is y. The two images of O formed by partial reflection in M_1 and reflection in M_2 (of rays passing through M_1) coincide. The magnitude of f is

(a) $\frac{y^2 - x^2}{2y}$ (b) $\frac{y^2 + x^2}{2y}$

(c) $y - x$ (d) $\frac{y^2 + x^2}{y - x}$

15. In a uniform magnetic field of 10^{-5} T in free space, the energy density is u. The electric field which will produce the same energy density in free space is

(a) 10^5 V/m (b) 3×10^3 V/m

(c) 10 V/m (d) 9×10^{-3} V/m

16. A block of mass m is placed on a horizontal surface. The coefficient of friction between them is μ. The block has to be moved by applying a single external force on it. The force may be applied in any direction. The minimum value of this force must be

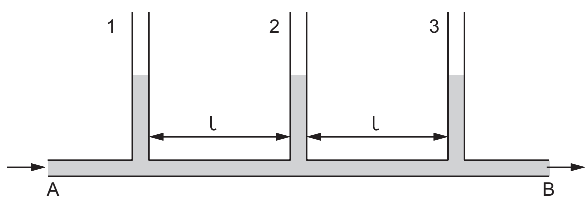
(a) mg, applied vertically upward, if $\mu > 1$

(b) μmg, applied horizontally, if $\mu < 1$

(c) $\dfrac{\mu mg}{\sqrt{\mu^2 + 1}}$ for all values of μ

(d) $\dfrac{\mu^2 mg}{\mu^2 + 1}$ for all values of μ

17.



An ideal liquid flows through the horizontal pipe AB, which is of uniform cross-section. The vertical pipes 1, 2 and 3 are equispaced. The liquid levels in these pipes are at heights h_1, h_2 and h_3 respectively above AB. Liquid flows from A to B in AB.

(a) $h_1 = h_2 = h_3$

(b) $h_2 = \frac{1}{2}(h_1 + h_3)$

(c) $h_2 > \frac{1}{2}(h_1 + h_3)$

(d) $h_2 < \frac{1}{2}(h_1 + h_3)$

18. A number of spherical conductors of different radii are given charge such that the charge density of each conductor is inversely proportional to its radius. The conductors will have

(a) the same potential

(b) the same potential energy

(c) the same charge

(d) potentials inversely proportional to their radii

19. A block of mass m slides down an inclined plane which makes an angle θ with the horizontal. The coefficient of friction between the block and the plane is μ. The force exerted by the block on the plane is

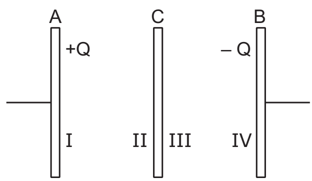
(a) $mg \cos \theta$

(b) $\sqrt{\mu^2 + 1}\ mg \cos \theta$

(c) $\dfrac{\mu mg \cos \theta}{\sqrt{\mu^2 + 1}}$

(d) $\mu mg \cos \theta$

20.



Plates A and B constitute an isolated, charged parallel-plate capacitor. The inner surfaces (I and IV) of A and B have charges $+Q$ and $-Q$ respectively. A third plate C with charge $+Q$ is now introduced midway between A and B. Which of the following statements is not correct?

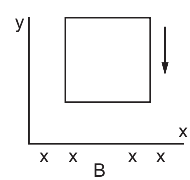
(a) The surfaces I and II will have equal and opposite charges.

(b) The surfaces III and IV will have equal and opposite charges.

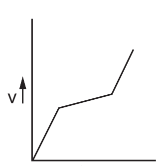
(c) The charge on surface III will be greater than Q.

(d) The potential difference between A and C will be equal to the potential difference between C and B.

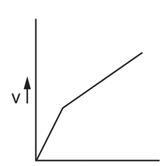
21. A square, conducting loop falls from rest in the xy-plane. There is a uniform magnetic field in the z-direction below the x-axis. The velocity v of the loop is plotted against time t. Which of the following best represents the resulting curve?

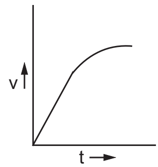


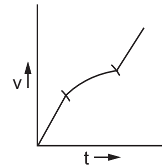
(a)



(b)



(c) 

(d) 

22. A wheel of radius r rolls without slipping on the ground, with speed v. When it is at a point P, a piece of mud flies off tangentially from its highest point, and lands on the ground at point Q. The distance PQ is

(a) $2v\sqrt{\frac{r}{g}}$

(b) $2\sqrt{2}\,v\sqrt{\frac{r}{g}}$

(c) $4v\sqrt{\frac{r}{g}}$

(d) $v\sqrt{\frac{r}{g}}$

23. An isolated parallel-plate capacitor of capacitance C has plates X and Y. If plate X is given charge Q, the potential difference between X and Y is

(a) zero

(b) $\frac{2Q}{C}$

(c) $\frac{Q}{C}$

(d) $\frac{Q}{2C}$

24. In the previous question, if Y is earthed, what amount of charge will flow from Y into the earth?

(a) zero

(b) Q

(c) $\frac{Q}{2}$

(d) $-\frac{Q}{2}$

25. Three stars A, B, C have surface temperatures T_A, T_B and T_C. A appears bluish, B appears reddish and C appears yellowish. We can conclude that

(a) $T_A > T_C > T_B$

(b) $T_A > T_B > T_C$

(c) $T_B > T_C > T_A$

(d) $T_C > T_B > T_A$

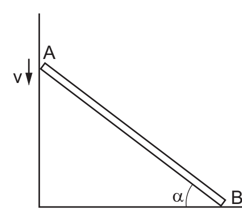
26. A horizontal rod rotates about a vertical axis through one end. A ring, which can slide along the rod without friction, is initially close to the axis and then slides to the other end of the rod. In this process, which of the following quantities will be conserved?
[L = angular momentum, E_T = total kinetic energy, E_R = rotational kinetic energy.]

(a) L only
(b) L and E_T only
(c) L and E_R only
(d) E_T only

27. Two electric lamps A and B radiate the same power. Their filaments have the same dimensions, and have emissivities e_A and e_B. Their surface temperatures are T_A and T_B. The ratio T_A/T_B will be equal to

(a) $\left(\frac{e_B}{e_A}\right)^{1/4}$
(b) $\left(\frac{e_B}{e_A}\right)^{1/2}$
(c) $\left(\frac{e_A}{e_B}\right)^{1/2}$
(d) $\left(\frac{e_A}{e_B}\right)^{1/4}$

28. The end A of a rod slides down a smooth wall and its end B slides on a smooth floor. When AB makes angle α with the horizontal, A has speed v. The speed of B must be



(a) $\frac{v}{\tan\alpha}$
(b) $v\tan\alpha$
(c) $\frac{v}{\cos\alpha}$
(d) $v\sin\alpha$

29. An electric dipole has moment $\vec{p} = p\vec{i}$. Two points which are at equal distances from the dipole, and far away from it, have electric intensities $E_1\vec{i}$ and $-E_2\vec{i}$. The ratio E_1/E_2 must be

(a) 1
(b) $\sqrt{2}$
(c) 2
(d) $\frac{1}{2}$

30. When a receiver of sound (e.g., microphone diaphragm or human eardrum) is receiving sound, the nature of its vibration is most likely to be

(a) free (b) forced

(c) resonance

(d) similar to that of stationary waves

31. A lens of power 16 D is used as a simple microscope. In order to obtain maximum magnification, at what distance from the lens should a small object be placed?

(a) 5 cm (b) 10 cm

(c) 16 cm (d) 25 cm

32. In the previous question, what is the range of the magnification that can be obtained?

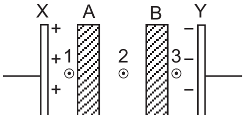
(a) 4 to infinity (b) 5 to infinity

(c) 4 to 5 (d) 5 to 6.25

33. A flat, rectangular coil is placed in a uniform magnetic field and rotated about an axis passing through its centre, parallel to its shorter edges and perpendicular to the field. The maximum emf induced is E. If the axis is shifted to coincide with one of the shorter edges, the maximum induced emf will be

(a) zero (b) $E/2$

(c) E (d) $2E$

34. 

Two identical dielectric slabs, A and B, are placed symmetrically between the plates X and Y of a charged parallel-plate capacitor. The electric intensity has magnitudes E_1, E_2 and E_3 at the points 1, 2 and 3.

(a) $E_1 > E_2 > E_3$ (b) $E_1 = E_3 < E_2$

(c) $E_1 = E_3 > E_2$ (d) $E_1 = E_2 = E_3$

35. A coil with resistance R is placed in a magnetic field. The flux linked with the coil is ϕ. If the magnetic field suddenly reverses in direction, how much charge will circulate in the coil?

(a) $\frac{\phi}{2R}$ (b) $\frac{\phi}{R}$

(c) $\frac{2\phi}{R}$ (d) zero

36. One mole gas is first cooled from 300 K to 150 K at constant volume and then heated from 150 K to 300 K at constant pressure. The net heat absorbed by the gas is

(a) zero (b) $150R$

(c) $300R$ (d) $450R$

37. In the X-rays produced by a Coolidge tube let λ_c be the cut-off wavelength, λ_α be the wavelength of the K_α line and λ_β be the wavelength of the K_β line.

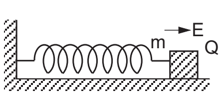
(a) $\lambda_\beta > \lambda_\alpha > \lambda_c$ (b) $\lambda_\alpha > \lambda_\beta > \lambda_c$

(c) $\lambda_\alpha > \lambda_c > \lambda_\beta$ (d) $\lambda_\beta > \lambda_c > \lambda_\alpha$

38. A capacitor of capacitance C is given charge Q and then connected in parallel to a coil of inductance L. There is no resistance in the circuit. When the charge on the capacitor becomes zero, the current in the coil will be

(a) $Q\sqrt{\frac{L}{C}}$ (b) $\frac{Q}{\sqrt{LC}}$

(c) $Q\sqrt{\frac{C}{L}}$ (d) zero

39. 

In the spring–block system shown, the block oscillates on a smooth horizontal surface with time period T. It is now given some charge Q and an electric field E is switched on, as shown. The block will now oscillate with time period T', where

(a) $T' > T$

(b) $T' = T$

(c) $T' < T$

(d) T' may be > or < T depending on the magnitudes of m, Q and E

40. An organ pipe filled with oxygen gas at 47°C resonates in its fundamental mode at a frequency 300 Hz. If it is now filled with nitrogen gas, at what temperature will it resonate at the same frequency, in the fundamental mode?

(a) 7°C (b) 27°C

(c) 87°C (d) 107°C

• *Straight-Objective Type 2* •

Choose the correct options (a, b, c or d). One or more options may be correct.

41. Two conducting spheres of unequal radii are given charge such that they have the same charge density. If they are now brought in contact,

(a) no charge will be exchanged between them

(b) charge will flow from the larger to the smaller sphere

(c) charge will flow from the smaller to the larger sphere

(d) some heat will be produced

42. Two particles are projected from the same point, with the same speed, in the same vertical plane, at different angles with the horizontal. A frame of reference is fixed to one particle. The position vector of the other particle, as observed from this frame, is $\vec{r}$. Which of the following statements are correct?

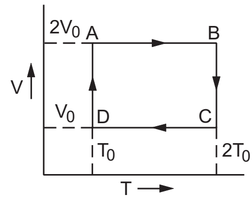
(a) $\vec{r}$ is a constant vector.

(b) $\vec{r}$ changes in magnitude and direction with time.

(c) The magnitude of $\vec{r}$ increases linearly with time; its direction does not change.

(d) The direction of $\vec{r}$ changes with time; its magnitude may or may not change, depending on the angles of projection.

43.



One mole of an ideal gas is taken through the cyclic process shown in the *V–T* diagram, where *V* = volume and *T* = absolute temperature of the gas. Which of the following statements is/are correct?

(a) Heat is given out by the gas.

(b) Heat is absorbed by the gas.

(c) The magnitude of the work done by the gas is $RT_0 \ln 2$.

(d) The magnitude of the work done by the gas is V_0T_0.

44. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges $+Q$ and $-Q$ respectively. A third plate C, identical to A and B, with charge $+Q$, is now introduced midway between A and B, parallel to them. Which of the following is/are correct?

(a) The charge on the inner face of B is now $-\frac{3Q}{2}$.

(b) There is no change in the potential difference between A and B.

(c) The potential difference between A and C is one-third of the potential difference between B and C.

(d) The charge on the inner face of A is now $\frac{Q}{2}$.

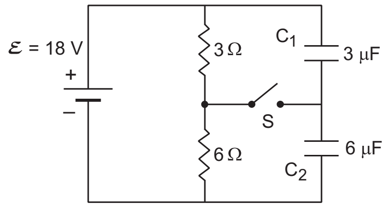
45. A ring of radius *R* has uniformly distributed charge. It spins about its axis with an angular velocity ω. P is a point on its axis at a distance *x* from its centre. The velocity of light in vacuum is *c*. The ratio of the magnetic field to the electric field at P is proportional to

(a) ω

(b) R^2

(c) $\frac{1}{x}$

(d) $\frac{1}{c^2}$

46. 

In the circuit shown, which of the following statements is/are correct?

(a) When S is open, charge on C_1 is 36 μC.

(b) When S is open, charge on C_2 is 36 μC.

(c) When S is closed, the charges on C_1 and C_2 do not change.

(d) When S is closed, charges on both C_1 and C_2 change.

47. Two sounds of equal amplitudes and frequencies, n_1 and n_2, reach a point together. The resultant wave can have which of the following forms (symbols have their usual meanings)?

(a) $y = A \sin\left[\left(\frac{n_1 - n_2}{2}\right)t\right] \sin\left[\left(\frac{n_1 + n_2}{2}\right)t\right]$

(b) $y = A \cos\left[\left(\frac{n_1 - n_2}{2}\right)t\right] \cos\left[\left(\frac{n_1 + n_2}{2}\right)t\right]$

(c) $y = A \sin[(n_1 - n_2)t] \cos[(n_1 + n_2)t]$

(d) $y = A \sin[(n_1 - n_2)t] \sin\left[\left(\frac{n_1 + n_2}{2}\right)t\right]$

48. A particle moving with kinetic energy 3 J makes an elastic head-on collision with a stationary particle which has twice its mass. During the impact,

(a) the minimum kinetic energy of the system is 1 J

(b) the maximum elastic potential energy of the system is 2 J

(c) momentum and total energy are conserved at every instant

(d) the ratio of kinetic energy to potential energy of the system first decreases and then increases

49. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct?

(a) The entire charge will appear on its outer surface.

(b) All points on the conductor will have the same potential.

(c) All points on its surface will have the same charge density.

(d) All points near its surface and outside it will have the same electric intensity.

50. A particle moves in the *xy* plane with a constant acceleration g in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following are correct?

(a) The x-component of its velocity is constant.

(b) At the origin, the y-component of its velocity is $a\sqrt{\frac{g}{2b}}$.

(c) At the origin, its velocity makes an angle $\tan^{-1}a$ with the x-axis.

(d) The particle moves exactly like a projectile.

51. A converging lens of focal length f is placed in front of a fixed object, at a distance f from it. The lens is then moved away from the object with a constant velocity. The velocity of the image will

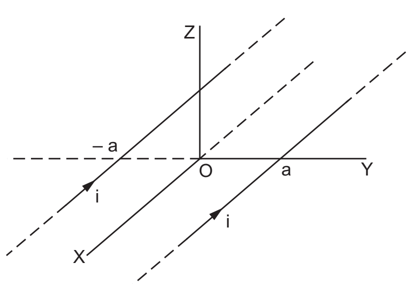
(a) be constant

(b) always be directed towards the object

(c) pass through a maximum

(d) be zero when the distance of the lens from the object is $2f$

52.



Two long, thin, parallel conductors carrying equal currents in the same direction are fixed parallel to the x-axis, one passing through $y = a$ and the other through $y = -a$. The resultant magnetic field due to the two conductors at any point is B. Which of the following are correct?

(a) $B = 0$ for all points on the x-axis.

(b) At all points on the y-axis, excluding the origin, B has only a z-component.

(c) At all points on the z-axis, excluding the origin, B has only a y-component.

(d) B cannot have an x-component.

53. A thin-walled, spherical conducting shell S of radius R is given charge Q. The same amount of charge is also placed at its centre C. Which of the following statements are correct?

(a) On the outer surface of S, the charge density is $\frac{Q}{2\pi R^2}$.

(b) The electric field is zero at all points inside S.

(c) At a point just outside S, the electric field is double the field at a point just inside S.

(d) At any point inside S, the electric field is inversely proportional to the square of its distance from C.

54. A man can swim with a velocity v relative to water. He has to cross a river of width d flowing with a velocity $u (u > v)$. The distance through which he is carried downstream by the river is x. Which of the following statements are correct?

(a) If he crosses the river in minimum time, $x = \frac{du}{v}$.

(b) x cannot be less than $\frac{du}{v}$.

(c) For x to be minimum, he has to swim in a direction making an angle of $\frac{\pi}{2} + \sin^{-1}(v/u)$ with the direction of the flow of water.

(d) x will be maximum if he swims in a direction making an angle of $\frac{\pi}{2} - \sin^{-1}(v/u)$ with the direction of the flow of water.

Answers

1. a	**2.** d	**3.** a	**4.** b	**5.** c
6. d	**7.** d	**8.** c	**9.** d	**10.** a
11. c	**12.** c	**13.** a	**14.** a	**15.** b
16. c	**17.** a	**18.** a	**19.** b	**20.** d
21. d	**22.** c	**23.** d	**24.** b	**25.** a
26. b	**27.** a	**28.** b	**29.** c	**30.** b
31. a	**32.** c	**33.** c	**34.** d	**35.** c
36. b	**37.** b	**38.** b	**39.** b	**40.** a
41. b, d	**42.** c	**43.** a, c	**44.** a, b, c, d	**45.** a, b, c, d
46. a, b, d	**47.** a, b	**48.** a, b, c, d	**49.** a, b	**50.** a, b, c, d
51. d	**52.** a, b, c, d	**53.** a, c, d	**54.** a, c	

Hints and Solutions to Selected Questions

1. For two conductors in series, their potential differences are proportional to their resistances. For two capacitors in series, their potential differences are inversely proportional to their capacitances. Hence, A and B are at the same potential and no charge will flow between them.

2. For the projectile, $R = \dfrac{u^2 \sin 2\theta}{g}$, $H = \dfrac{u^2 \sin^2 \theta}{2g}$.

Time of flight $= t = \dfrac{2u \sin \theta}{g}$.

Initial horizontal velocity $= u \cos \theta$.

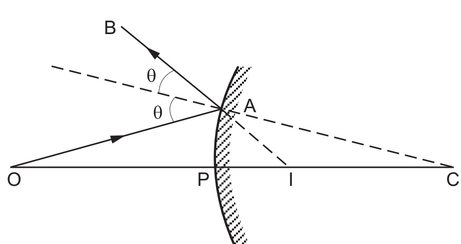
When horizontal acceleration $g/2$ is present,

$$\text{the horizontal range} = (u \cos \theta)t + \frac{1}{2}\left(\frac{g}{2}\right)t^2$$

$$= (u \cos \theta)\frac{2u \sin \theta}{g} + \frac{g}{4}\left(\frac{4u^2 \sin^2 \theta}{g^2}\right)$$

$$= R + 2H.$$

3.



A ray of light aimed from B towards I will always pass through O, irrespective of the value of θ or of the magnitudes of OP and PC.

4. For a particle undergoing SHM with an amplitude A and angular frequency ω, the maximum acceleration $= \omega^2 A$.

Here, the maximum force on the particle $= QE_0$.

$\therefore$ its maximum acceleration $= \dfrac{QE_0}{m} = \omega^2 A$

or $A = \dfrac{QE_0}{m\omega^2}$.

6. See the hint to Q. **27** and **28** on p. 4–24.

7. The only horizontal forces acting on the two bodies are those due to friction of magnitude μmg each, in opposite directions. Hence, they have accelerations μg each, in opposite directions.

8. The duration of one beat is from one maximum to the next maximum of intensity. In this case, it is 0.4 s.

$\therefore$ the number of beats per second = 2.5 = the difference in frequencies.

9. The sections up to A and the sections beyond A form pairs which produce equal magnetic fields at P.

10. The sections of the conductors on either side of P form pairs which produce equal and opposite magnetic fields at P.

11. Time period of rotation (T) and distance (r) are related as $T^2 \propto r^3$. Also, when the angular velocity doubles, the time period becomes half.

12. Let v = the velocity of the plank after the impact.

Using conservation of momentum,

$$mu = mv + mfu$$

or $v = u(1 - f)$.

$\therefore$ the velocity of the bullet relative to the plank $= fu - v$.

13. Due to M_1, an image is formed at a distance x from M_1, i.e., at a distance $x - y$ behind M_2. Thus, for M_2,

$$u = -(x + y),\ v = x - y.$$

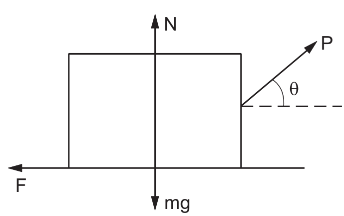
Use $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

14. See the previous solution. Here, the image formed by M_2 is real, at a distance $y - x$ from it.

15. $u_B = \frac{B^2}{2\mu_0} = u_E = \frac{1}{2}\varepsilon_0 E^2$

or $\quad E = \frac{B}{\sqrt{\mu_0 \varepsilon_0}} = cB.$

16.



$mg = N + P\sin\theta$

or $\quad N = mg - P\sin\theta.$

$P\cos\theta = F_{\text{lim}} = \mu N = \mu mg - \mu P\sin\theta$

or $\quad P[\cos\theta + \mu\sin\theta] = \mu mg$

or $\quad P = \frac{\mu mg}{\cos\theta + \mu\sin\theta}.$

For P to be minimum, $\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$

or $\quad -\sin\theta + \mu\cos\theta = 0 \quad$ or $\quad \tan\theta = \mu.$

$$\therefore \quad P_{\min} = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \mu\frac{\mu}{\sqrt{\mu^2+1}}} = \frac{\mu mg}{\sqrt{\mu^2+1}}.$$

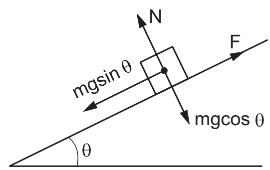
17. As the tube is horizontal and the velocity of flow is the same at all points, the liquid pressure is constant.

18. $V = \frac{Q}{C} = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_0 R} = \frac{R\sigma}{\varepsilon_0}$.

Here, $\sigma \propto \frac{1}{R}$ or $\sigma R = \text{constant}$.

$\therefore$ all the conductors are at the same potential.

19.



The force exerted by the block on the plane

= the force exerted by the plane on the block

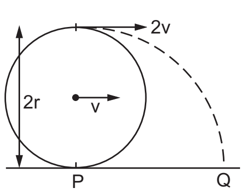
$= \sqrt{N^2 + F^2} = \sqrt{N^2 + \mu^2 N^2} = N\sqrt{1 + \mu^2}$

$= mg\cos\theta\sqrt{1 + \mu^2}$.

20. See the solution to Q. **45** on p. 5–40.

21. At every position, the slope of the v–t graph is equal to the acceleration. This will be equal to g, (a) before the coil enters the magnetic field, and (b) after it has entered the field completely. When part of the coil is in the field, the Ampere force will cause an acceleration less than g. Also, during this phase, as v is still increasing, the Ampere force will increase, and hence the acceleration will decrease gradually.

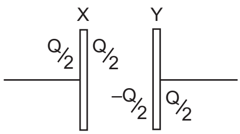
22.



Considering the vertical motion from the highest point to Q,

$-2r = \frac{1}{2}(-g)t^2$ or $t = 2\sqrt{\frac{r}{g}}$.

$\therefore$ PQ $= (2v)t = 4v\sqrt{\frac{r}{g}}$.

23. 

In the isolated capacitor the charge distribution will be as shown. The potential difference between the plates depends only on the charges on the inner surfaces, this will be $\frac{1}{C}\left(\frac{Q}{2}\right)$.

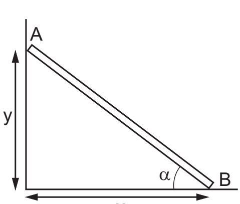
24. When Y is earthed, the free charges on it will run off into the earth. This will cause the entire $+Q$ charge on X to be attracted to its inner surface, resulting in $-Q$ charge on the inner surface of Y. The total free charge on Y then becomes $+Q$, which will run off into the earth.

25. The appearance of a star depends on the wavelength (λ_m) at which it radiates maximum energy. This depends inversely on the surface temperature (T) of the star. Now, $\lambda_{blue} < \lambda_{yellow} < \lambda_{red}$. Thus, star A (bluish) radiates the shortest λ and must be at the highest temperature, while star B (reddish) must be at the lowest temperature.

26. As no external torque acts on the system, its angular momentum is conserved. Since there is no loss of energy due to friction, the total energy is conserved. However, the ring acquires some translational kinetic energy as it slides outwards, and hence rotational kinetic energy is not conserved.

27. The power radiated by a filament is $P = e(\sigma T^4)$(area), where e = emissivity, σ = Stefan–Boltzmann constant and T = surface temperature.

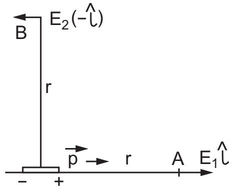
Here, eT^4 = constant or $e_A T_A^4 = e_B T_B^4$.

28. 

$$x^2 + y^2 = AB^2 = \text{constant}$$

or $2x\dot{x} + 2y\dot{y} = 0$. Also, $\dot{y} = -v$.

$\therefore\ \dot{x} = v_B = (-\dot{y})(y/x) = v \tan\alpha$.

29. 

The electric intensity has the same direction as $\vec{p}$ in the end-on position (A), and a direction opposite to $\vec{p}$ in the broadside-on position (B).

Now, $E = \frac{p}{r^3}\sqrt{3\cos^2\theta + 1}$.

At A, $E = E_1 = \frac{p}{r^3}\sqrt{3 \times 1 + 1}$; at B, $E = E_2 = \frac{p}{r^3}\sqrt{0 + 1}$.

30. When sound is incident on a receiver, it vibrates at the frequency of the incident sound. This is forced vibration.

31. $f = \frac{100 \text{ cm}}{16} = 6.25$ cm.

Maximum magnification is obtained when the image is formed at $D = 25$ cm.

Thus, $v = -25$ cm.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ gives $u = -5$ cm.

32. The maximum magnification $= 1 + \frac{D}{f} = 5$.

The minimum magnification $= \frac{D}{f} = 4$.

33. The flux linked with the coil does not depend upon the position of the axis of rotation. The induced emf depends only on the rate of change of this flux.

34. Applying Gauss's theorem to the points 1, 2, 3 gives the result $E_1 = E_2 = E_3$.

35. The total charge circulating in a coil is equal to

$\frac{\text{total change in flux}}{\text{total resistance}}$.

Here, reversing the magnetic field causes a change of flux by 2ϕ.

$\therefore\ Q = \frac{2\phi}{R}$.

36. For the first process, at constant volume,

$\Delta Q_1 = nC_V \, \Delta T = C_V(-150)$.

For the first process, at constant pressure,

$\Delta Q_2 = nC_p \, \Delta T = C_p(150)$.

$\therefore \; \Delta Q_1 + \Delta Q_2 = (C_p - C_V)150 = 150R$.

37. The cut-off wavelength appears when the entire energy of the incident electron gets converted to a photon. Hence, λ_c is the shortest possible wavelength. The K_β transition, from M shell to K shell, involves more energy than the K_α transition, from L shell to K shell. Hence, $\lambda_\beta < \lambda_\alpha$.

38. The initial energy stored in the capacitor is $\dfrac{Q^2}{2C}$.

When this energy gets transferred to the coil, carrying current i,

$\dfrac{1}{2}Li^2 = \dfrac{Q^2}{2C}$.

39. A constant force acting on a system undergoing SHM does not alter its time period.

40. $n = \dfrac{V}{2l}$ and $V \propto \sqrt{\dfrac{T}{M}}$. $\therefore \; \dfrac{T_1}{M_1} = \dfrac{T_2}{M_2}$.

41. Let the spheres have radii r_1 and r_2 ($r_1 > r_2$).

$$V_1 = \frac{4\pi r_1^2 \sigma}{4\pi\varepsilon_0 r_1} = \frac{r_1 \sigma}{\varepsilon_0}, \qquad V_2 = \frac{4\pi r_2^2 \sigma}{4\pi\varepsilon_0 r_2} = \frac{r_2 \sigma}{\varepsilon_0}$$

$\therefore \quad V_1 > V_2$.

$\therefore$ charge will flow from the larger to the smaller sphere.

Any flow of charge causes loss of energy and hence heat is produced.

42. Let u be the speed of projection and θ_1 and θ_2 be the angles of projection.

Let $\vec{r_1}$ and $\vec{r_2}$ be the position vectors of the two particles in a ground frame.

$$\vec{r_1} = (u\cos\theta_1)t\,\vec{i} + \left[(u\sin\theta_1)t - \frac{1}{2}gt^2\right]\vec{j}.$$

$$\vec{r_2} = (u\cos\theta_2)t\,\vec{i} + \left[(u\sin\theta_2)t - \frac{1}{2}gt^2\right]\vec{j}.$$

The position vector of one particle with respect to another is

$$\vec{r} = \vec{r_1} - \vec{r_2} = [u(\cos\theta_1 - \cos\theta_2)t]\,\vec{i} + [u(\sin\theta_1 - \sin\theta_2)t]\,\vec{j}$$

$$= at\,\vec{i} + bt\,\vec{j} = [a\,\vec{i} + b\,\vec{j}]t,$$

where a and b are constants.

43. No work is done in the processes AB and CD (constant volume).

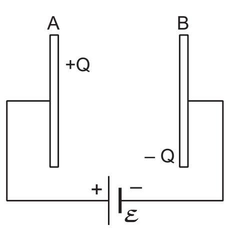
$W_{\rm DA} = RT_0 \ln 2$

$$W_{\rm BC} = R(2T_0)\ln\left(\frac{V_0}{2V_0}\right) = -2RT_0 \ln 2.$$

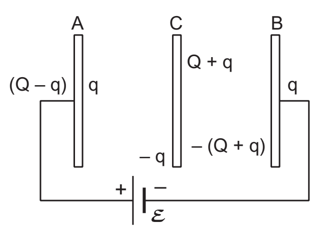
Total work done by the gas in the cycle $= W = -RT_0 \ln 2$.

As $\Delta U = 0$, $Q = W = -RT_0 \ln 2$, i.e., heat is given out.

44.



(a)
Initial



(b)
After plate C is introduced

Let C_0 = the capacitance of A + B system. Then, $Q = \mathcal{E}C_0$.

Also, the capacitance of A + C system = the capacitance of C + B system = $2C_0$.

In (b), $\dfrac{q}{2C_0} + \dfrac{Q+q}{2C_0} = \mathcal{E} = \dfrac{Q}{C_0}$

or $\quad q + Q + q = 2Q \quad$ or $\quad q = \dfrac{Q}{2}\cdot$

$$V_{A} - V_{C} = \frac{q}{2C_0} = \frac{Q}{4C_0}\cdot$$

$$V_{C} - V_{B} = \frac{Q+q}{2C_0} = \frac{Q + Q/2}{2C_0} = \frac{3Q}{4C_0}\cdot$$

45. $E = \dfrac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}\cdot$

Equivalent current $= i = \left(\dfrac{\omega}{2\pi}\right)Q.$

$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 Q\omega R^2}{4\pi (R^2 + x^2)^{3/2}}\cdot$$

$\therefore \quad \dfrac{B}{E} = \dfrac{\omega R^2}{xc^2}\cdot$

46. When S is open, C_1 and C_2 are in series, with equivalent capacitance $C = 2\ \mu F$.

$\therefore$ charge on C_1 = charge on $C_2 = 36\ \mu C$.

p.d. across the 3-Ω resistor = 6 V.

p.d. across $C_1 = \dfrac{36\ \mu C}{3\ \mu F} = 12$ V.

Hence, when S is closed, charges on C_1 and C_2 must change.

48. TIP In a head-on elastic collision between two particles, the kinetic energy becomes minimum and the potential energy becomes maximum at the instant when the two move with a common velocity. The momentum and the total energy are conserved at every instant.

Let m and u be the mass and initial velocity respectively of the first particle, $2m$ be the mass of the second particle, and v be the common velocity. Then,

$$\frac{1}{2}mu^2 = 3 \text{ J}.$$

$mu = (m + 2m)v \quad$ or $\quad v = \dfrac{u}{3}\cdot$

$\therefore$ the minimum KE of the system $= \frac{1}{2}(3m)\left(\frac{u}{3}\right)^2 = \left(\frac{1}{2}mu^2\right)\frac{1}{3} = 1$ J.

$\therefore$ the maximum PE of the system = 2 J.

50. $y = ax - bx^2$

or $\quad \dot{y} = a\dot{x} - 2bx\dot{x}$

or $\quad \ddot{y} = a\ddot{x} - 2bx\ddot{x} - 2b\dot{x}^2 = -g.$

Now, $\ddot{x} = 0$ as there is no acceleration in the x-direction.

$\therefore \quad \dot{x} = \sqrt{\frac{g}{2b}} =$ constant.

At $x = 0$, $\dot{y} = a\dot{x} = a\sqrt{\frac{g}{2b}}$.

At $x = 0$, $\tan\theta = \frac{\dot{y}}{\dot{x}} = a$ or $\theta = \tan^{-1}(a)$.

51. Let u = object distance, v = image distance.

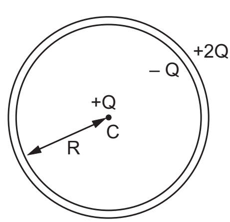
$x = |u| + v$ = the distance of the image from the fixed object.

The velocity of the image $= \dot{x}$.

x becomes minimum for $|u| = v = 2f$.

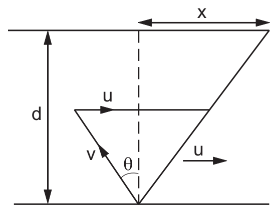
Hence, $\dot{x} = 0$.

53.



$-Q$ charge must be induced on the inner surface of the shell. Hence $+2Q$ charge appears on its outer surface.

54.



The time required for crossing $= t = \dfrac{d}{v\cos\theta}\cdot$

$x = (u - v\sin\theta)t = \dfrac{ud}{v}\sec\theta - d\tan\theta.$

For x to be minimum, $\dfrac{dx}{d\theta} = 0.$

This gives $\sin\theta = \dfrac{v}{u}\cdot$

□

2

Miscellaneous Questions—2

• *Assertion–Reason Type* •

Choose the correct option (a, b, c or d). Only one option is correct.

1. STATEMENT-1: A geostationary satellite must be located in the equatorial plane, i.e., at some point vertically above the equator.
STATEMENT-2: The only external force acting on an earth satellite is directed towards the centre of the earth.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

2. STATEMENT-1: In an elastic collision in one dimension between two bodies, neither of which was at rest before collision, total momentum remains the same before, during and after the collision.
STATEMENT-2: In an elastic collision in one dimension between two bodies, neither of which was at rest before collision, total kinetic energy remains the same before, during and after the collision.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

3. STATEMENT-1: The gravitational potential energy of a solid sphere is negative.
STATEMENT-2: Two masses attract each other.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

4. STATEMENT-1: The electrostatic potential energy of a sphere with uniformly distributed negative charge is positive.
STATEMENT-2: Two similar charges repel each other.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

5. STATEMENT-1: When an uncharged parallel-plate capacitor is charged by connecting it to a cell, the heat produced in the circuit is equal to the energy stored in the capacitor.
STATEMENT-2: The charge on a parallel-plate capacitor means the equal and opposite charges on its inner surfaces.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

6. STATEMENT-1: When a charged particle enters a magnetic field from outside, it cannot complete one rotation inside the field.
STATEMENT-2: The entry and exit of a charged particle into and out of a uniform magnetic field are symmetrical.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

7. STATEMENT-1: It is possible for a charged particle to move in a circular path around a uniformly charged long conductor.
STATEMENT-2: The electrostatic force on the moving particle is directed towards the conductor.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

8. STATEMENT-1: It is not possible for a charged particle to move in a circular path around a long straight conductor carrying current.
STATEMENT-2: The electromagnetic force on a moving particle is normal to its plane of rotation.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

9. STATEMENT-1: When two sounds of slightly different frequencies are heard together, periodic variations in intensity (called *beats*) are observed. Similar phenomenon is not observed when two lights of slightly different wavelengths reach a point together.

STATEMENT-2: Sound waves are longitudinal in nature, while light waves are transverse in nature.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

10. STATEMENT-1: When a sphere falls under gravity or moves up due to buoyancy forces in a fluid, its velocity becomes constant after some time.
STATEMENT-2: The force of viscosity is proportional to velocity.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

11. STATEMENT-1: When two sounds of equal frequencies and slightly different intensities are heard together, beats are heard.
STATEMENT-2: Beats are caused by alternate constructive and destructive interferences between two sounds.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

12. STATEMENT-1: It is necessary to define two molar heat capacities for a gas.
STATEMENT-2: Work is done by a gas when its volume changes.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

13. STATEMENT-1: The thermal conductivity of a rectangular slab of a material can be defined only under steady-state conditions, i.e., when the temperatures of the end faces do not change with time.
STATEMENT-2: The rate of conduction of heat through a rectangular slab is proportional to the difference in temperature between its end faces.
(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

14. STATEMENT-1: In high-quality optical devices, such as camera, binocular, periscope, etc., prisms are used in place of plane mirrors to reflect light.
STATEMENT-2: In plane mirrors, reflections occur both in the front and rear (silvered) surfaces.
(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

15. STATEMENT-1: Real images cannot be formed by reflection of light in a convex mirror.
STATEMENT-2: Parallel rays incident on a convex mirror must diverge after reflection.
(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

16. STATEMENT-1: When an electrical heater is switched on, the colour of the filament gradually changes from red to yellow to almost white.

STATEMENT-2: Wien's displacement law states that $\lambda_m T = b$ (constant).

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

17. STATEMENT-1: The specific heat capacity of a gas at constant pressure is always greater than its specific heat capacity at constant volume.

STATEMENT-2: Work is done by a gas when it expands.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

18. STATEMENT-1: When two identical strings stretched to slightly different tensions vibrate together, the loudness of the sound heard changes periodically.

STATEMENT-2: Interference can occur in all wave motions under suitable conditions.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

19. STATEMENT-1: A single lens will have two different focal lengths if the media on its two sides have different refractive indices.
STATEMENT-2: The focal length of a lens can be defined in two different ways.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

20. STATEMENT-1: When a closed organ pipe vibrates, the pressure of the gas at the closed end remains constant.
STATEMENT-2: In a stationary-wave system, displacement nodes are pressure antinodes, and displacement antinodes are pressure nodes.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

21. STATEMENT-1: When a coil is connected to a cell, no current flows through it initially.
STATEMENT-2: When a coil is connected to a cell, the initial emf induced in it is equal to the emf of the cell.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

22. STATEMENT-1: In a moving train, a small potential difference arises across the axles of the wheels due to the earth's magnetic field. This potential difference vanishes at the equator.

STATEMENT-2: At the equator, the earth's magnetic field is horizontal.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

23. STATEMENT-1: In an X-ray tube, the wavelengths of the characteristic X-rays depend on the metal used as target.
STATEMENT-2: Metals of large atomic numbers are best suited for the production of X-rays.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

24. STATEMENT-1: When α- and β-particles pass through external electric fields, β-particles are deflected much more than α-particles.
STATEMENT-2: β-particles have much larger velocities than α-particles.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

25. STATEMENT-1: The activity of a radioactive sample decreases linearly with time.
STATEMENT-2: The number of active nuclei present in a radioactive sample decreases exponentially with time.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

Answers

1. a	**2.** c	**3.** a	**4.** a	**5.** b
6. a	**7.** a	**8.** c	**9.** b	**10.** a
11. d	**12.** a	**13.** d	**14.** a	**15.** d
16. a	**17.** a	**18.** a	**19.** a	**20.** d
21. a	**22.** a	**23.** b	**24.** b	**25.** d

□

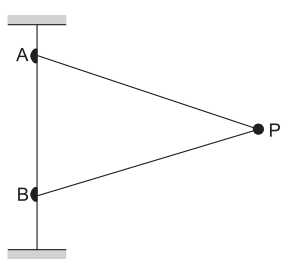
3

Miscellaneous Questions—3

• *Linked-Comprehension Type* •

This section contains a number of paragraphs. Based upon each paragraph, three multiple-choice questions have to be answered. Each question has four choices (a, b, c, d), out of which only one is correct.

- The particle P of mass m is attached to two light, rigid rods AP and BP of length l each. A and B are hinges on a fixed, vertical axis. The system APB can rotate freely about this axis. The angle ABP = the angle BAP = θ. The tensions in AP and BP are T_1 and T_2 respectively.



1. When the system is at rest, which of the following is not correct?

(a) At P, the direction of T_1 is from P to A.

(b) At P, the direction of T_2 is from B to P.

(c) The rods AP and BP together exert a net force and a net torque on AB.

(d) $T_1 \neq T_2$.

2. When the system is made to rotate about AB with angular velocity ω, which of the following is not correct?

(a) T_1 will always be greater than T_2.

(b) $T_1 - T_2 = m\omega^2 l$ for small values of ω.

(c) T_2 will become zero for $\omega^2 = g/l \cos\theta$.

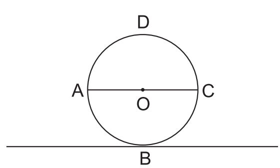
(d) The direction of T_2 will always be from P to B.

3. The system is now rotated by 90° so that AB becomes horizontal and P is located vertically below AB. What is the minimum horizontal velocity that must be imparted to P, normal to the plane of the figure, such that it moves in a complete circular path in a vertical plane with AB as the axis?

(a) $2\sqrt{gl \sin\theta}$ (b) $2\sqrt{gl \cos\theta}$

(c) $(5/2)\sqrt{gl \sin\theta}$ (d) None of these

- In a ring ABCD of radius *r*, the lower half ABC has mass *m* and the upper half ADC has mass 2*m*. In both parts, the masses are distributed evenly. The ring is initially at rest on a horizontal surface, as shown. O is the centre of the ring.



4. Let C_1 denote the centre of mass of the section ABC and C_2 denote the centre of mass of the section ADC. The distance C_1C_2 is equal to

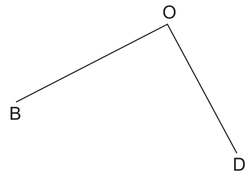
(a) r (b) $2r/3$ (c) $2\pi r/5$ (d) $4r/\pi$

5. The ring is now pushed very slightly and begins to roll on the horizontal surface without slipping. When it has made half a rotation, i.e., B is vertically above D, its angular velocity ω will be given by (where $\beta = g/\pi r$)

(a) $\omega^2 = 3\beta/2$ (b) $\omega^2 = 4\beta/3$

(c) $\omega^2 = 8\beta/5$ (d) $\omega^2 = 9\beta/4$

6.



The ring is now folded along the diameter AC, such that the plane of the section ABC is normal to the plane of the section ADC. (The angle BOD = 90°). It is then placed on a thin, fixed horizontal wire, i.e., the diameter AC lies along the wire. The angle made by DO with the vertical will now be

(a) $\tan^{-1}(2/3)$ (b) $\tan^{-1}(1/2)$

(c) 30° (d) 60°

- A biconvex lens made of material with refractive index n_2. The radii of curvatures of its left surface and right surface are R_1 and R_2. The media on its left and right have refractive indices n_1 and n_3 respectively. The first and second focal lengths of the lens are respectively f_1 and f_2.

7. The ratio, f_1/f_2, of the two focal lengths is equal to

(a) n_1/n_3 (b) $(n_1 - 1)/(n_3 - 1)$

(c) $(n_1 + 1)/(n_3 + 1)$ (d) $(n_2 - n_3)/(n_2 - n_1)$

8. Assume that $n_1 = n_3$. Which of the following statements is not correct?

(a) $f_1 = f_2$.

(b) f_2 is inversely proportional to $n_3 - 1$.

(c) If R_1 and R_2 are unequal, the focal length would depend on the direction in which light travels through the lens.

(d) f_1 may be negative if $n_1 > n_2$.

9. Assume that $R_1 = R_2$, $n_1 \neq n_3$. The ratio, f_1/f_2, of the two focal lengths is equal to

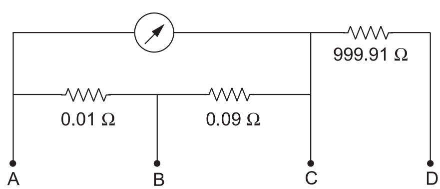
(a) 1 (b) n_1/n_3

(c) n_3/n_1 (d) $(n_3 - 1)/(n_1 - 1)$

- Electrical multimeters, or multitesters, are widely used by technicians when working with electrical and electronic circuits. In this instrument, a single milliammeter connected through different resistances is used to measure currents, potential differences and resistances over different ranges.

  Current always enters the multimeter at the same terminal, A, and then passes through the milliammeter as well as through other resistances, placed in series with or parallel to it. The choice of the terminal at which current leaves the instrument decides its role (ammeter, voltmeter, etc.) and its range (maximum current or voltage which it can measure).

  The milliammeter shown in the circuit has a coil (or internal) resistance of 0.9 Ω and gives the full-scale deflection for a current of 10 mA.



10. If A and B are used as the terminals of the multimeter, i.e., current enters at A and leaves at B, it will function as an ammeter of range

(a) 10 A (b) 1 A
(c) 100 mA (d) 10 mA

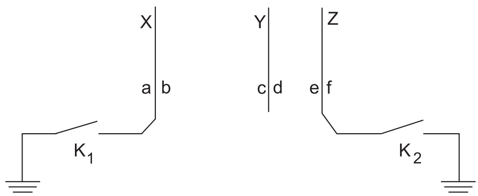
11. If A and C are used as the terminals of the multimeter, i.e., current enters at A and leaves at C, it will function as an ammeter of range

(a) 10 A (b) 1 A
(c) 100 mA (d) 10 mA

12. If A and D are used as the terminals of the multimeter, i.e., current enters at A and leaves at D, it will function as a voltmeter of which range?

(a) 1 V
(b) 10 V
(c) 100 V
(d) It will not function either as a voltmeter or as an ammeter or a milliammeter.

•



The figure shows three identical parallel conducting plates X, Y and Z. The separation between X and Y is 2*d*, and that between Y and Z is *d*. The six surfaces of the three plates are labeled *a*, *b*, *c*, *d*, *e* and *f*, as shown. The key K_1 can connect X to ground, and the key K_2 can connect Z to ground. Initially, both keys are open. Y has *Q* charge; X and Z have no charge.

13. If K_1 is closed and K_2 remains open, the charges on the surfaces *a*, *b*, *c*, *d*, *e* and *f* will be respectively

(a) 0, –*Q*, *Q*, 0, 0 and 0

(b) 0, –*Q*/2, *Q*/2, *Q*/2, –*Q*/2 and 0

(c) 0, –*Q*/3, *Q*/3, 2*Q*/3, –2*Q*/3 and 0

(d) –*Q*/3, *Q*/3, 2*Q*/3, –2*Q*/3, –*Q*/3 and *Q*/3

14. If K_2 is closed and K_1 remains open, the charges on the surfaces *a*, *b*, *c*, *d*, *e* and *f* will be respectively

(a) 0, 0, 0, *Q*, –*Q* and 0

(b) 0, –*Q*/3, *Q*/3, 2*Q*/3, –2*Q*/3 and 0

(c) 0, –2*Q*/3, 2*Q*/3, *Q*/3, –*Q*/ 3 and 0

(d) –*Q*/3, *Q*/3, *Q*/3, 2*Q*/3, –*Q*/3 and *Q*/3

15. If both K_1 and K_2 are closed, the charges on the surfaces *a*, *b*, *c*, *d*, *e* and *f* will be respectively

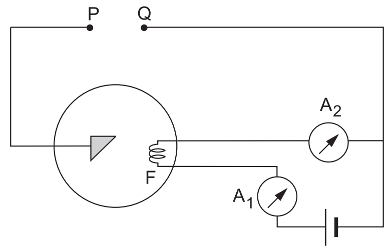
(a) 0, *Q*, –*Q*, 0, 0 and 0

(b) 0, 0, 0, *Q*, –*Q* and 0

(c) 0, –2*Q*/3, 2*Q*/3, *Q*/3, –*Q*/3 and 0

(d) 0, –*Q*/3, *Q*/3, 2*Q*/3, –2*Q*/3 and 0

•



The diagram shows the basic setup for the production of X-rays. A_1 and A_2 are two ammeters, reading 2.55 A and 2.566 A respectively. F is a filament which is also the cathode. The potential difference applied between P and Q is 50 000 V. Assume that all X-ray photons have the maximum possible energy and that one X-ray photon is emitted for every 100 electrons incident on the target. You may assume that the kinetic energy of the other electrons reappear as heat in the tube.

16. The number of X-ray photons produced per second is approximately

(a) 10^{12} (b) 10^{15}
(c) 10^{18} (d) 10^{21}

17. The momentum of each X-ray photon is approximately

(a) 3×10^{-17} kg m s^{-1} (b) 3×10^{-20} kg m s^{-1}
(c) 3×10^{-23} kg m s^{-1} (d) 3×10^{-26} kg m s^{-1}

18. The rate at which heat is produced in the X-ray tube is approximately

(a) 10 W (b) 40 W
(c) 200 W (d) 800 W

Answers

1. d	**2.** d	**3.** a	**4.** d	**5.** c
6. b	**7.** a	**8.** c	**9.** b	**10.** b
11. c	**12.** c	**13.** a	**14.** a	**15.** d
16. b	**17.** c	**18.** d		

□

4

Miscellaneous Questions—4

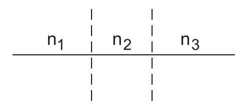
• *Matrix-Matching Type* •

Match the quantities in column A with those in column B, darkening the appropriate bubbles in the given 4×4 matrix.

1. Columns A and B describe some definitions, descriptions and symbolic representations used in current electricity.

Column A	*Column B*
(i) The reading of a voltmeter of finite resistance connected directly to a cell	(a) Less than the emf of the cell
(ii) The line integral of the electric intensity $(\vec{E})$ over a closed path, i.e., $\int \vec{E} \cdot \vec{dl}$, in a circuit which does not have a cell	(b) Greater than the emf of the cell
(iii) The potential difference between the terminals of a cell in which current flows in a direction opposite to its emf	(c) Zero
(iv) Current flowing through a cell whose emf is being measured by a potentiometer, in the position of balance	(d) Its emf divided by its internal resistance

2. All questions refer to the diagram given below. The dotted lines represent the two surfaces of a lens that may be plane or curved. R_1 and R_2 are the radii of curvature of the surfaces on the left and right respectively. The refractive index of the medium to the left of the lens is n_1, that of the material of the lens itself is n_2, and that of the medium to the right of the lens is n_3. The first and second focal lengths of the lens are f_1 and f_2 respectively.



Column A contains certain conditions and column B contains some results which may follow from such conditions.

Column A	***Column B***
(i) $n_1 = n_3$	(a) Both f_1 and f_2 are either positive or negative
(ii) $R_1 > 0$, $R_2 < 0$, with $n_2 < n_1, n_3$	(b) f_1 is positive but f_2 is negative
(iii) R_1 and R_2 have opposite signs, with $n_2 > n_1, n_3$	(c) $f_1 = f_2$
(iv) R_1 and R_2 are both negative, with $\vert R_1 \vert < \vert R_2 \vert$ and $n_2 > n_1 = n_3$	(d) Both f_1 and f_2 are negative

3. Interference can be observed in wave motions under suitable conditions. Consider only interference in light and sound waves, between two waves at a time.

Column A	***Column B***
(i) Requires coherent sources	(a) Interference in light and sound
(ii) Intensity may increase and decrease periodically	(b) Interference in light only
(iii) Only one real source can be used at a time	(c) Interference in sound only
(iv) Two different real sources may be used at a time	(d) Neither in sound nor in light

4. We identify and understand a sound on the basis of certain characteristics, given in column A. These characteristics depend on certain measurable parameters of the sound. These parameters and related units are given in column B.

Column A	***Column B***
(i) Loudness	(a) Waveform
(ii) Pitch	(b) Decibel
(iii) Tone	(c) Frequency
(iv) Intensity	(d) Watt/metre^2

5. In the production of X-rays by a Coolidge tube, the changes which can be made in its operation are listed in column A. The possible effects of such changes are listed in column B.

Column A	***Column B***
(i) Change in the current flowing through the filament (emitter or cathode)	(a) Harder or softer X-rays are produced
(ii) Change in the potential difference applied across the tube	(b) Wavelengths of the characteristic X-rays change
(iii) Change in the metal used as the target	(c) Intensity of the X-rays change
(iv) Rotating the tube from a horizontal position to a vertical position	(d) cut of wavelengths changes

6. In the Young's double-slit experiment, some variations which can be made are listed in column A. The possible effects of such changes are listed in column B.

Column A	***Column B***
(i) Lights of two different wavelengths are incident on the double slits	(a) Completely dark fringes are not formed on the screen

(ii) White light is incident on the slits	(b) The entire visible spectrum is formed on both sides of the centre of the screen
(iii) The two slits are of unequal width	(c) Two fringe patterns overlap on the screen
(iv) A thin film is placed in front of one of the slits	(d) Bright white fringe is formed at the centre of the screen

7. Some properties and observations related to sound are given in column A, while the processes, physical quantities and units related to them are given in column B.

Column A	*Column B*
(i) Loudness	(a) Interference
(ii) Identifying the source of a sound	(b) Decibel
(iii) Beats	(c) Waveform
(iv) Coherent sources	(d) Energy crossing unit area normally per unit time

8. Some relations and laws related to fluids are given in column A, while the physical reasons behind them are given in column B.

Column A	*Column B*
(i) Stokes' law	(a) Surface potential energy
(ii) Equation of continuity	(b) Force of viscosity
(iii) Bernoulli's theorem	(c) Conservation of mass
(iv) Velocity of efflux	(d) Conservation of energy

9. In the motion of charged particles through a region containing either only uniform electric field ($\vec{E}$) or only uniform magnetic field ($\vec{B}$) or both, certain properties are listed in column A and their possible reasons or explanations are given in column B. (Here, $\vec{v}$ denotes the velocity of the particle and *force* means the force acting on the particle.)

Column A	***Column B***
(i) $\vec{v}$ cannot remain constant in $\vec{E}$ only	(a) $\vec{v} = \vec{E}/\vec{B}$
(ii) $\vec{v}$ may remain constant if only $\vec{B}$ is present	(b) The force acting on the particle must always be normal to its velocity
(iii) $\vec{v}$ must remain constant in if only $\vec{B}$ is present	(c) The force acting on the particle is of constant magnitude
(iv) $\vec{v}$ may remain constant if both $\vec{E}$ and $\vec{B}$ are present	(d) There must be some component of the force acting on the particle in the direction of its motion at some time

Answers

1.

	a	b	c	d
(i)	●	○	○	○
(ii)	○	○	○	○
(iii)	○	●	○	○
(iv)	○	○	●	○

2.

	a	b	c	d
(i)	○	○	●	○
(ii)	○	○	○	●
(iii)	●	○	○	○
(iv)	○	○	○	●

3.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	●	○
(iii)	○	●	○	○
(iv)	○	○	●	○

4.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	●	○
(iii)	●	○	○	○
(iv)	○	○	○	●

5.

	a	b	c	d
(i)	○	○	●	○
(ii)	●	○	○	●
(iii)	○	●	○	○
(iv)	○	○	○	○

6.

	a	b	c	d
(i)	○	○	●	○
(ii)	○	●	○	●
(iii)	●	○	○	○
(iv)	○	○	○	○

7.

	a	b	c	d
(i)	○	●	○	●
(ii)	○	○	●	○
(iii)	●	○	○	○
(iv)	○	○	○	○

8.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	●	○
(iii)	○	○	○	●
(iv)	○	○	○	●

9.

	a	b	c	d
(i)	○	○	○	●
(ii)	○	○	○	○
(iii)	○	●	○	○
(iv)	●	○	○	○

□

Part 9

Practice Test Papers

Practice Test Papers

In the following pages, six model test papers are provided with the following objectives:

(a) to assess your preparation,

(b) to develop your skills of time management, and

(c) to familiarize you with the actual examination-hall situation.

• *Instructions* •

The time for answering each test paper is one hour. You may attempt questions in any order within this time limit. Use a separate blank sheet handy for your rough work. Do not use calculators, log tables or slide rules.

Each test paper contains 24 questions. There is negative marking. For every two wrong answers, the marks of one correctly answered question will be deducted.

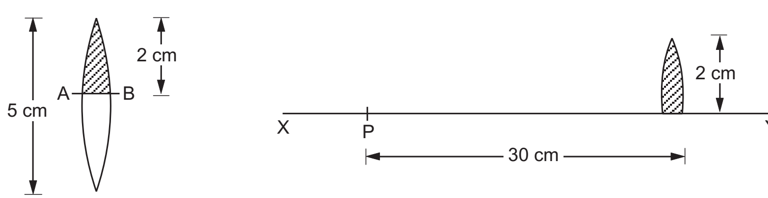
1

Practice Worksheet—1

• Straight-Objective Type •

This section contains seventeen multiple-choice questions numbered 1–17. Each question has four choices (a, b, c and d), out of which only one is correct.

1.



A converging lens of 20 cm focal length and 5 cm diameter is cut along the line AB. The part of the lens shown shaded in the diagram is now used to form an image of a point object P placed 30 cm away from it on the line XY, which is perpendicular to the plane of the lens. The image of P will be formed

(a) on XY

(b) 1 cm below XY

(c) 1.5 cm below XY

(d) 0.5 cm above XY

2. A hydrogen atom in an excited state emits a photon which has the longest wavelength of the Paschen series. Further emissions from the atom cannot include the

(a) longest wavelength of the Lyman series

(b) second-longest wavelength of the Lyman series

(c) longest wavelength of the Balmer series

(d) second-longest wavelength of the Balmer series

3. The two ends of a long cylinder containing gas are maintained at different temperatures. Which of the following quantities will have the same value throughout the cylinder?

(a) Pressure

(b) Density

(c) The ratio of pressure to density

(d) The number of molecules per unit volume

4. Two conducting spheres of radii r and $3r$ initially have charges $3q$ and q respectively. Their separation is much larger than their radii. If they are joined by a conductor of high resistance, the force between them will

(a) increase continuously

(b) decrease continuously

(c) first increase and then decrease

(d) first decrease and then increase

5. The headlight of a train of length 200 m is switched on when it starts from rest with an acceleration of 0.5 m s^{-2}. After 20 s, its tail-light is switched on. An observer in a frame moving with a constant velocity parallel to the railway track observes that the two events occur at the same place. The velocity of this frame is

(a) 5 m s^{-1} in a direction opposite to the train's motion

(b) 10 m s^{-1} in a direction opposite to the train's motion

(c) 5 m s^{-1} in the same direction as the train's motion

(d) 10 m s^{-1} in the same direction as the train's motion

6. The xz-plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A to B. Its directions in the two media are given by the unit vectors $\hat{r}_A = a\vec{i} + b\vec{j}$ and $\hat{r}_B = \alpha\vec{i} + \beta\vec{j}$ respectively, where $\vec{i}$ and $\vec{j}$ are unit vectors in the x- and y-directions. Then,

(a) $\mu_1 a = \mu_2 \alpha$ (b) $\mu_1 \alpha = \mu_2 a$

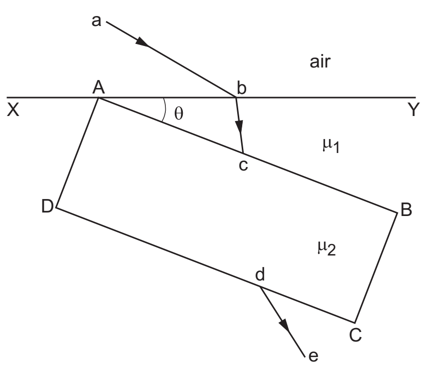
(c) $\mu_1 b = \mu_2 \beta$ (d) $\mu_1 \beta = \mu_2 b$

7. A thin uniform rod of length l is pivoted at its upper end. It is free to swing in a vertical plane. Its time period for oscillations of small amplitude is

(a) $2\pi\sqrt{l/g}$ (b) $2\pi\sqrt{2l/3g}$

(c) $2\pi\sqrt{3l/2g}$ (d) $2\pi\sqrt{l/2g}$

8.



A ray of light ab passing through air enters a liquid of refractive index μ_1 at the boundary XY. In the liquid, the ray is shown as bc. The angle between ab and bc (angle of deviation) is δ. The ray then passes through a rectangular slab ABCD of refractive index μ_2 ($\mu_2 > \mu_1$) and emerges from the slab as the ray de. The angle between XY and AB is θ. The angle between ab and de is

(a) δ (b) $\delta + \theta$

(c) $\delta + \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$ (d) $\delta + \theta - \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

9. When unit mass of water boils to become steam at 100°C, it absorbs Q amount of heat. The densities of water and steam at 100°C are ρ_1 and ρ_2 respectively, and the atmospheric pressure is p_0. The increase in the internal energy of the water is

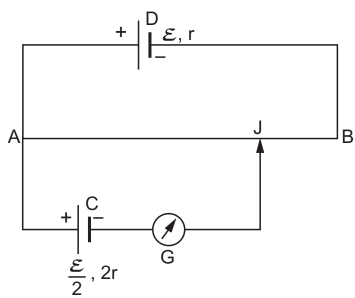
(a) Q

(b) $Q + p_0\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$

(c) $Q + p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$

(d) $Q - p_0\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

10.



In the figure, the potentiometer wire AB of length L and resistance $9r$ is joined to the cell D of emf $\mathcal{E}$ and internal resistance r. The emf of the cell C is $\mathcal{E}/2$ and its internal resistance is $2r$. The galvanometer G will show no deflection when the length AJ is

(a) $\frac{4L}{9}$

(b) $\frac{5L}{9}$

(c) $\frac{7L}{18}$

(d) $\frac{11L}{18}$

11. A solid sphere of radius R rests on a smooth horizontal surface. It receives a horizontal impulse at a height h above the surface. If the sphere is to begin rolling without slipping at once, h must be equal to

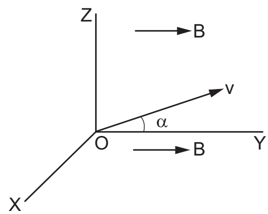
(a) $\frac{5R}{3}$

(b) $\frac{5R}{4}$

(c) $\frac{7R}{5}$

(d) $\frac{3R}{2}$

12.



In a region of space, a uniform magnetic field B exists in the y-direction. A proton is fired from the origin, with its initial velocity v making a small angle α with the y-direction in the yz-plane. In the subsequent motion of the proton,

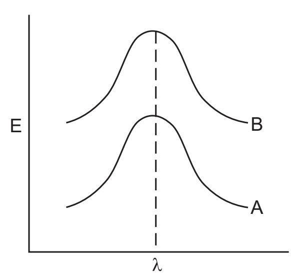
(a) its x-coordinate can never be positive

(b) its x- and z-coordinates cannot both be zero at the same time

(c) its z-coordinate can never be negative

(d) its y-coordinate will be proportional to the square of its time of flight

13.

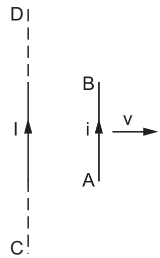


Two bodies A and B have the same surface area and mass. The bodies have absolute temperatures T_A and T_B, emissivities e_A and e_B, and specific heat capacities s_A and s_B. The intensity, E, of radiation near a given wavelength is shown plotted against the wavelength, λ, of radiation for both the bodies. Which of the following is possible?

(a) $T_A = T_B, e_A = e_B, s_A \neq s_B$ (b) $T_A = T_B, e_A \neq e_B, s_A = s_B$

(c) $T_A \neq T_B, e_A = e_B, s_A = s_B$ (d) $T_A \neq T_B, e_A = e_B, s_A \neq s_B$

14.



A short conductor AB is held parallel to a long thin conductor CD. They carry currents in the same direction. AB is moved (by an external agent) with a constant velocity v, away from CD, in the plane containing AB and CD. AB remains parallel to CD during the motion. How does the power (P) delivered by the external agent depend on time t?

(a) $P \propto t$.

(b) $P \propto \frac{1}{t}$.

(c) $P \propto \frac{1}{\sqrt{t}}$.

(d) P is independent of t.

15. An ideal gas at room temperature has 10^5 molecules per cubic centimetre. Its pressure is of the order of

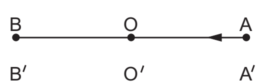
(a) 10^{-15} atm

(b) 10^{-10} atm

(c) 10^{-5} atm

(d) 1 atm

16.



Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time

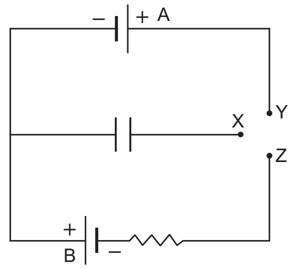
(a) $T/8$

(b) $3T/8$

(c) $T/6$

(d) $4T/3$

17.



In the circuit shown above, the cells are ideal and of equal emfs, the capacitance of the capacitor is C and the resistance of the resistor is R. X is first joined to Y and then to Z. After a long time, the total heat produced in the resistor will be

(a) equal to the energy finally stored in the capacitor

(b) half the energy finally stored in the capacitor

(c) twice the energy finally stored in the capacitor

(d) four times the energy finally stored in the capacitor

• Assertion–Reason Type •

This section contains three questions numbered 18–20. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

18. STATEMENT-1: A real image is formed when two rays, starting from a point, and moving along different paths, meet again at some other point.

STATEMENT-2: A real image can never be formed by reflection in a convex mirror or refraction in a concave lens.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

19. STATEMENT-1: Beats will be heard if a tuning fork is sounded along with a sonometer wire with the same fundamental frequency as the tuning fork.

STATEMENT-2: Beats are caused by alternate constructive and destructive interference between two sounds.

(a) Statement-1 is True, Statement- 2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

20. STATEMENT-1: The Doppler effect occurs in all wave motions.

STATEMENT-2: The Doppler effect can be explained by the principle of superposition of waves.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

• *Linked-Comprehension Type* •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (21–23) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- The results obtained in electrostatics using Gauss's law can in some cases be used to obtain results in gravitation, and vice versa. This is because both follow the inverse-square law of distance. We use the following three rules:
  (1) For charge in electrostatics, the corresponding quantity in gravitation is mass.
  (2) Electric intensity of electrostatics corresponds to gravitational intensity, or acceleration due to gravity, in gravitation.
  (3) The constant $k = (4\pi\varepsilon_0)^{-1}$ of electrostatics corresponds to $-G$ in gravitation.

21. The gravitational intensity near a thin infinite sheet with mass σ per unit area has the magnitude

(a) $4\pi G\sigma$ (b) $(1/4\pi)G\sigma$

(c) $2\pi G\sigma$ (d) $2\pi\sigma/G$

22. The gravitational intensity at a distance r from a long thread having λ mass per unit length has the magnitude

(a) $\lambda G/r$ (b) $2\lambda G/r$

(c) $\lambda G/2r$ (d) $\lambda G/2\pi r$

23. The acceleration due to gravity at the surface of the earth is g, and at a distance r from the centre of the earth is gr/R, where R is the radius of the earth. Inside a uniformly charged sphere of radius R and charge Q, the electric intensity at a distance r $(r < R)$ from the centre has the magnitude

(a) $kQ\{1 - (r/R)^2\}$ (b) kQr/R^3

(c) kQR/r^3 (d) $kQ(R - r)/R^3$

• *Matrix-Matching Type* •

Match the situations described in column A with their possible results in column B, and indicate your answers by darkening appropriate bubbles in the 4 × 4 matrix given in the answer sheet.

24. Light rays are incident on devices which may cause either reflection or refraction or both. The natures of the incident light and the devices are described in column A. Some possible results of this on the rays are given in column B.

Column A	*Column B*
(i) A ray of white light is incident on one face of an equilateral glass prism	(a) Divergent beam
(ii) A ray of white light is incident at an angle on a thick glass sheet	(b) Total internal reflection

(iii) A ray of white light passes from an optically denser medium to an optically rarer medium	(c) Lateral shift
(iv) A parallel beam of monochromatic light passes symmetrically through a glass sphere	(d) Dispersion

Answers

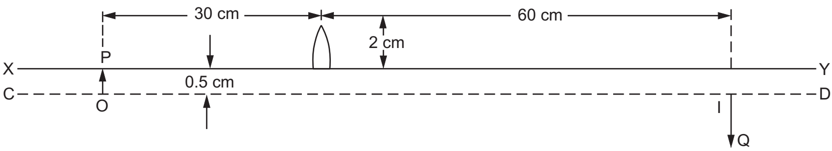
1. c	**2.** d	**3.** a	**4.** c	**5.** a
6. a	**7.** b	**8.** a	**9.** b	**10.** b
11. c	**12.** a	**13.** b	**14.** b	**15.** a
16. b	**17.** d			
18. c	**19.** d	**20.** c		
21. c	**22.** a	**23.** b		

24.

	a	b	c	d
(i)	●	●	○	●
(ii)	○	○	●	○
(iii)	○	○	○	●
(iv)	○	○	○	○

Hints and Solutions to Selected Questions

1.



When a part of a lens is used to form an image, it forms the same image as the whole lens. Also, the optic axis of this part is the same as the optic axis of the entire lens. Here, CD is the optic axis of the original lens and hence also of the part of the lens used here. P will then behave as an object placed 0.5 cm away from CD, i.e., as an object OP of height 0.5 cm.

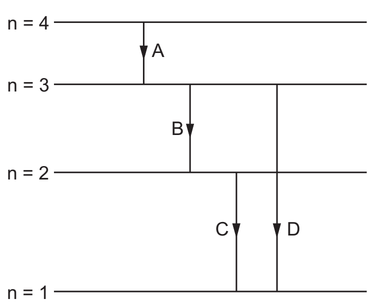
Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get $v = 60$ cm.

Magnification, $M = \left|\frac{v}{u}\right| = 2$ for the image IQ.

Thus $\frac{IQ}{OP} = 2$ or $IQ = 1$ cm.

Q is the image of P, formed 1 cm below CD or 1.5 cm below XY.

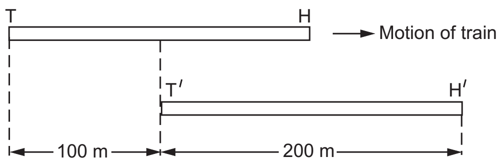
2.



The photon emitted comes from the transition A. Further transitions possible are B (longest λ of the Balmer series), C (longest λ of the Lyman series) and D (second-longest λ of the Lyman series).

4. Charge will flow from the smaller to the larger sphere till they reach the same potential. As their capacitances are proportional to their radii, charge $2q$ will be transferred. The system will pass through the condition when the charges are equal. The force of interaction becomes maximum at this point.

5.



The distance travelled by the train in 20 s is

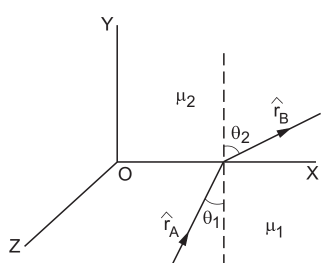
$$\frac{1}{2} \times (0.5\ \text{m s}^{-2}) \times (20\ \text{s})^2 = 100\ \text{m}.$$

$\therefore$ the distance between the two events H and T′ is 100 m.

The observer has to move 100 m in 20 s in a direction opposite to that of the train.

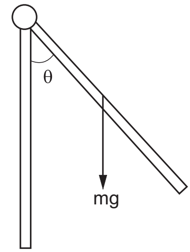
6. $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2.$

$$\sin\theta_1 = \frac{a}{\sqrt{a^2 + b^2}} = a.$$



$$\sin\theta_2 = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} = \alpha.$$

$\therefore \quad \mu_1 a = \mu_2 \alpha.$



7. $T = mg\left(\frac{l}{2}\sin\theta\right) \simeq \frac{1}{2}mgl\theta = -I\alpha$

or $\quad \frac{1}{2}mgl\theta = -\frac{1}{3}ml^2\alpha$

or $\quad \alpha = -\left(\frac{3g}{2l}\right)\theta.$

Put $\Omega^2 = \frac{3g}{2l}\cdot$

$\therefore \quad \alpha = -\Omega^2\theta.$

This represents an angular SHM with the time period

$$\frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{2l}{3g}}\cdot$$

8. When a ray of light passes through a slab with parallel faces, the emergent ray is parallel to the incident ray.

9. The work done by unit mass of water during expansion to form steam at constant pressure p_0 is

$$W = p_0\,\Delta V = p_0(V_{\text{steam}} - V_{\text{water}}) = p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\cdot$$

$$\Delta U = Q - W = Q - p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\cdot$$

10. In the position of balance,

current in AB is $i = \frac{\mathcal{E}}{10r}\cdot$

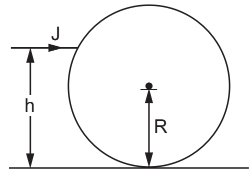
p.d. across AB is $i \times 9r = \frac{9\mathcal{E}}{10}\cdot$

p.d. per unit length of AB is $\frac{9\mathcal{E}}{10L}\cdot$

For balance,

$$\frac{\mathcal{E}}{2} = (\text{AJ})\,\frac{9\mathcal{E}}{10L} \quad \text{or} \quad \text{AJ} = \frac{5L}{9}\cdot$$

11.



Let J be the impulse, m the mass of the sphere, v the linear velocity and ω the angular velocity just after the impact.

$J = mv$ and $v = R\omega$ for pure rolling.

$$J(h - R) = I\omega = \left(\frac{2}{5}mR^2\right)\frac{v}{R} = \frac{2}{5}mvR = \frac{2}{5}JR$$

or $\quad 5(h - R) = 2R \quad$ or $\quad 5h = 7R \quad$ or $\quad h = \frac{7}{5}R.$

13. The wavelength at which the intensity of radiation is the maximum depends only on the temperature. As this wavelength is the same for both, their temperatures must be equal. The difference in intensities can then be due only to a difference in their emissivities.

14. The distance between the conductors is $x = vt$.

The magnetic field due to CD on AB is $= B = \frac{\mu_0 I}{4\pi x}\cdot$

The force on AB is $F = Bil$.

Power, $P = Fv = Bilv = \left(\frac{\mu_0 I}{4\pi x}\right) ilv = \frac{\mu_0 Iilv}{4\pi vt} = \frac{\mu_0 Iil}{4\pi}\cdot\frac{1}{t}\cdot$

$\therefore \quad P \propto \frac{1}{t}\cdot$

15. $p = nkT = (10^5 \times 10^6\ \text{m}^{-3})\ (1.38 \times 10^{-23}\ \text{J K}^{-1})\ (300\ \text{K})$

$$\simeq 10^{-10}\ \text{N m}^{-2} = 10^{-10}\ \text{Pa} \simeq 10^{-15}\ \text{atm}.$$

16. For the particle starting from A,

$$x_1 = a\cos\omega t.$$

For the particle starting from O′ and moving to the left,

$$x_2 = -a\sin\omega t.$$

The particles will cross when

$$x_1 = x_2 \quad \text{or} \quad -a\sin\omega t = a\cos\omega t$$

$$\text{or} \quad \tan\omega t = -1 \quad \text{or} \quad \omega t = \frac{3\pi}{4}$$

$$\text{or} \quad \left(\frac{2\pi}{T}\right)t = \frac{3\pi}{4} \quad \text{or} \quad t = \frac{3T}{8}.$$

17. The charge on the capacitor will change from $-\mathcal{E}C$ to $+\mathcal{E}C$, where $\mathcal{E}$ = emf of each cell.

$\therefore$ total charge flowing through the cell B is $2\mathcal{E}C$.

$\therefore$ total energy supplied by B is $2\mathcal{E}^2C$.

The energy finally stored in the capacitor is $E = \frac{1}{2}\mathcal{E}^2C$.

$\therefore$ heat produced $= 2\mathcal{E}^2C = 4E$.

□

2

Practice Worksheet—2

• *Straight-Objective Type* •

This section contains seventeen multiple-choice questions numbered 1–17. Each question has four choices (a, b, c and d), out of which only one is correct.

1. A converging glass lens with a refractive index of 1.5 has a focal length of f in air. When it is completely immersed in a liquid of refractive index 2, its focal length and nature will be respectively

(a) $2f$ and converging (b) $3f$ and converging

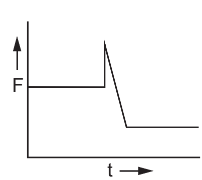
(c) $2f$ and diverging (d) $3f$ and diverging

2. A large hollow metal sphere of radius R has a small opening at the top. Small drops of mercury each of radius r and charged to a potential V fall into the sphere. The potential of the sphere becomes V' after N drops fall into it. Then,

(a) $V' < V$ for all N (b) $V' = V$ for $N = 1$

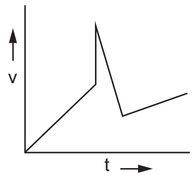
(c) $V' = V$ for $N = R/r$ (d) $V' = V$ for $N = (R/r)^{1/3}$

3.

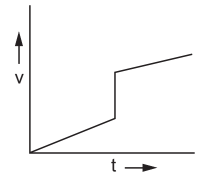


The force *F* acting on a particle plotted against time *t* is shown in the figure given. Its velocity *v* is plotted against *t* in the following figures. Which of these represents the resulting curve best?

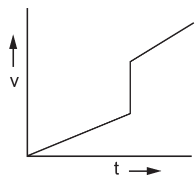
(a)



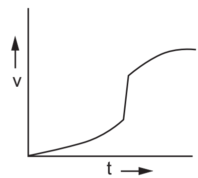
(b)



(c)



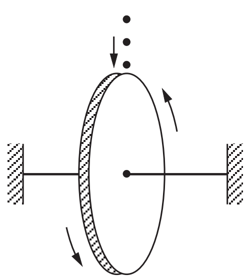
(d)



4. When a body is placed in surroundings at a constant temperature of 20°C and heated by a 10-W heater, its temperature remains constant at 40°C. If the temperature of the body is now raised from 20°C to 80°C in 5 min at a uniform rate, the total heat it will lose to the surroundings will be

(a) 3000 J (b) 3600 J (c) 4500 J (d) 5400 J

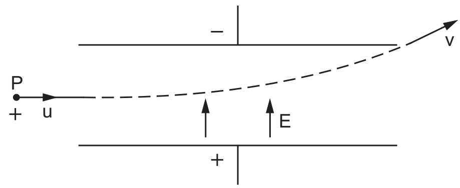
5.



A disc of mass m_0 rotates freely about a fixed horizontal axis passing through its centre. A thin cotton pad is fixed to its rim, which can absorb water. The mass of water dripping onto the pad per unit time is μ. After what time will the angular velocity of the disc get reduced to half its initial value?

(a) $\frac{2m_0}{\mu}$ (b) $\frac{3m_0}{\mu}$ (c) $\frac{m_0}{\mu}$ (d) $\frac{m_0}{2\mu}$

6.



A charged particle P enters the region between two parallel plates with a velocity u in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v. Taking α as a constant, v will depend on u as

(a) $v = \alpha u$

(b) $v = \sqrt{u^2 + \alpha u}$

(c) $v = \sqrt{u^2 + \frac{\alpha}{u}}$

(d) $v = \sqrt{u^2 + \frac{\alpha}{u^2}}$

7. A spaceship orbits the earth at a constant speed along a circular path. When an astronaut inside the spaceship releases an object, it does not move away from him. Which of the following is the most accurate reason for this?

(a) The astronaut and the object move along the same circular path due to the earth's gravitational pull.

(b) An object moving in a circular path round the earth experiences no gravitational pull.

(c) The gravitational forces on the object due to the spaceship exactly balance the gravitational pull on it due to the earth.

(d) The gravitational pull on the object due to the earth is very weak at a large distance from the earth.

8. The radius of gyration of a square plate of side length l about a diagonal is

(a) $\frac{l}{3}$

(b) $\frac{l}{3\sqrt{2}}$

(c) $\frac{l}{2\sqrt{3}}$

(d) $\frac{l}{6}$

9. The total energy of a hydrogen atom in its ground state is –13.6 eV. If the potential energy in the first excited state is taken as zero then the total energy in the ground state will be

(a) –3.4 eV (b) 3.4 eV
(c) –6.8 eV (d) 6.8 eV

10. When x amount of heat is given to a gas at constant pressure, it performs $x/3$ amount of work. The average number of degrees of freedom per molecule of the gas is

(a) 3 (b) 4
(c) 5 (d) 6

11. In a coaxial cable, a thin straight conductor is fixed along the axis of an outer hollow conductor. The two carry equal currents flowing in opposite directions. Let X denote the region between the two conductors and Y denote the region outside the outer conductor. The magnetic field is zero in

(a) Y but not in X (b) X but not in Y
(c) both X and Y (d) neither X nor Y

12. At ordinary temperatures, the molecules of a diatomic gas have only translational and rotational kinetic energies. At high temperatures, they may also have vibrational energy. As a result of this, compared to lower temperatures, a diatomic gas at higher temperatures will have

(a) lower molar heat capacity
(b) higher molar heat capacity
(c) lower isothermal compressibility
(d) higher isothermal compressibility

13. A parallel-plate capacitor containing a dielectric slab is connected to a cell. The slab is then taken out of the capacitor slowly. Disregard the forces of gravity and friction. For this process, which of the following statements is incorrect?

(a) The external agent pulling the slab out will have to perform some work.
(b) The potential energy of the capacitor will decrease.

(c) The cell will receive some energy.

(d) The work done on or by the external agent will be equal to the energy supplied or absorbed by the cell.

14. A car is initially at rest, 330 m away from a stationary observer. It begins to move towards the observer with an acceleration of 1.1 m s^{-2}, sounding its horn continuously. 20 s later, the driver stops sounding the horn. The velocity of sound in air is 330 m s^{-1}. The observer will hear the sound of the horn for a duration of

(a) 20 s
(b) 21 s
(c) $20\frac{2}{3}$ s
(d) $19\frac{1}{3}$ s

15. A converging lens of focal length f is placed just above a water surface, parallel to the surface, without touching it. A point source, S, of light is placed inside the water, vertically below the lens, at a depth f from it. This arrangement will produce

(a) a parallel beam of light emerging from the lens
(b) a real image of S in air
(c) a virtual image of S in water
(d) a virtual image of S in air

16. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross section of the tank is N times the area of the hole, where N is a large number. Neglect the mass of the tank itself. The initial acceleration of the tank is

(a) $\frac{g}{2N}$
(b) $\frac{g}{\sqrt{2N}}$
(c) $\frac{g}{N}$
(d) $\frac{g}{2\sqrt{N}}$

17. A string of length l, fixed at both ends, vibrates in its fundamental mode. Transverse waves can travel along the string with a velocity V. Sound waves travel in air with a velocity v. The wavelength, in air, of the sound produced by the string is

(a) $\frac{2lv}{V}$
(b) $\frac{2lV}{v}$
(c) $\frac{lV}{v}$
(d) $2l$

• Assertion–Reason Type •

This section contains three questions numbered 18–20. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

18. STATEMENT-1: Any process which occurs very rapidly is likely to be adiabatic.
STATEMENT-2: Exchange of heat between a system and its surroundings is not instantaneous.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

19. STATEMENT-1: An open organ pipe can be used as a musical instrument but not a closed organ pipe.
STATEMENT-2: The fundamental frequency of an open organ pipe is twice the fundamental frequency of a closed organ pipe of the same length.

(a) Statement -1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

20. STATEMENT-1: Electric or magnetic lines of force do not intersect.
STATEMENT-2: The tangent drawn to a line of force gives the direction of the intensity at that point.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

• Linked-Comprehension Type •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (21–23) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- A parallel-plate capacitor is clamped to a stand such that its plates are vertical. It remains connected to a cell through a centre-zero galvanometer. A metal sheet which is parallel to the plates of the capacitor and whose thickness is slightly less than the separation between these plates is now allowed to fall under gravity though the plates, without touching them.

21. Which of the following correctly describes the deflection(s) of the galvanometer?

(a) It will show a constant deflection to one side only, as long as the sheet is passing through the plates.

(b) It will show a variable deflection to one side only, as long as the sheet is passing through the plates.

(c) It will show deflections first to one side and then to the other.

(d) It will not show any deflections if the metal sheet does not have any charge on it.

22. Which of the following correctly describes the motion of the metal plate as it passes through the plates of the capacitor?

(a) It will move with a constant acceleration equal to g the acceleration due to gravity.

(b) It will move with a constant acceleration less than g.

(c) It will move with a constant acceleration greater than g.

(d) It will move with variable acceleration less than the acceleration due to gravity.

23. In the entire process of the metal sheet passing through the plates, which of the following statements will not be correct?

(a) Its gain in kinetic energy will be less than its loss in gravitational potential energy.
(b) Some heat will be produced in the electrical circuit.
(c) The cell will lose some electrical energy but never gain any energy.
(d) The current through the cell will reverse in direction at some point.

• *Matrix-Matching Type* •

24. Certain radioactive processes are listed in column A, and some properties and processes related to them are listed in column B.

Column A	*Column B*
(i) α decay	(a) Increase in atomic number
(ii) β decay	(b) Decrease in atomic number
(iii) γ emission	(c) No change in atomic number
(iv) K capture	(d) Emission of energy

Answers

1. c	**2.** a	**3.** b	**4.** c	**5.** d
6. d	**7.** a	**8.** c	**9.** c	**10.** b
11. a	**12.** b	**13.** d	**14.** a	**15.** c
16. c	**17.** a			
18. a	**19.** b	**20.** a		
21. c	**22.** d	**23.** c		

24.

	a	b	c	d
(i)	○	●	○	●
(ii)	●	○	○	●
(iii)	○	○	●	●
(iv)	○	●	○	○

Hints and solutions to Selected Questions

1. Let $k = \frac{1}{R_1} - \frac{1}{R_2}$.

$\therefore \quad \frac{1}{f} = (1.5 - 1)k$

and $\frac{1}{f'} = \left(\frac{1.5}{2} - 1\right)k = \left(\frac{-0.5}{2}\right)\frac{2}{f} = -\frac{0.5}{f}$.

$\therefore \quad f' = -2f.$

2. Capacitance of each mercury drop is $4\pi\varepsilon_0 r$.

Charge on each mercury drop is $4\pi\varepsilon_0 rV$.

Total charge on N drops is $Q = 4\pi\varepsilon_0 rVN$.

Capacitance of the hollow sphere is $4\pi\varepsilon_0 R$.

When it acquires Q charge, its potential is $V' = \frac{Q}{4\pi\varepsilon_0 R}$.

For $V' = V$, $\quad 4\pi\varepsilon_0 RV = 4\pi\varepsilon_0 rVN \quad$ or $\quad N = \frac{R}{r}$.

4. Let θ be the temperature of the body. It is given that the temperature of the surroundings is $\theta_0 = 20°\text{C}$.

Rate of loss of heat, $\frac{dQ}{dt} = k(\theta - \theta_0)$.

For $\theta = 40°\text{C}$,

$$\frac{dQ}{dt} = 10\ \text{W} = k(40°\text{C} - 20°\text{C}) \quad \text{or} \quad k = \frac{10\ \text{W}}{20°\text{C}} = \frac{1}{2}\ \text{W/°C}.$$

When the temperature of the body is raised uniformly, its temperature is given by

$$\theta = 20°\text{C} + \left(\frac{80°\text{C} - 20°\text{C}}{300\ \text{s}}\right)t = 20°\text{C} + \left(\frac{1°\text{C}}{5\ \text{s}}\right)t, \text{ where } t = \text{time}.$$

$$\therefore \quad \frac{dQ}{dt} = k(\theta - \theta_0) = \left(\frac{1}{2}\ \text{W/°C}\right)\left(\frac{1°\text{C}}{5\ \text{s}}\right)t = \left(\frac{1}{10}\ \text{W/s}\right)t.$$

$$\therefore \quad Q = \int_{0\text{ s}}^{300\text{ s}} \left(\frac{1}{10}\text{ W/s}\right) t\, dt = \left(\frac{1}{10}\text{ W/s}\right)\frac{(300\text{ s})^2 - (0\text{ s})^2}{2}$$

$$= \left(\frac{1}{20}\text{ W/s}\right)(90\,000\text{ s}^2)$$

$$= 4500\text{ W s}$$

$$= 4500\text{ J.}$$

5. $L = I_0\omega_0 = \dfrac{I\omega_0}{2}$ or $2I_0 = I$

or $2\left(\dfrac{1}{2}m_0r^2\right) = \dfrac{1}{2}m_0r^2 + (\mu t)r^2$

or $t = \dfrac{m_0}{2\mu}\cdot$

6. Let l be the length of each plate and $u_x = u =$ constant.

$\therefore$ the time of travel between the plates is given by $t = \dfrac{l}{u}\cdot$

Let a be the constant acceleration in the y-direction.

$\therefore$ $v_y = at$ when the particle emerges from the plates.

$$\therefore \quad v^2 = u_x^2 + v_y^2 = u^2 + a^2t^2 = u^2 + a^2\frac{l^2}{u^2} = u^2 + \frac{\alpha}{u^2}\cdot$$

9. The total energy in the first excited state is $-\dfrac{13.6}{4}$ eV $= -3.4$ eV.

This consists of the kinetic energy of 3.4 eV and the potential energy of –6.8 eV. In order to take the PE here as zero, we add 6.8 eV to all energy levels. The total energy in the ground state then becomes (–13.6 + 6.8) eV = –6.8 eV.

12. Vibrational energy involves additional degrees of freedom. The molar heat capacity is proportional to the number of degrees of freedom.

14. Let the car begin to move and sound its horn at time t_0.

The sound reaches the observer in 1 s.

$\therefore$ the observer begins to hear the sound at time $t_1 = t_0 + 1$ s.

The displacement of the car in 20 s is $\left(\frac{1}{2}\right)(1.1 \text{ m s}^{-2})(20 \text{ s})^2 = 220$ m.

∴ at time $t_2 = t_0 + 20$ s, the horn is switched off and the car is 110 m away from the observer. The sound emitted by the horn at the instant it is switched off will travel to the observer in a further time (1/3) s.

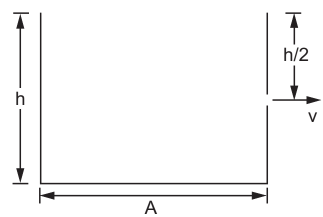
∴ the observer stops hearing the sound at time $t_3 = t_2 + \frac{1}{3}$ s.

∴ the observer hears sound for a duration of

$$t = t_3 - t_1 = t_2 + \frac{1}{3}\text{ s} - t_1 = (t_0 + 20\text{ s}) + \frac{1}{3}\text{ s} - (t_0 + 1\text{ s}) = 19\frac{1}{3}\text{ s}.$$

15. As seen from the lens, S is at a distance less than f from the lens.

16. $v = \sqrt{2g \cdot \frac{h}{2}} = \sqrt{gh}.$



Let α be the area of the hole.

The volume of the liquid discharged per unit time is $v\alpha$.

The mass of the liquid discharged per unit time is $\rho v\alpha$.

The momentum of this liquid per unit time is $\rho v^2\alpha$, which gives the force exerted by the liquid.

∴ $F = \rho\alpha v^2 = \rho\alpha gh.$

The mass of the liquid in the tank is $m = Ah\rho$.

Initial acceleration of the tank, $\frac{F}{m} = \frac{\alpha g}{A} = \frac{g}{N}\cdot$

17. The frequency of sound emitted is $n = \frac{V}{2l}\cdot$

The wavelength of sound in air is $\frac{v}{n} = \frac{2lv}{V}\cdot$

□

3

Practice Worksheet—3

• *Straight-Objective Type* •

This section contains seventeen multiple-choice questions numbered 1–17. Each question has four choices (a, b, c and d), out of which only one is correct.

1. A flat coil of area A and n turns is placed at the centre of a ring of radius r ($r^2 >> A$) and resistance R. The coil and the ring are coplanar. When the current in the coil increases from zero to i, the total charge circulating in the ring is

(a) $\dfrac{\mu_0 nAi}{2rR}$ (b) $\dfrac{\mu_0 nAi}{r^2 R}$

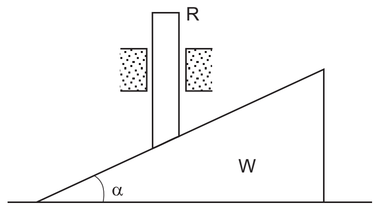
(c) $\dfrac{\mu_0 nAi}{2\pi r}$ (d) $\dfrac{\mu_0 n^2 i}{4\pi rR}$

2. A satellite is in a circular orbit very close to the surface of a planet. At some point it is given an impulse along its direction of motion, causing its velocity to increase η times. It now goes into an elliptical orbit, with the planet at the centre of the ellipse. The maximum possible value of η for this to occur is

(a) 2 (b) $\sqrt{2}$

(c) $\sqrt{2}+1$ (d) $\dfrac{1}{\sqrt{2}-1}$

3.



In the arrangement shown above, all surfaces are frictionless. The rod R is constrained to move vertically. The vertical acceleration of R is a_1 and the horizontal acceleration of the wedge W is a_2. The ratio a_1/a_2 is equal to

(a) tan α (b) cot α (c) sin α (d) cos α

4. Three infinitely long thin conductors are joined at the origin of coordinates and lie along the *x*-, *y*- and *z*-axes. A current *i* flowing along the conductor lying along the *x*-axis divides equally into the other two at the origin. The magnetic field at the point $(0, -a, 0)$ has magnitude

(a) $\frac{\mu_0 i}{4\pi a}$ (b) $\frac{3\mu_0 i}{4\sqrt{2}\pi a}$ (c) $\frac{\sqrt{5}\mu_0 i}{8\pi a}$ (d) $\frac{\sqrt{3}\mu_0 i}{2\pi a}$

5. An engine whistling at a constant frequency n_0 and moving with a constant velocity goes past a stationary observer. As the engine crosses him, the frequency of the sound heard by him changes by a factor *f*. The actual difference in the frequencies of the sound heard by him before and after the engine crosses him is

(a) $\frac{1}{2}n_0(1-f^2)$ (b) $\frac{1}{2}n_0\left(\frac{1-f^2}{f}\right)$

(c) $n_0\left(\frac{1-f}{1+f}\right)$ (d) $\frac{1}{2}n_0\left(\frac{1-f}{1+f}\right)$

6. Which of the following units does not have the same dimensions as the henry?

(a) joule $(\text{ampere})^{-2}$ (b) tesla $(\text{metre})^2$ $(\text{ampere})^{-2}$

(c) ohm second (d) $(\text{farad})^{-1}$ $(\text{second})^{-1}$

7. Assume that helium obeys the Bohr theory exactly. Which of the following transitions in helium will not give rise to a spectral line which has the same wavelength as some spectral line in the hydrogen spectrum?

(a) From $n = 4$ to $n = 2$ (b) From $n = 6$ to $n = 2$

(c) From $n = 8$ to $n = 4$ (d) From $n = 6$ to $n = 3$

8. A ball of density ρ_0 falls from rest from a point P onto the surface of a liquid of density ρ in time T. It enters the liquid, stops, moves up, and returns to P in a total time $3T$. Neglect the viscosity, surface tension and splashing. The ratio ρ/ρ_0 is equal to

(a) 1.5 (b) 2 (c) 3 (d) 4

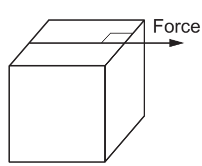
9. When an object is placed in front of a concave mirror of focal length f, a virtual image is produced with a magnification of 2. To obtain a real image with a magnification of 2, the object has to be moved by a distance equal to

(a) $\frac{f}{2}$ (b) $\frac{2f}{3}$ (c) f (d) $\frac{3f}{2}$

10. A charged particle of specific charge s moves undeflected through a region of space containing mutually perpendicular and uniform electric and magnetic fields, E and B. When the field E is switched off, the particle will move in a circular path of radius

(a) $\frac{E}{Bs}$ (b) $\frac{Es}{B}$ (c) $\frac{Es}{B^2}$ (d) $\frac{E}{B^2 s}$

11.



A solid cube is placed on a horizontal surface. The coefficient of friction between them is μ, where $\mu < 1/2$. A variable horizontal force perpendicular to one edge and passing through the midpoint of that edge is applied on the cube's upper face. The maximum acceleration with which it can move without toppling is

(a) μg (b) $2\mu g$

(c) $g(1-2\mu)$ (d) $\left(\mu+\frac{1}{2}\right)g$

12. A radioactive sample with half-life = T emits α-particles. Its total activity is A_i at some time and A_f at a later time. The number of α-particles emitted by the sample between these two points in time is

(a) $A_i - A_f$ (b) $\frac{T}{\ln 2}(A_i - A_f)$

(c) $\frac{\ln 2}{T}(A_i - A_f)$ (d) $\frac{T}{\ln 2}\left(\frac{1}{A_f}-\frac{1}{A_i}\right)$

13. A uniform rod of mass m, hinged at its upper end, is released from rest from a horizontal position. When it passes through the vertical position, the force on the hinge is

(a) $\frac{3}{2}mg$ (b) $2mg$ (c) $\frac{5}{2}mg$ (d) $3mg$

14. In solar radiation, the intensity of radiation is maximum around the wavelength λ_m. If R is the radius of the sun and c is the velocity of light, the mass lost by the sun per unit time is proportional to

(a) $\frac{R^2}{\lambda^4 c^2}$ (b) $\frac{R^2}{\lambda^2 c^2}$

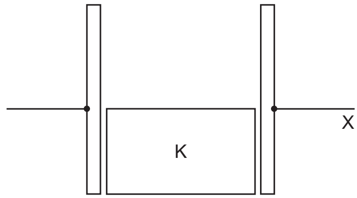
(c) $\frac{R^3}{\lambda^4 c^3}$ (d) $\frac{R^3}{\lambda^4 c^2}$

15. In an isolated, charged, parallel-plate air capacitor, the charge per unit area on each plate has a magnitude of σ. A dielectric slab having the dielectric constant K is now introduced between the plates. The induced charge per unit area on the surface of the dielectric will have magnitude

(a) $\frac{\sigma}{K}$ (b) $\sigma(K-1)$

(c) $\sigma\left(1-\frac{1}{K}\right)$ (d) $\frac{\sigma}{K+1}$

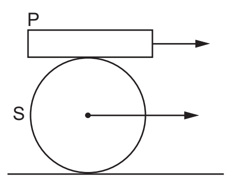
16.



A parallel-plate capacitor is charged from a cell and then isolated from it. A dielectric slab of dielectric constant K is now introduced in the region between the plates, filling half of it. The electric intensity in the dielectric is E_1 and that in air is E_2.

(a) $E_1 = E_2$
(b) $E_1 = \frac{E_2}{K}$
(c) $E_1 = \left(1 - \frac{1}{k}\right) E_2$
(d) $E_1 = \frac{E_2}{K - 1}$

17.



A plank P is placed on a solid cylinder S, which rolls on a horizontal surface. The two are of equal mass. There is no slipping at any of the surfaces in contact. The ratio of the kinetic energy of P to that of S is

(a) 1 : 1 (b) 2 : 1 (c) 8 : 3 (d) 11 : 8

• Assertion–Reason Type •

This section contains three questions numbered 18–20. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

18. STATEMENT-1: In characteristic X-rays, K_α X-rays are of smaller wavelengths than K_β X-rays for the same element.

STATEMENT-2: Characteristics X-rays are produced by transitions of orbital electrons in the target atom.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

19. STATEMENT-1: In radioactivity, the nature of a sample can be understood by its half life or average life but not by its total life.
STATEMENT-2: The total life of any radioactive sample is infinite.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

20. STATEMENT-1: In a gas, any rapid change must be adiabatic, whereas a slow change may be adiabatic.
STATEMENT-2: In a p–V diagram, the magnitude of the slope is greater for an adiabatic process than for an isothermal process.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

• *Linked-Comprehension Type* •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (21–23) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- When we hear a sound, we understand it mainly on the basis of three parameters, called *intensity*, *pitch* and *tone*. These quantities, in turn, depend on certain measurable quantities associated with the sound.

21. Which of the following statements regarding the intensity and loudness of a sound is not correct?

(a) The unit of intensity is W/m^2.

(b) The unit of loudness is the decibel.

(c) The loudness of a sound is directly proportional to its intensity.

(d) The intensity depends on the frequency, amplitude, density of the medium and the velocity of sound in the medium.

22. Which of the following statements regarding the pitch of a sound is not correct?

(a) The pitch of a sound depends only on its frequency.

(b) The difference between the frequencies of two sounds is called their 'interval'.

(c) If two sounds differ by N octaves, the ratio of their frequencies is 2^N.

(d) Different notes on a musical scale differ only in pitch.

23. Which of the following statements regarding the tone of a sound is not correct?

(a) The tone of a sound depends on its waveform.

(b) We identify the source of a sound by its tone.

(c) The tone of a sound improves if most of its harmonics are present along with the fundamental.

(d) The second harmonic of a sound can also be called its second overtone.

• *Matrix-Matching Type* •

Match the quantities in column A with those in column B, darkening the appropriate bubbles in the given 4×4 matrix.

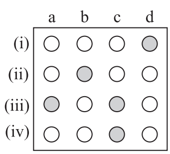
24. In the experimental setup for a photocell, the wavelength of the light incident on the cathode is initially 0.6 times the threshold wavelength for the material of the cathode. Certain changes in the experimental setup are given in column A and their possible effects are given in column B.

Column A	*Column B*
(i) The intensity of the incident light is doubled but the frequency remains unaltered	(a) Photocurrent remains the same
(ii) Both the intensity and wavelength of the incident light are doubled	(b) Photocurrent falls to zero
(iii) The intensity of the incident light is doubled and its wavelength is made half	(c) Stopping potential increases
(iv) The intensity of the incident light remains the same and the wavelength is made half	(d) Photocurrent increases

Answers

1. a	**2.** b	**3.** a	**4.** c	**5.** b
6. d	**7.** d	**8.** c	**9.** c	**10.** d
11. c	**12.** b	**13.** c	**14.** a	**15.** c
16. a	**17.** c			
18. d	**19.** a	**20.** b		
21. c	**22.** b	**23.** d		

24.



Hints and Solutions to Selected Questions

1. See the solution of Q. **53** on p. 5-137 to find M. Then use the formula

$$\text{total charge flow} = \frac{\text{total change in flux}}{\text{resistance}}.$$

2. The initial velocity of the satellite is given by $v = \sqrt{\frac{GM}{R}}$.

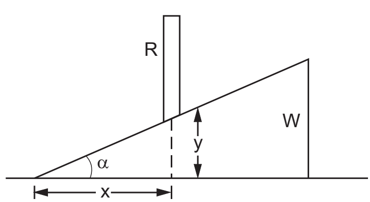
The satellite will remain in the elliptical orbit as long as $v < v_e$, where v_e is the escape velocity.

Also, $v_e = \sqrt{\frac{2GM}{R}}$.

$\therefore \quad \eta v \le v_e \quad$ or $\quad \eta \cdot \sqrt{\frac{GM}{R}} \le \sqrt{\frac{2GM}{R}}$

or $\quad \eta \le \sqrt{2} \quad$ or $\quad \eta_{max} = \sqrt{2}$.

3.



When W moves to the right by x, R moves down by y.

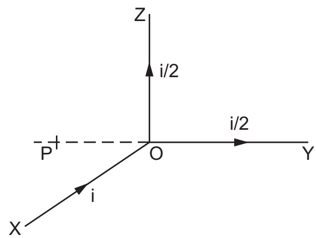
$\tan\alpha = \frac{y}{x} \quad$ or $\quad y = x \tan\alpha$

or $\quad \ddot{y} = \ddot{x} \tan\alpha$.

Now, $\ddot{y} = a_1$ and $\ddot{x} = a_2$.

$\therefore \quad a_1 = a_2 \tan\alpha$.

4.



Let P be the point (0, –a, 0), with OP = a.

The field at P due to the current i along the x-axis is $\frac{\mu_0 i}{4\pi a}\vec{k}$ and that due to the current $i/2$ along the z-axis is $\frac{\mu_0 i}{8\pi a}\vec{i}$.

The resultant field at P is $\frac{\mu_0 i}{4\pi a}\sqrt{1^2 + (1/2)^2}$.

5. Let v be the velocity of the engine and V be the velocity of sound. The frequency of the sound heard as the engine approaches the observer is given by $n_1 = \dfrac{n_0}{1 - \frac{v}{V}}$.

The frequency of the sound heard as the engine recedes from the observer is given by $n_2 = \dfrac{n_0}{1 + \frac{v}{V}}$.

It is given that $n_2 = fn_1$.

$\therefore \quad \dfrac{n_0 V}{V + v} = f\dfrac{n_0 V}{V - v} \quad$ or $\quad V - v = fV + fv$

or $\quad V(1 - f) = v(1 + f) \quad$ or $\quad \dfrac{v}{V} = \dfrac{1 - f}{1 + f}$.

The difference in the frequencies of the sound heard is

$$n_1 - n_2 = n_1(1 - f) = \left(\frac{n_0}{1 - \frac{1-f}{1+f}}\right)(1 - f) = \frac{n_0(1 - f^2)}{2f}.$$

7. For a transition from $n = p$ to $n = q$ in hydrogen, the photon energy is $\varepsilon_1 = 13.6\left(\frac{1}{q^2} - \frac{1}{p^2}\right)$ eV.

For a transition from $n = a$ to $n = b$ in helium, the photon energy is $\varepsilon_2 = 4 \times 13.6\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$ eV $= 13.6\left\{\left(\frac{2}{b}\right)^2 - \left(\frac{2}{a}\right)^2\right\}$ eV.

For photons of the same energy to appear in the two spectra,

$$\frac{1}{q^2} - \frac{1}{p^2} = \left(\frac{2}{b}\right)^2 - \left(\frac{2}{a}\right)^2.$$

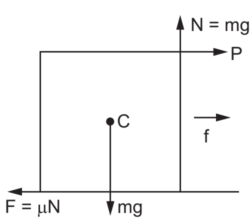
This will be satisfied if $b = 2q$ and $a = 2p$.

8. The downward and upward motions must be symmetrical. Hence, the total time spent by the ball inside the liquid is T, i.e., the downward motion of the ball inside the liquid takes time $T/2$. Hence, its upward acceleration inside the liquid is $2g$.

Let its volume be V. Then, inside the liquid,

$$V\rho g - V\rho_0 g = V\rho_0(2g) \quad \text{or} \quad \rho = 3\rho_0.$$

11. See the solutions of Q. **29** and **30** on p. 1-96.



Let a be the length of each edge of the cube.

At the position of toppling, taking the torque about C, we have

$$\left(\mu mg \times \frac{a}{2}\right) + \left(P \times \frac{a}{2}\right) = mg \times \frac{a}{2}$$

or $\quad P = mg - \mu mg.$

Let f = acceleration.

$$P - \mu mg = mf$$

or $\quad mg - 2\mu mg = mf \quad$ or $\quad f = g(1 - 2\mu).$

12. Let N_i and N_f be the initial number and the final number of active nuclei present.

$\therefore \quad A_i = \lambda N_i \quad$ and $\quad A_f = \lambda N_f.$

The number of α-particles emitted = the number of nuclei disintegrating

$$= N_i - N_f$$

$$= \frac{1}{\lambda}(A_i - A_f) = \frac{T}{\ln 2}(A_i - A_f).$$

13. $mg \cdot \frac{l}{2} = \frac{1}{2} I\omega^2 = \frac{1}{2} \cdot \frac{ml^2}{3} \omega^2$

or $\omega^2 = \frac{3g}{l}$.

Now, $N - mg = m\omega^2 \frac{l}{2} = m \cdot \frac{3g}{l} \cdot \frac{l}{2} = \frac{3}{2} mg$

or $N = \frac{5}{2} mg.$

14. Let T be the temperature of the surface of the sun.

∴ Wien constant, $b = \lambda_m T$.

The energy lost by the sun per unit time is $(\sigma T^4)(4\pi R^2) = mc^2$.

16. The potential difference between the plates is the same in both halves of the region.

In a uniform electric field,

the potential difference = electric intensity × distance.

As the distance between the plates is also the same in both halves, the electric intensity must be the same in both halves.

17. Let v be the velocity of the centre of mass of the cylinder S. Then, the velocity of P is $2v$.

∴ KE of P is $\frac{1}{2}(m)(2v)^2 = 2mv^2$.

KE of S is $\frac{1}{2} mv^2\left(1 + \frac{k^2}{r^2}\right) = \frac{1}{2} mv^2\left(1 + \frac{1}{2}\right) = \frac{3}{4} mv^2$.

∴ The ratio becomes

$$2mv^2 : \frac{3}{4} mv^2 = 8 : 3.$$

□

4

IIT Questions—1

• Straight-Objective Type •

This section contains fifteen multiple-choice questions numbered 1–15. Each question has four choices (a, b, c and d), out of which only one is correct.

1. Electrons each having the energy 80 keV are incident on the tungsten target of an X-ray tube. K-shell electrons of tungsten have –72.5 keV energy. The X-rays emitted by the tube contain only

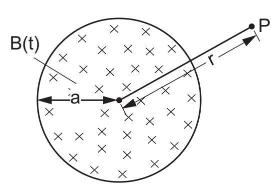
(a) a continuous X-ray spectrum (bremsstrahlung) with a minimum wavelength of ~0.0155 nm

(b) a continuous X-ray spectrum (bremsstrahlung) with all wavelengths

(c) the characteristic X-ray spectrum of tungsten

(d) a continuous X-ray spectrum (bremsstrahlung) with a minimum wavelength of ~0.0155 nm and the characteristic X-ray spectrum of tungsten

2.

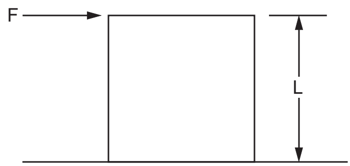


A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at the point P at a distance r from the centre of the circular region

(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$

3.

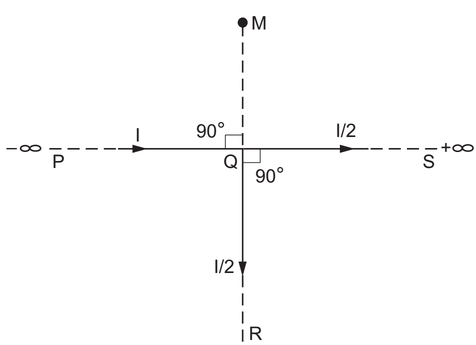


A cubical block of side length L rests on a rough horizontal surface having the coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

(a) infinitesimal (b) $\frac{mg}{4}$

(c) $\frac{mg}{2}$ (d) $mg(1 - \mu)$

4.



An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field strength due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field strength at M is now H_2. The ratio H_1/H_2 is given by

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) 2

5. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s then the ratio f_1/f_2 is

(a) $\frac{18}{19}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{19}{18}$

6. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

7. In a double-slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern,

(a) the intensities of both the maxima and minima increase
(b) the intensity of the maxima increases and the minima has the zero intensity
(c) the intensity of the maxima decreases but that of the minima increases
(d) the intensity of the maxima decreases and the minima has the zero intensity

8. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and the gravity is neglected, the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

9. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$

(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

10. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction then

(a) the positive ions deflect towards the $+y$-direction and negative ions towards the $-y$-direction

(b) all the ions deflect towards the $+y$-direction

(c) all the ions deflect towards the $-y$-direction

(d) the positive ions deflect towards the $-y$-direction and negative ions towards the $+y$-direction

11. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true?

(a) Its kinetic energy increases, and the potential and total energies decrease.

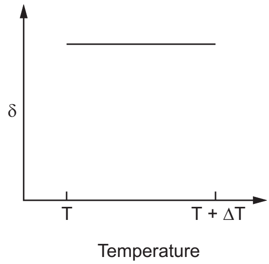
(b) Its kinetic energy decreases but the potential energy increases, and thus the total energy remains the same.

(c) Its kinetic and total energies decrease, and the potential energy increases.

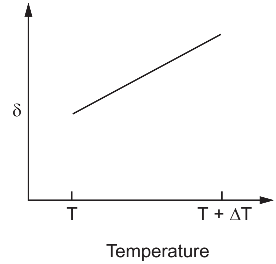
(d) Its kinetic, potential and total energies decrease.

12. An ideal gas is initially at a thermodynamic temperature T and has a volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. Which of the following graphs shows how the quantity $\delta = \Delta V/(V\Delta T)$ varies with temperature?

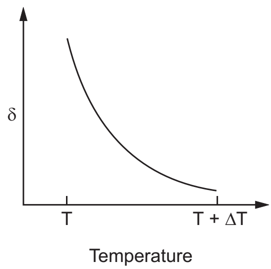
(a)



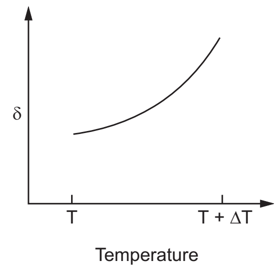
(b)



(c)

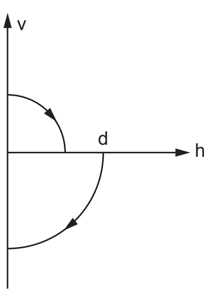


(d)

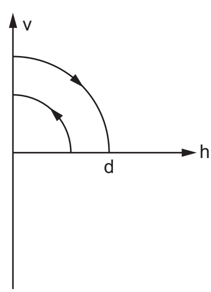


13. A ball is dropped vertically from a height *d* above the ground. It hits the ground and bounces up vertically to a height *d*/2. Neglecting the subsequent motion and the air resistance, how its velocity *v* varies with the height *h* above the ground?

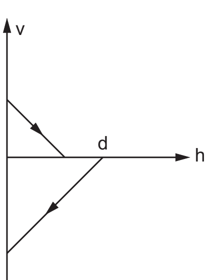
(a)



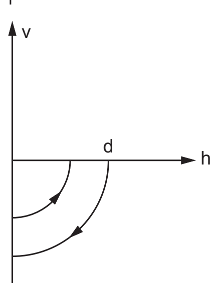
(b)



(c)

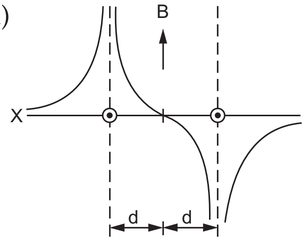


(d)

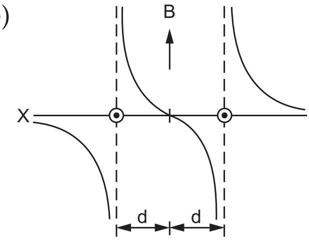


14. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the line XX′ is given by

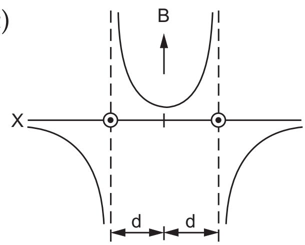
(a)



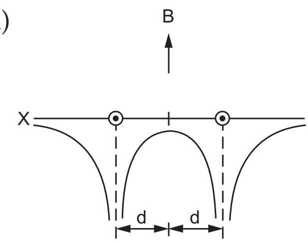
(b)



(c)



(d)



15. Two vibrating strings of the same material but of lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes—the one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is equal to

(a) 2 (b) 4 (c) 8 (d) 1

• *Assertion–Reason Type* •

This section contains two questions numbered 16 and 17. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

16. STATEMENT-1: For an elastic collision between two bodies, the relative speed of the bodies after the collision is equal to the relative speed before the collision.

STATEMENT-2: In an elastic collision, the linear momentum of the system is conserved.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-2.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is True, Statement-2 is True.

17. STATEMENT-1: A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal, and the same block is made to go up on the surface with the same initial velocity v. The decrease in mechanical energy in the second situation is smaller than that in the first situation.
STATEMENT-2: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

• *Linked-Comprehension Type* •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (18–20) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

18. The ratio x_1/x_2 equals

(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

19. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$ (c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

20. The loss of kinetic energy during the above process is

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$ (c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{2t}$

• *Matrix-Matching Type* •

Match the physical quantities in column A with the units in column B, and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix.

21. Some physical quantities are given in column A and some possible SI units in which these quantities may be expressed are given in column B.

Column A	*Column B*
(i) GM_eM_s, where G = universal gravitational constant, M_e = mass of the earth, M_s = mass of the sun	(a) V C m
(ii) $\frac{3RT}{M}$, where R = universal gas constant, T = absolute temperature, M = molar mass	(b) kg m^3 s^{-2}

(iii) $\frac{F^2}{q^2B^2}$, where
F = force,
q = charge,
B = magnetic flux density

(c) $m^2\ s^{-2}$

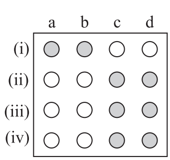
(iv) $\frac{GM_e}{R_e}$, where
G = universal gravitational constant,
M_e = mass of the earth,
R_e = radius of the earth

(d) $F\ V^2\ kg^{-1}$

Answers

1. d	**2.** b	**3.** c	**4.** c	**5.** d
6. c	**7.** a	**8.** a	**9.** a	**10.** c
11. a	**12.** c	**13.** a	**14.** b	**15.** d
16. b	**17.** c			
18. c	**19.** a	**20.** b		

21.



Hints and Solutions to Selected Questions

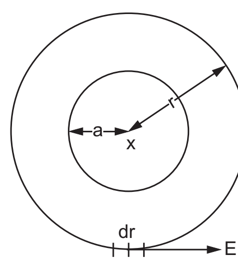
1. As the energy of the incident electrons is greater than the magnitude of the energy of the K-shell electrons, the target atoms will have vacancies in the K shell (K-shell electrons will be knocked out). This will cause emission of the entire characteristic spectrum of tungsten. Along with this, continuous *X*-rays will always be present, with

$$\lambda_c = \frac{1242 \text{ nm eV}}{80 \times 10^3 \text{ eV}} \cong 0.0155 \text{ nm}.$$

2. Construct a concentric circle of radius *r*. The induced electric field (*E*) at any point on this circle is equal to that at P. For this circle,

$$\oint \vec{E} \cdot \vec{dr} = 2\pi r E = \text{induced emf} = \frac{d\Phi}{dt} = \pi a^2 \dot{B}(t).$$

As the RHS is independent of *r*, therefore $E \propto \frac{1}{r}$.

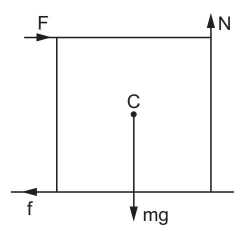


3. When *F* is applied, the normal reaction (*N*) of the floor moves to the right. The cube topples when *N* reaches its edge.

Here, $N = mg$ and the force of friction $= f = F$.

Taking torque about the centre C,

$$F \times \frac{L}{2} + f \times \frac{L}{2} = N \times \frac{L}{2} \quad \text{or} \quad 2F = mg.$$



4. The magnetic field strength at M due to QR is zero in all cases. When QS carries $\frac{I}{2}$ current, the field strength at M due to QS is $\frac{H_1}{2}$. Then

$$H_2 = H_1 + \frac{H_1}{2} = \frac{3H_1}{2} \quad \text{or} \quad \frac{H_1}{H_2} = \frac{2}{3}.$$

5. $n_o = \dfrac{n_s V}{V - v_s}$

Here, $f_1 = \dfrac{n_s V}{340 \text{ m/s} - 34 \text{ m/s}}$ and $f_2 = \dfrac{n_s V}{340 \text{ m/s} - 17 \text{ m/s}}$.

$\therefore \quad \dfrac{f_1}{f_2} = \dfrac{340 \text{ m/s} - 17 \text{ m/s}}{340 \text{ m/s} - 34 \text{ m/s}} = \dfrac{19}{18}$.

6. The effective current is $i = q \cdot \dfrac{\omega}{2\pi}$ and the area is $A = \pi r^2$.

The magnetic moment is $\mu = Ai = \dfrac{1}{2} q\omega r^2$.

The angular momentum is $L = I\omega = mr^2\omega$.

$\therefore \quad \dfrac{\mu}{L} = \dfrac{q}{2m}$.

7. In interference between waves of equal amplitudes a, the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. For waves of unequal amplitudes a and A $(A > a)$, the minimum intensity is nonzero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$.

8. The linear acceleration of the bead is $a = L\alpha$.

$\therefore$ the reaction force on the bead due to the rod is $N = ma = mL\alpha$.

After time t, the angular velocity of the bead is $\omega = \alpha t$.

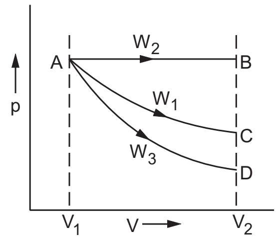
$\therefore$ the centripetal acceleration of the bead is $\omega^2 L = \alpha^2 t^2 L$.

$\therefore$ the force of friction at limiting position is $\mu N = \mu m L\alpha$.

$\therefore$ for slipping,

$$\mu m L\alpha = m\alpha^2 t^2 L \quad \text{or} \quad t = \sqrt{\mu/\alpha}.$$

9.



The three processes are plotted on a p–V diagram. AB is isobaric, AC is isothermal and AD is adiabatic. In each case, the work done is equal to the area under the curve.

12. $pV = nRT$. At constant pressure, $p\Delta V = nR\Delta T$.

Dividing, $\dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$ or $\delta = \dfrac{\Delta V}{V\Delta T} = \dfrac{1}{T}$ or $\delta \propto \dfrac{1}{T}$.

13. v is negative when the ball is falling and positive when it bounces up. Also, $v = -\sqrt{2g(d-h)}$ for downward motion and $v = \sqrt{g(d-2h)}$ for upward motion.

15. $\nu = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}} = \dfrac{1}{2l}\sqrt{\dfrac{T}{A\rho}} = \dfrac{1}{2l}\sqrt{\dfrac{T}{\pi R^2\rho}} = \dfrac{1}{2lR}\sqrt{\dfrac{T}{\pi\rho}}$.

As T and ρ are the same for both wires, $\nu \propto \dfrac{1}{lR}$.

Here, $lR = L \times 2r = 2L \times r$. $\therefore$ $\nu_1 = \nu_2$.

□

5

IIT Questions—2

• *Straight-Objective Type* •

This section contains sixteen multiple-choice questions numbered 1–16. Each question has four choices (a, b, c and d), out of which only one is correct.

1. A quantity X is given by $\varepsilon_0 L \dfrac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of vacuum, L is a length, ΔV is a potentital difference and Δt is a time interval. The dimensional formula for X is the same as that for

(a) resistance (b) charge

(c) voltage (d) current

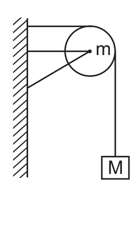
2. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by

(a) $\sqrt{2}Mg$

(b) $\sqrt{2}mg$

(c) $\sqrt{(M+m)^2+m^2}\cdot g$

(d) $\sqrt{(M+m)^2+M^2}\cdot g$



3. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v_1}$ and $\vec{v_2}$, respectively, at time $t = 0$. They collide at

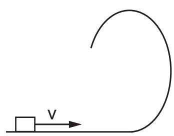
$t = t_0$. Their velocities become $\vec{v}_1{}'$ and $\vec{v}_2{}'$ at $t = 2t_0$ while still moving in air. The value of $|(m_1 \vec{v}_1{}' + m_2 \vec{v}_2{}') - (m_1 \vec{v}_1 + m_2 \vec{v}_2)|$ is

(a) zero (b) $(m_1 + m_2)gt_0$

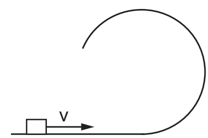
(c) $2(m_1 + m_2)gt_0$ (d) $\frac{1}{2}(m_1 + m_2)gt_0$

4. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in the case

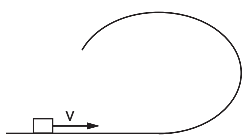
(a)



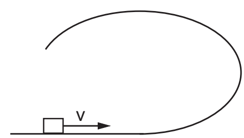
(b)



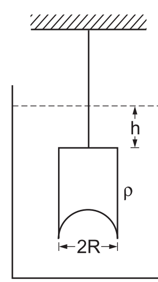
(c)



(d)



5. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ. It stays vertical inside the liquid. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$

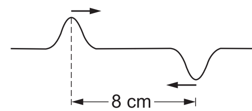
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

6. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and the energy is E_1, and in another experiment the displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and the energy is E_2. Then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$

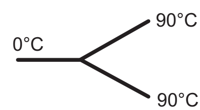
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

7. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other, as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s, the total energy of the pulses will be



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

8. Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be



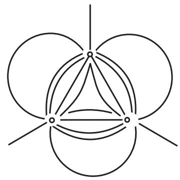
(a) 45°C (b) 60°C
(c) 30°C (d) 20°C

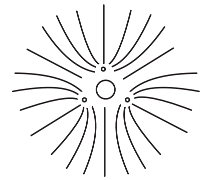
9. When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, the ratio k_1/k_2 can be expressed as

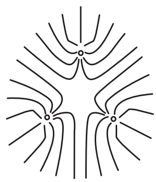
(a) $\dfrac{1 + (60°C)\gamma_{Fe}}{1 + (60°C)\gamma_{Hg}}$ (b) $\dfrac{1 - (60°C)\gamma_{Fe}}{1 + (60°C)\gamma_{Hg}}$

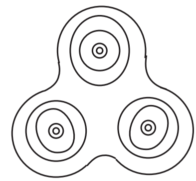
(c) $\dfrac{1 + (60°C)\gamma_{Fe}}{1 - (60°C)\gamma_{Hg}}$ (d) $\dfrac{1 + (60°C)\gamma_{Hg}}{1 + (60°C)\gamma_{Fe}}$

10. Three positive charges each having the value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as

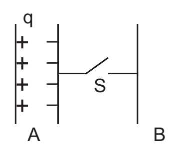
(a) 

(b) 

(c) 

(d) 

11. Consider the situation shown in the figure. The capacitor A has a charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) zero (b) $q/2$

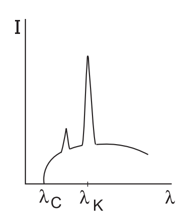
(c) q (d) $2q$

12. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic flux density at the centre is

(a) $\dfrac{\mu_0 NI}{b}$ (b) $\dfrac{2\mu_0 NI}{a}$

(c) $\dfrac{\mu_0 NI}{2(b-a)} \ln \dfrac{b}{a}$ (d) $\dfrac{\mu_0 I^N}{2(b-a)} \ln \dfrac{b}{a}$

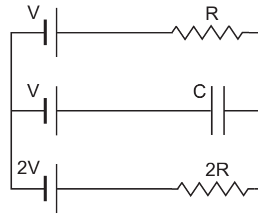
13. The intensity of X-rays from a Coolidge tube is plotted against the wavelength (λ), as shown in the figure. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased,



(a) $\lambda_K - \lambda_C$ increases (b) $\lambda_K - \lambda_C$ decreases

(c) λ_K increases (d) λ_K decreases

14. In the given circuit, with steady current, the potential drop across the capacitor must be



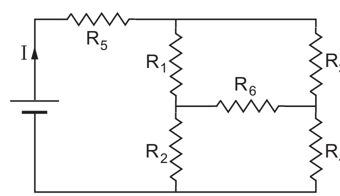
(a) V

(b) $\frac{V}{2}$

(c) $\frac{V}{3}$

(d) $\frac{2V}{3}$

15. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



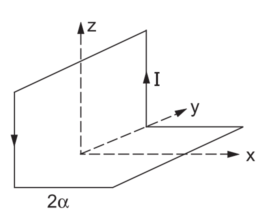
(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4 = R_5R_6$

16. A nonplanar loop of a conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point P$(a, 0, a)$ points in the direction



(a) $\frac{1}{\sqrt{2}}(-\vec{j} + \vec{k})$

(b) $\frac{1}{\sqrt{3}}(-\vec{j} + \vec{k} + \vec{i})$

(c) $\frac{1}{\sqrt{3}}(\vec{i} + \vec{k} + \vec{k})$

(d) $\frac{1}{\sqrt{2}}(\vec{i} + \vec{k})$

• *Assertion–Reason Type* •

This section contains two questions numbered 17 and 18. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

17. STATEMENT-1: If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.
STATEMENT-2: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

18. STATEMENT-1: The formula connecting u, v and f for a spherical mirror is valid only for those mirrors whose sizes are very small compared to their radii of curvature.
STATEMENT-2: The laws of reflection are strictly valid for plane surfaces but not for large spherical surfaces.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

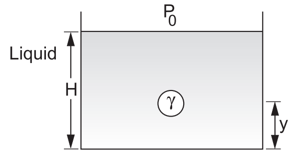
(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

• Linked-Comprehension Type •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (19–21) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- A small spherical monatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (see the figure below). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0. The height of the liquid is H and the atmospheric pressure is P_0 (neglect the surface tension).



19. As the bubble moves upwards, besides the buoyancy force, which of the following forces is/are acting on it?

(a) Only the force of gravity

(b) The force of gravity and the force due to the presure of the liquid

(c) The force of gravity, the force due to the presure of the liquid, and the force due to the viscosity of the liquid

(d) The force of gravity and the force due to the viscosity of the liquid

20. When the gas bubble is at a height y from the bottom, its temperature is

(a) $T_0\left(\dfrac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{2/5}$ (b) $T_0\left(\dfrac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{2/5}$

(c) $T_0\left(\dfrac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{3/5}$ (d) $T_0\left(\dfrac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{3/5}$

21. The buoyancy force acting on the gas bubble is (assume R is the molar gas constant)

(a) $\rho_l nRgT_0 \dfrac{(P_0+\rho_l gH)^{2/5}}{(P_0+\rho_l gy)^{7/5}}$

(b) $\dfrac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{2/5}\{P_0+\rho_l g(H-y)\}^{3/5}}$

(c) $\rho_l nRgT_0 \dfrac{(P_0+\rho_l gH)^{3/5}}{(P_0+\rho_l gy)^{8/5}}$

(d) $\dfrac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{3/5}\{P_0+\rho_l g(H-y)\}^{2/5}}$

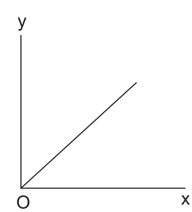
• Matrix-Matching Type •

Match the set of parameters given in column A with the graphs given in column B. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

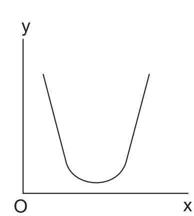
22. Column A gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in column B.

Column A | Column B

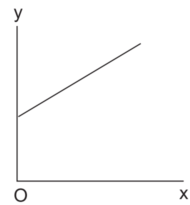
(i) The potential energy of a simple pendulum (y-axis) as a function of its displacement (x-axis)

(a) 

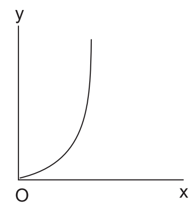
(ii) Displacement (y-axis) as a function of time (x-axis) for a one-dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

(b) 

(iii) The range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle

(c) 

(iv) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)

(d) 

Answers

1. d **2.** d **3.** c **4.** a **5.** d
6. c **7.** b **8.** b **9.** a **10.** c
11. a **12.** c **13.** a **14.** c **15.** c
16. d
17. b **18.** a
19. d **20.** b **21.** b
22.

	a	b	c	d
(i)	●	○	○	●
(ii)	○	●	○	●
(iii)	○	○	○	●
(iv)	○	●	○	○

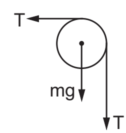
Hints and Solutions to Selected Questions

1. $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q}{L}$.

$\therefore \quad [\varepsilon_0 LV] = [\varepsilon_0 L\Delta V] = [Q].$

$\therefore \quad [X] = \left[\dfrac{\varepsilon_0 L\Delta V}{\Delta t}\right] = \left[\dfrac{Q}{\Delta t}\right] = \text{[current]}.$

2. For the block of mass M to be in equilibrium, the tension in the string is $T = Mg$. The force on the pulley by the clamp must balance all the other forces acting on it, i.e., the resultant of the forces shown in the figure. This has a magnitude of $\sqrt{(Mg + mg)^2 + (Mg)^2} = g\sqrt{(M+m)^2 + M^2}$.



3. The momentum of the two-particle system at $t = 0$ is given by

$$\vec{p_i} = m_1\vec{v_1} + m_2\vec{v_2}.$$

A collision between the two does not affect the total momentum of the system.

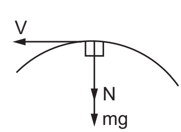
A constant external force $(m_1 + m_2)g$ acts on the system. The impulse given by this force in time $t = 0$ to $t = 2t_0$ is $(m_1 + m_2)g \times 2t_0$.

$\therefore$ the absolute change in momentum in this interval is

$$|(m_1\vec{v_1}' + m_2\vec{v_2}') - (m_1\vec{v_1} + m_2\vec{v_2})| = 2(m_1 + m_2)gt_0.$$

4. The blocks will have the same speed, say, equal to V, at the highest point of each track, as they all rise to the same height. Let R be the radius of curvature of a track and N be the normal reaction of the track at the highest point of the track.

Centripetal force $= N + mg = \dfrac{mV^2}{R}$.



N will be maximum when R is minimum. This occurs when the track is most sharply curved.

5. Net upward buoyancy force on the cylinder
= weight of liquid displaced by it
= $\rho g V$
= (upward force on the bottom) – (downward force on the top)
$\therefore$ the force on the bottom is $\rho g V + (h\rho g)\pi R^2 = \rho g(V + \pi R^2 h)$.

6. A stationary wave has the equation of the form

$$y = A \sin kx \sin \omega t.$$

Here, for y_1, we have $k_1 = \frac{\pi}{L}$, $\omega_1 = \omega$.

$$\therefore \quad v_1 = \frac{\omega_1}{k_1} = \frac{\omega L}{\pi}.$$

For y_2, we have $k_2 = \frac{2\pi}{L}$, $\omega_2 = 2\omega$.

$$\therefore \quad v_2 = \frac{\omega_2}{k_2} = \frac{\omega L}{\pi} = v_1.$$

Thus, the wave velocities are the same in both cases. Also, they have the same amplitude. The frequency for y_2 is twice the frequency for y_1.

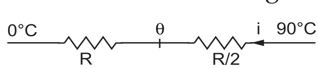
Now, $\because$ energy $\propto$ (frequency)^2,

$\therefore$ $E_2 = 4E_1$.

7. After 2 s, the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy. Its entire energy must be kinetic.

8. As the rods are made of the same material and have the same dimensions, they have the same thermal resistance, say R. Let θ be the temperature of the junction.

The circuit becomes as shown in the figure below.



$$i = \frac{90°C - \theta}{R/2} = \frac{\theta - 0°C}{R}$$

or $\quad 180°C - 2\theta = \theta \quad$ or $\quad \theta = 60°C$.

9. Let ρ_{Fe} and ρ_{Hg} denote the densities of iron and mercury at 0°C, and m be the mass of the block.

$\therefore$ the volume of the block is $\dfrac{m}{\rho_{Fe}}$ and that of displaced mercury is $\dfrac{k_1 m}{\rho_{Fe}}$.

$$\therefore \left(\frac{k_1 m}{\rho_{Fe}}\right)\rho_{Hg} = m \quad \text{or} \quad k_1 = \frac{\rho_{Fe}}{\rho_{Hg}}. \quad \text{Also, } \rho_t = \frac{\rho_0}{1+\gamma t}.$$

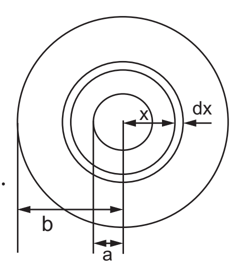
$$\therefore \quad k_2 = \frac{\rho_{Fe}}{1+(\gamma_{Fe}\times 60°\text{C})} \times \frac{1+(\gamma_{Hg}\times 60°\text{C})}{\rho_{Hg}} = k_1 \frac{1+(60°\text{C})\gamma_{Hg}}{1+(60°\text{C})\gamma_{Fe}}$$

$$\text{or} \quad \frac{k_1}{k_2} = \frac{1+(60°\text{C})\gamma_{Fe}}{1+(60°\text{C})\gamma_{Hg}}.$$

10. The tangent drawn to a line of force at any point on it must give the direction of the electric intensity at that point. Consider a point midway between any two of the charges. Here, the resultant intensity is only that due to the third charge and must point away from the third charge. This is satisfied only in the sketch (c).

11. The $\pm q$ charges appearing on the inner surfaces of A are bound charges. As B is without charge initially and is isolated, the charges on A will not be affected on closing the switch S. No charge will flow into B.

12. The number of turns per unit length is $\dfrac{N}{b-a}$.



Consider an elemental ring of radius x and width dx.

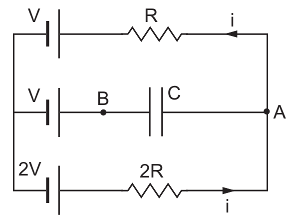
The number of turns in the ring is $dN = \dfrac{N\,dx}{b-a}$.

The magnetic field at the centre due to the ring is $dB = \dfrac{\mu_0 (dN) I}{2x} = \dfrac{\mu_0 I}{2} \cdot \dfrac{N\,dx}{b-a} \cdot \dfrac{1}{x}$.

$$\therefore \text{ the field at the centre is } \int dB = \frac{\mu_0 N I}{2(b-a)} \int_a^b \frac{dx}{x} = \frac{\mu_0 N I}{2(b-a)} \cdot \ln \frac{b}{a}.$$

13. λ_K does not depend on the accelerating voltage. λ_C decreases with increase in accelerating voltage. Thus, as the accelerating voltage is increased, the difference $\lambda_K - \lambda_C$ will increase.

14. Moving anticlockwise from A,



$$iR + V - 2V + 2iR = 0$$

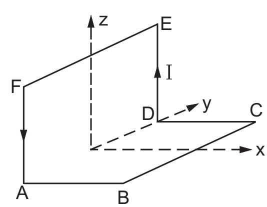
or $3iR = V$ or $i = \frac{V}{3R}$.

$$V_A - V_B = iR + V - V = iR$$

$$= \frac{V}{3} = \text{potential drop across C.}$$

15. As I is independent of R_6, no current flows through R_6. This requires that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4. This must satisfy the condition $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, as in the Wheatstone bridge.

16. The conductors BC and EF will together produce a magnetic field at P which lies in the xz-plane and is equally inclined to the x- and z-axes. The conductors AB and CD will together produce a magnetic field at P directed along the z-axis and of the magnitude B_0 (say). The conductors DE and FA will produce a field at P with the same magnitude B_0, and directed along the x-axis. Thus, the field at P will have equal components in the x- and z-directions, and no component in the y-direction. It must therefore point in the direction $(1/\sqrt{2})(\vec{i} + \vec{k})$.



□

6

IIT Questions—3

• Straight-Objective Type •

This section contains sixteen multiple-choice questions numbered 1–16. Each question has four choices (a, b, c and d), out of which only one is correct.

1. A siren placed at a railway platform is emitting sound of 5 kHz frequency. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of the train B to that of the train A is

(a) 242 : 252 (b) 2 : 1 (c) 5 : 6 (d) 11 : 6

2. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m s^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(a) 30 m s^{-1} (b) 20 m s^{-1} (c) 10 m s^{-1} (d) 5 m s^{-1}

3. A geostationary satellite orbits around the earth in a circular orbit of radius 36 000 km. Then, the time period of a spy satellite orbiting a few hundred kilometres above the earth's surface (R_e = 6400 km) will approximately be

(a) $\frac{1}{2}$ h (b) 1 h (c) 2 h (d) 4 h

4. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass *M*, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of *M* is

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) $\frac{1}{25}$ kg

5. A particle of *m* mass and *q* charge moves with a constant velocity *v* along the positive *x* direction. It enters a region containing a uniform magnetic field *B* directed along the negative *z* direction, extending from $x = a$ to $x = b$. The minimum value of *v* required so that the particle can just enter the region $x > b$ is

(a) qbB/m (b) $q(b - a)B/m$

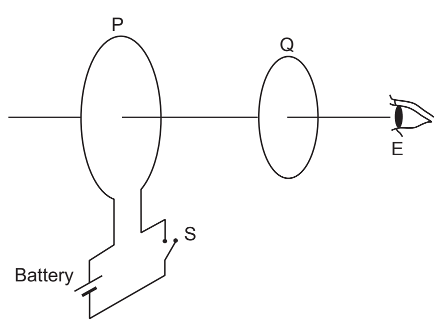
(c) qaB/m (d) $q(b + a)B/2m$

6. A long straight wire along the *z*-axis carries a current *I* in the negative *z*-direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is

(a) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{(y\vec{i} - x\vec{j})}{(x^2 + y^2)}$ (b) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{(x\vec{i} + y\vec{j})}{(x^2 + y^2)}$

(c) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{(x\vec{j} - y\vec{i})}{(x^2 + y^2)}$ (d) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{(x\vec{i} - y\vec{j})}{(x^2 + y^2)}$

7. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are



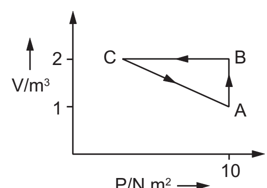
(a) respectively clockwise and anticlockwise
(b) both clockwise
(c) both anticlockwise
(d) respectively anticlockwise and clockwise

8. Two identical capacitors have the same capacitance C. One of them is charged to a potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

9. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is



(a) –5 J (b) –10 J

(c) –15 J (d) –20 J

10. An ideal black body at room temperature is thrown into a furnace. It is observed that

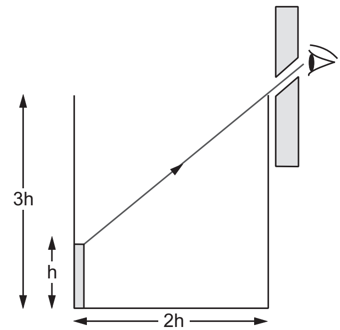
(a) initially it is the darkest body and at later times the brightest
(b) it is the darkest body at all times
(c) it cannot be distinguished at all times
(d) initially it is the darkest body and at later times it cannot be distinguished

11. Which of the following processes represents a gamma decay?

(a) ${}^{A}_{Z}X + \gamma \to {}_{Z-1}{}^{A}X + a + b$ (b) ${}^{A}_{Z}X + {}^{1}_{0}n \to {}^{A-3}_{Z-2}X + c$

(c) ${}^{A}_{Z}X \to {}^{A}_{Z}X + f$ (d) ${}^{A}_{Z}X + {}^{0}_{-1}e \to {}_{Z-1}{}^{A}X + g$

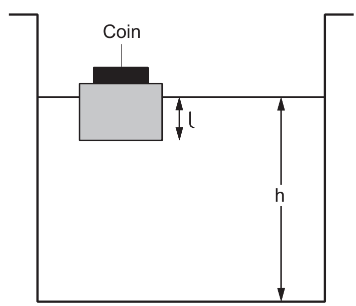
12. An observer can see through a pinhole the top end of a thin rod of height *h*, placed as shown in the figure. The beaker height is *3h* and its radius is *h*. When the beaker is filled with a liquid up to a height *2h*, he can see the lower end of the rod. Then the refractive index of the liquid is



(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

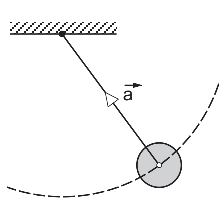
13. A wooden block, with a coin placed on its top, floats in water, as shown in the figure. The distances *l* and *h* are shown there. After some time the coin falls into the water. Then



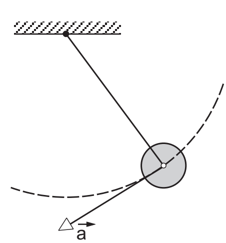
(a) *l* decreases and *h* increases
(b) *l* increases and *h* decreases
(c) both *l* and *h* increase
(d) both *l* and *h* decrease

14. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in the figure

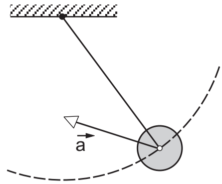
(a)



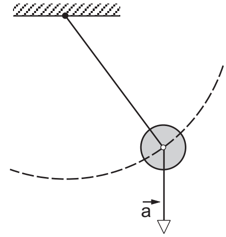
(b)



(c)



(d)



15. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(a) up the incline while ascending and down the incline while descending

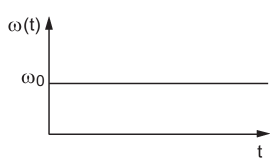
(b) up the incline while ascending as well as descending

(c) down the incline while ascending and up the incline while descending

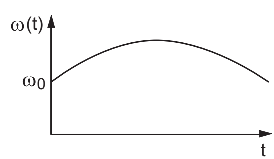
(d) down the incline while ascending as well as descending

16. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

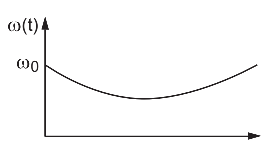
(a)



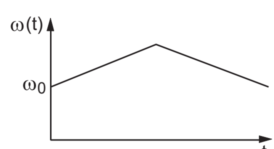
(b)



(c)



(d)



• *Assertion–Reason Type* •

This section contains two questions numbered 17 and 18. Each question contains an assertion (Statement-1) and a reason (Statement-2). Each question has four choices (a, b, c and d), out of which only one is correct.

17. STATEMENT-1: For an observer looking out through the window of a fast-moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

STATEMENT-2: If the observer and the object are moving with the velocities V_1 and V_2 respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $V_1 - V_2$.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

18. STATEMENT-1: It is easier to pull a heavy object than to push it on a level ground.

STATEMENT-2: The magnitude of frictional force depends on the nature of the two surfaces in contact.

(a) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 **is not** a correct explanation for Statement-1.

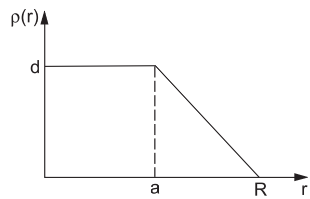
(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

• *Linked-Comprehension Type* •

This section contains a paragraph. Based upon this paragraph, three multiple-choice questions (19–21) have to be answered. Each question has four choices (a, b, c and d), out of which only one is correct.

- The nuclear charge (Ze) is nonuniformly distributed within a nucleus of radius R. The charge density, i.e. charge per unit volume, $\rho(r)$ is dependent only on the radial distance r from the centre of the nucleus, as shown in the figure. The electric field is only along the radial direction.



19. The electric field at $r = R$ is

(a) independent of a

(b) directly proportional to a

(c) inversely proportional to a

(d) none of these

20. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is

(a) $\dfrac{3Ze}{4\pi R^3}$ (b) $\dfrac{3Ze}{\pi R^3}$ (c) $\dfrac{4Ze}{3\pi R^3}$ (d) $\dfrac{Ze}{3\pi R^3}$

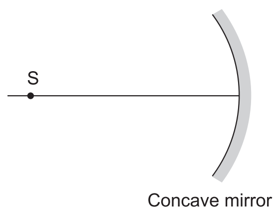
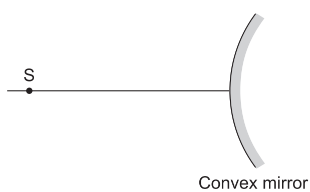
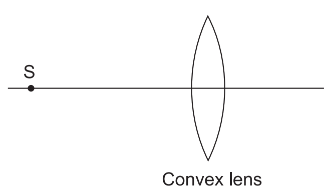
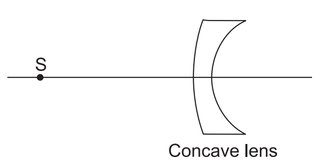
21. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies that

(a) $a = 0$ (b) $a = \dfrac{R}{2}$ (c) $a = R$ (d) $a = \dfrac{2R}{3}$

• Matrix-Matching Type •

Match the optical devices shown in column A with the images described in column B.

22. Column A shows spherical mirrors and lenses. Column B lists images which may be formed by these devices.

Column A	*Column B*
(i) 	(a) Real image
(ii) 	(b) Virtual image
(iii) 	(c) Magnified image
(iv) 	(d) Image at infinity

Answers

1. b **2.** c **3.** c **4.** a **5.** b

6. a **7.** d **8.** c **9.** a **10.** d

11. c **12.** b **13.** d **14.** c **15.** b

16. b

17. b **18.** b

19. d **20.** b **21.** c

22.

	a	b	c	d
(i)	●	●	●	●
(ii)	○	●	○	○
(iii)	●	●	●	●
(iv)	○	●	○	○

Hints and Solutions to Selected Questions

1. When an observer approaches a stationary source, which is emitting sound of frequency n_s, he hears a frequency n, given by

$$n = n_s\left(1 + \frac{v}{V}\right),$$

where v = velocity of the observer and V = velocity of sound.

Here, let v_A and v_B be the velocities of the trains A and B.

For A, we have $n = 5.5 \text{ kHz} = (5 \text{ kHz})\left(1 + \frac{v_A}{V}\right)$ or $\frac{v_A}{V} = 0.1$.

For B, we have $n = 6 \text{ kHz} = (5 \text{ kHz})\left(1 + \frac{v_B}{V}\right)$ or $\frac{v_B}{V} = 0.2$.

$$\therefore \quad \frac{v_B}{v_A} = 2.$$

2. $$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(10 \text{ kg})(14 \text{ m s}^{-1}) + (4 \text{ kg})(0)}{10 \text{ kg} + 4 \text{ kg}} = 10 \text{ m s}^{-1}.$$

3. For a satellite in a circular orbit around the earth, the time period T depends on the radius r as $T^2 \propto r^3$. Here, for the geostationary satellite, $T_1 = 24$ h and $r_1 = 36\,000$ km. For the spy satellite, $r_2 = 6400$ km and time period $= T_2$.

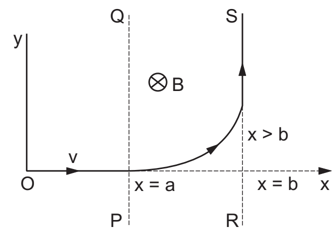
$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} \quad \text{or} \quad T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (24 \text{ h})\left(\frac{6400 \text{ km}}{36\,000 \text{ km}}\right)^{3/2} \simeq 2 \text{ h}.$$

4. The frequency of vibration of a string is given by $n = \frac{N}{2l}\sqrt{\frac{T}{m}}$,

where N = number of loops (or segments) = number of antinodes. The other symbols have their usual meanings.

$$n = \frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}} \quad \text{or} \quad M = 25 \text{ kg}.$$

5. In the figure, the z-axis points out of the paper, and the magnetic field is directed into the paper, existing in the region between PQ and RS. The particle moves in a circular path of radius r in the magnetic field. It can just enter the region $x > b$ for $r \geq b - a$.



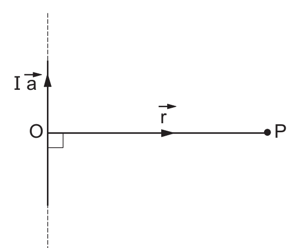
Now, $mv = Bqr$

or $r = \frac{mv}{Bq} \geq b - a$

or $v \geq \frac{q(b-a)B}{m}$

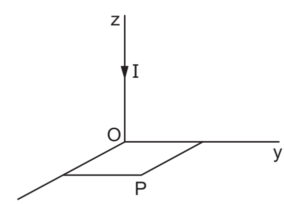
or $v_{min} = \frac{q(b-a)B}{m}$.

6. To find the magnetic field at a point P due to a long straight wire carrying current I, let $\vec{a}$ be a unit vector in the direction of current flow , and $\vec{r}$ be the vector, normal to $\vec{a}$, from the wire to P. Then the magnetic field at P is



$$\vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{\vec{a} \times \vec{r}}{r^2}$$

Here, $\vec{a} = -\vec{k}$, $\vec{r} = x\vec{i} + y\vec{j}$.



$$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{(-\vec{k}) \times (x\vec{i} + y\vec{j})}{r^2}$$

$$= \left(\frac{\mu_0 I}{2\pi}\right) \frac{(y\vec{i} - x\vec{j})}{(x^2 + y^2)}.$$

7. When S is closed, the clockwise current I_P flowing through P produces a magnetic field B directed from E to Q to P. This field B, by Lenz's law, causes an anticlockwise current I_{Q1} in Q. When S is opened, B decreases. This induces a clockwise current I_{Q2} in Q.

8. The initial energy of the system is given by $E_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$.

When the capacitors are joined, they reach a common potential equal to $\frac{\text{total charge}}{\text{total capacity}} = \frac{CV_1 + CV_2}{2C} = \frac{V_1 + V_2}{2} = V$.

The final energy of the system is $E_f = \frac{1}{2}(2C)V^2$.

$\therefore$ the decrease in energy is $E_i - E_f = \frac{1}{4}C(V_1 - V_2)^2$.

9. From the first law of thermodynamics, $Q = \Delta U + W$. For a cyclic process, $\Delta U = 0$. Also, $\phi = 5$ J.

$W = W_{AB} + W_{BC} + W_{CA}$.

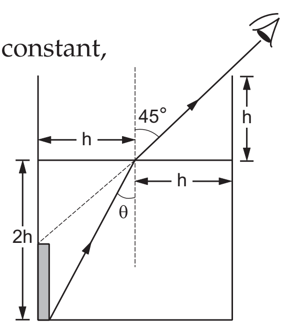
$W_{AB} = p\Delta V = (10 \text{ N m}^{-2})(1 \text{ m}^3) = 10 \text{ J}$ and $W_{BC} = 0$.

$\therefore \quad 5 \text{ J} = 10 \text{ J} + W_{CA} \quad$ or $\quad W_{CA} = -5 \text{ J}$.

10. A closed furnace behaves as a black body.

11. In a gamma decay process, there is no change in either A or Z.

12. The line of sight of the observer remains constant, making an angle of 45° with the normal.

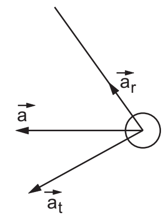


$$\sin\theta = \frac{h}{\sqrt{h^2 + (2h)^2}} = \frac{1}{\sqrt{5}}$$

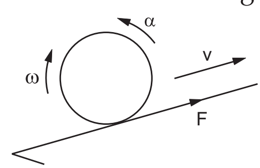
$$\mu = \frac{\sin 45°}{\sin\theta} = \frac{1/\sqrt{2}}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$$

13. When the coin slips into the water, the wooden block moves up and l decreases. When the coin was floating, it displaced water equal to its own weight. When inside the water, it displaces water equal to its own volume. As its density is greater than that of water, it displaced more water in the first case. Hence, h decreases when it falls into the water.

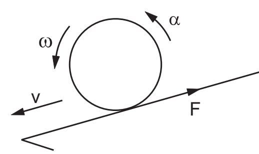
14. In the given position, the bob is moving along a circular path with the same speed and hence has some radial acceleration $\vec{a}_r$. It also has some tangential acceleration $\vec{a}_t$. As $\vec{a}$ is the resultant of these two, it will be directed somewhere between them.



15. When the cylinder rolls up the incline, its angular velocity ω is clockwise and decreasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (*F*) acting up the incline.



When the cylinder rolls down the incline, its angular velocity ω is anticlockwise and increasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (*F*) acting up the incline.



16. The angular momentum (*L*) of the system is conserved, i.e., $L = I\omega =$ constant.

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, *I* first decreases and then increases. As a result, ω will first increase and then decrease. Also, the change in ω will be a nonlinear function of time.

□

7

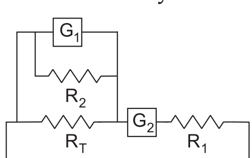
IIT Questions—4

• Straight-Objective Type 1 •

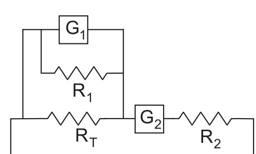
This section contains eight multiple-choice questions numbered 1–8. Each question has four choices (a, b, c and d), out of which only one is correct.

1. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

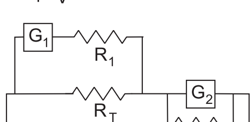
(a)



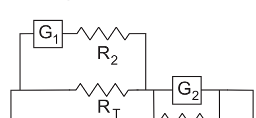
(b)



(c)



(d)



2. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W, and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

(a) $\dfrac{1}{R_{100}} = \dfrac{1}{R_{40}} + \dfrac{1}{R_{60}}$ (b) $R_{100} = R_{40} + R_{60}$

(c) $R_{100} > R_{60} > R_{40}$ (d) $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$

3. A real gas behaves like an ideal gas if its

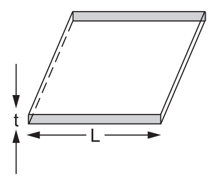
(a) pressure and temperature are both high

(b) pressure and temperature are both low

(c) pressure is high and temperature is low

(d) pressure is low and temperature is high

4. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



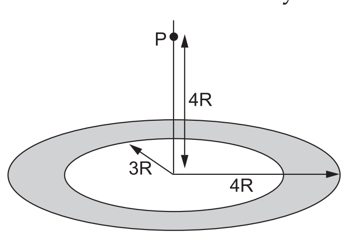
(a) directly proportional to L

(b) directly proportional to t

(c) independent of L

(d) independent of t

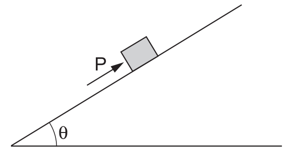
5. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is

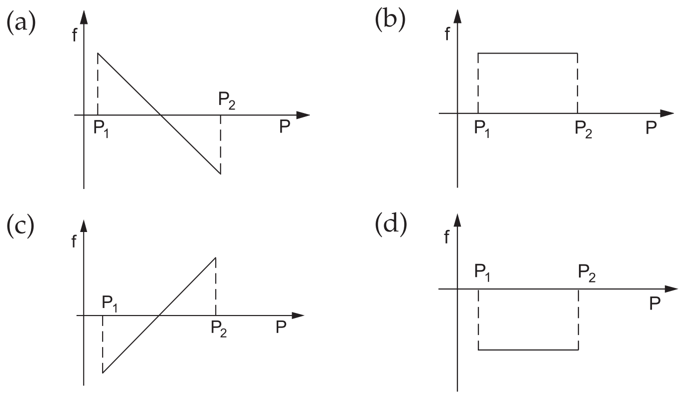


(a) $\dfrac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\dfrac{2GM}{7R}(4\sqrt{2}-5)$

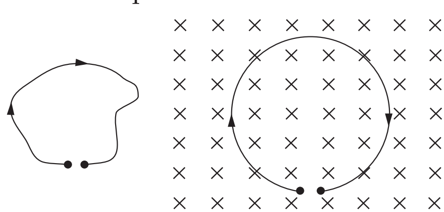
(c) $\dfrac{GM}{4R}$ (d) $\dfrac{2GM}{5R}(\sqrt{2}-1)$

6. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like





7. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(a) IBL (b) $\frac{IBL}{\pi}$ (c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

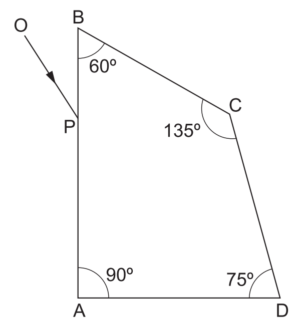
8. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

• Straight-Objectcive Type 2 •

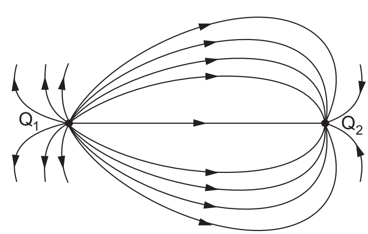
This section contains five multiple-choice questions numbered 9–13. Each question has four choices (a, b, c and d), out of which one or more may be correct.

9. A ray *OP* of monocromatic light is incident on the face *AB* of prism *ABCD* near vertex *B* at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



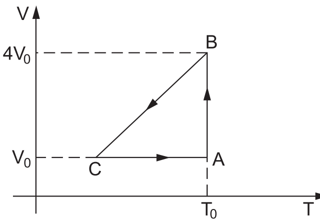
(a) The ray gets totally internally reflected at face *CD.*
(b) The ray comes out through face *AD.*
(c) The angle between the incident ray and the emergent ray is 90°.
(d) The angle between the incident ray and the emergent ray is 120°.

10. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the *x*-axis are shown in the figure. These lines suggest that



(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

11. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following.



(a) Internal energies at A and B are the same

(b) Work done by the gas in process AB is $P_0V_0 \ln 4$

(c) Pressure at C is $\frac{P_0}{4}$

(d) Temperature at C is $\frac{T_0}{4}$

12. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m s^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

(a) Total momentum of the system is 3 kg m s^{-1}

(b) Momentum of 5 kg mass after collision is 4 kg m s^{-1}

(c) Kinetic energy of the centre of mass is 0.75 J

(d) Total kinetic energy of the system is 4 J

13. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 second for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

(a) Error ΔT in measuring T, the time period, is 0.05 seconds

(b) Error ΔT in measuring T, the time period, is 1 second

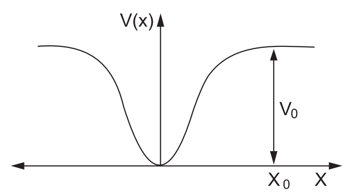
(c) Percentage error in the determination of g is 5%

(d) Percentage error in the determination of g is 2.5%

• Linked-Comprehension Type •

This section contains two paragraphs. Based upon the first paragraph, three multiple-choice questions (14–16), have to be answered. Based upon the second paragraph, 2 multiple-choice questions (17–18), have to be answered. Each questions has four choices (a, b, c and d), out of which only one is correct.

- When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily-using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider, a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



14. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$

(c) $V_0 > E > 0$ (d) $E > V_0$

15. For periodic motion of small amplitude A, the time period T of this particle is proportional to

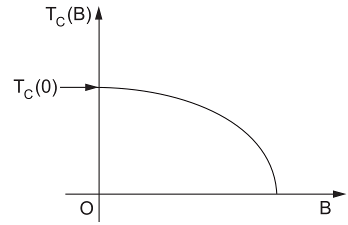
(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

16. The acceleration of this particle for $|x| > X_0$ is

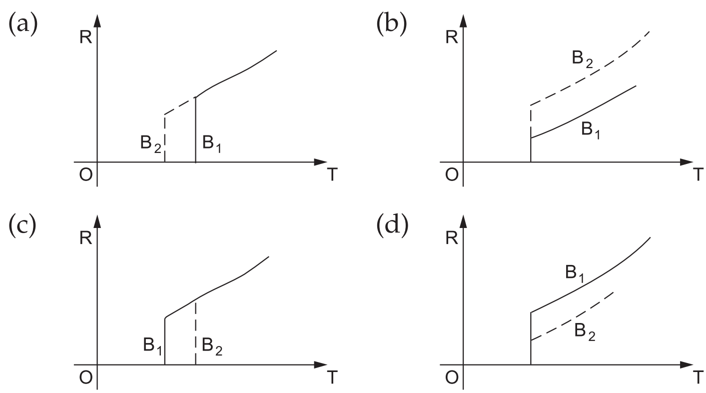
(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$

(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

- Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.



17. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?



18. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(a) $B = 5$ Tesla, $T_C(B) = 80$ K

(b) $B = 5$ Tesla, $75\text{ K} < T_C(B) < 100$ K

(c) $B = 10$ Tesla, $75\text{ K} < T_C(B) < 100$ K

(d) $B = 10$ Tesla, $T_C(B) = 70$ K

• *Integer Type* •

This section contains ten questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

20. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ N m^{-2}, the value of n is

21. A binary star consists of two stars A (mass $2.2 M_S$) and B (mass $11 M_S$), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

22. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11} g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km s^{-1}, the escape speed on the surface of the planet in km s^{-1} will be

23. A piece of ice (heat capacity $= 2100$ J kg^{-1} °C^{-1} and latent heat $= 3.36 \times 10^5$ J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

24. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 m s^{-1}.

25. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

26. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

27. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\, J_2$ then the value of R in Ω is

28. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Answers

1. c	**2.** d	**3.** d	**4.** c	**5.** a
6. a	**7.** c	**8.** b	**9.** a, b, c	**10.** a, d
11. a, b	**12.** a, c	**13.** a, c	**14.** c	**15.** b

16. d	**17.** b	**18.** b	**19.** 5	**20.** 4
21. 6	**22.** 3	**23.** 8	**24.** 7	**25.** 6
26. 3	**27.** 4	**28.** 9		

Hints and Solutions to Selected Questions

1. G_1 acts as a voltmeter due to the high resistance R_1 in series. The small resistance R_2 in parallel with G_2 acts as a shunt.

2. Power $P = \dfrac{V^2}{R}$ $\quad \dfrac{1}{R_{100}} = \dfrac{1}{R_{60}} + \dfrac{1}{R_{40}}$.

As resistance increases with temperature, the equality does not hold exactly. Hence, $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$ is the best option.

3. At low pressures, the number of molecules per unit volume decreases, so that the "point mass" approximation holds. Also, at high temperatures, the average speeds increase.

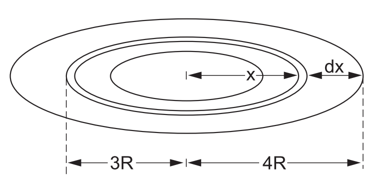
4. As the sheet is square, $R = \rho \dfrac{l}{A} = \rho \dfrac{L}{Lt} = \dfrac{\rho}{t}$.

5. Mass per unit area $\sigma = \dfrac{M}{\pi[(4R)^2 - (3R)^2]}$,

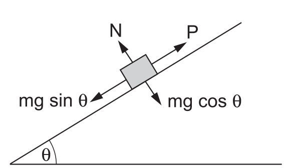
area of ring $= 2\pi x dx$

mass of ring $= dm = 2\pi\sigma x dx$

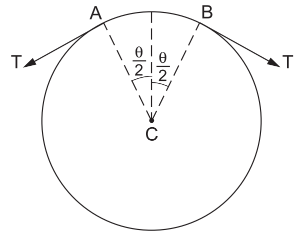
$$V_P = -G\int_{3R}^{4R} \frac{dm}{[(4R)^2 + x^2]^{½}}$$



6. Force of friction, f, becomes zero when $P = mg\sin\theta$. For $P < mg\sin\theta$, f acts upwards along the incline.



7. $L = 2\pi r$ or $r = \dfrac{L}{2\pi}$.



Length of element $AB = dl = r\theta = \left(\dfrac{L}{2\pi}\right)\theta$.

Outward ampere force on $AB = BIdl$.

Inward force on AB due to tension T

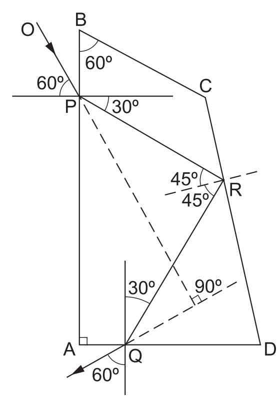
$= 2T\sin(\theta/2) = T\theta$.

Equating forces for equilibrium, $T = \dfrac{BIL}{2\pi}$.

8. Impedance $Z = \left[R^2 + \left(\dfrac{1}{\omega C}\right)^2\right]^{1/2}$, $\quad I = \dfrac{E}{Z}$

When ω increases, Z decreases, I increases.
As power $\propto I^2$, bulb glows brighter.

9. At P, $\sin 60° = \sqrt{3}\sin\gamma$, $\therefore \quad \gamma = 30°$.



$\sin C = \dfrac{1}{\sqrt{3}}$, $\quad C < 45°$.

Internal reflection occurs at R. Angle between initial and final rays = angle of deviation = 90°.

10. Q_1 is positive, Q_2 is negative. Also, $|Q_1| > |Q_2|$.

Lines of force start from a positive charge and end on a negative charge, and are denser near a larger charge.

11. Internal energy $\propto$ absolute temperature. $\therefore \quad U_A = U_B$.

In the isothermal process AB,

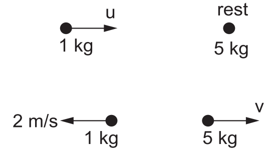
$W = nRT\ln(V_f / V_i) = p_0 V_0 \ln(V_B / V_A)$.

12. $u = -2 + 5v$

$v + 2 = -1(0 - u) = u$

Solving, $u = 3\ \text{m/s}$, $\quad v = 1\ \text{m/s}$

KE of CM $= \frac{1}{2}(6\ \text{kg})(0.5\ \text{m/s})^2 = 0.75$ J.



13. Error in measuring, $T = \frac{2\text{s}}{40} = 0.05\ \text{s} = \Delta T$

$T \propto \frac{1}{\sqrt{g}} \qquad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{1}{20} \qquad \frac{\Delta g}{g} \times 100\% = 5\%.$

14. For an oscillating system. The kinetic energy must be zero periodically, for a finite value of x. Also, $E < V_0$.

15. $V = \alpha x^4$, $\quad \therefore \quad [\alpha] = \text{ML}^{-2}\text{T}^{-2}$.

The only expression which has the dimension $\text{M}^0\text{L}^0\text{T}$ is (b).

16. For $x > X_0$, V is constant. Hence, force = 0 and therefore acceleration = 0.

17. Critical temperature decreases with increase in magnetic field.

18. Only data which agrees with the theory.

19. Phase difference $= \frac{\pi}{2}$.

20. $\omega = \sqrt{YA/mL}$.

21. $\frac{L_T}{L_B} = \frac{m_1 r_1^2}{m_2 r_2^2} + 1 = 6.$

22. $\frac{v_2}{v_1} = \sqrt{R_2^2 \rho_2 / R_1^2 \rho_1}$.

23. $Q = ms\theta + mL.$

24. $n_o = n_S\left(\dfrac{u_o + u_S}{u_s - u_o}\right) \quad (n_o)_1 - (n_o)_2 = (1.2/100)n_S.$

25. $M = |v/u| = \dfrac{f}{u+f}.$

26. $\lambda = \dfrac{h}{p} \quad \text{cf } V = \dfrac{p^2}{2m}.$

27. $J = I^2R = \left(\dfrac{\text{emf}}{R}\right)^2 R.$

28. $\lambda_m T = \text{constant}.$

Rate of radiated energy $= (4\pi R^2)(\sigma T^4).$

❑

8

IIT Questions—5

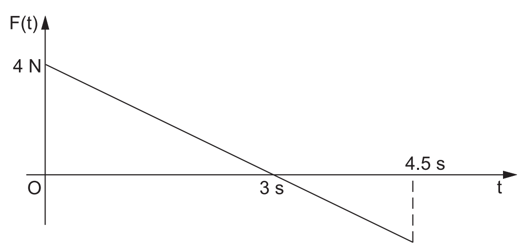
• Straight-Objective •

This section contains six multiple-choice questions numbered 1–6. Each question has four choices (a, b, c and d), out of which only one is correct.

1. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 m s^{-1}, the mass of the string is

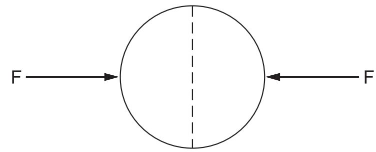
(a) 5 grams (b) 10 grams (c) 20 grams (d) 40 grams

2. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J (c) 5.06 J (d) 14.06 J

3. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$ (c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

4. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5$ V m^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m s^{-1}. Given $g = 9.8$ m s^{-2}, viscosity of the air $= 1.8\times 10^{-5}$ N s m^{-2} and the density of oil $= 900$ kg m^{-3}, the magnitude of q is

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C

(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

5. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(a) virtual and at a distance of 16 cm from the mirror

(b) real and at a distance of 16 cm from the mirror

(c) virtual and at a distance of 20 cm from the mirror

(d) real and at a distance of 20 cm from the mirror

6. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier calipers, the least count is

(a) 0.02 mm (b) 0.05 mm (c) 0.1 mm (d) 0.2 mm

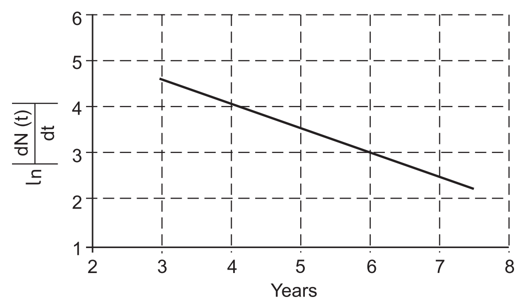
• *Integer Type* •

This section contains five questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. To determine the half-life of a radioactive element, a student plots a graph of

$\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at

time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is

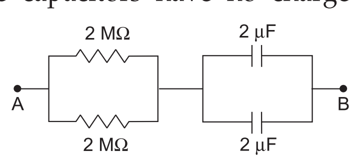


8. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour?

9. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

10. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

11. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?
[Take: $\ln 5 = 1.6$, $\ln 3 = 1.1$]



• Linked-Comprehension Type •

This section contains two paragraphs. Based upon each of the paragraphs, three multiple-choice questions (12–14) and (15–17), have to be answered.

- When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

12. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(a) $2\pi rT$ (b) $2\pi RT$ (c) $\frac{2\pi r^2 T}{R}$ (d) $\frac{2\pi R^2 T}{r}$

13. If $r = 5\times10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-2}, $T = 0.11$ N m^{-1}, the radius of the drop when it detaches from the dropper is approximately

(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m

(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

14. After the drop detaches, its surface energy is

(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J

(c) 5.4×10^{-6} J (d) 8.1×10^{-6} J

- The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

15. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

16. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then, the moment of inertia of CO molecule about its centre of mass is close to (take $h = 2\pi\times 10^{-34}$ J s)

(a) 2.76×10^{-46} kg m^2 (b) 1.87×10^{-46} kg m^2

(c) 4.67×10^{-47} kg m^2 (d) 1.17×10^{-47} kg m^2

17. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. $= \frac{5}{3}\times 10^{-27}$ kg, is close to

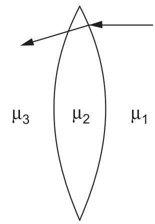
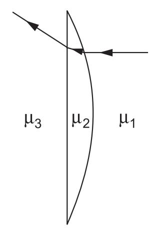
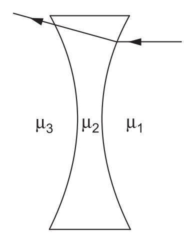
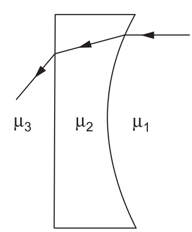
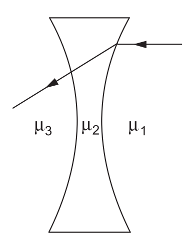
(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

• *Matrix-Matching Type* •

Match the set of parameters given in Column A with the figures given in Column B. Indicate your answer by darkening the appropriate bubbles of the matrix given in the ORS.

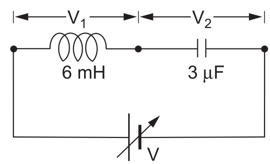
18. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **Column B**. A ray traversing these media is also shown in the figures. In **Column A** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column B**.

Column A	*Column B*
(a) $\mu_1 < \mu_2$	(p) 
(b) $\mu_1 > \mu_2$	(q) 
(c) $\mu_2 = \mu_3$	(r) 
(d) $\mu_2 > \mu_3$	(s) 
	(t) 

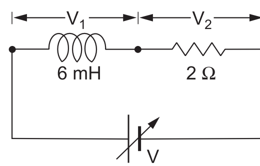
19. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column B**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **Column A**. Match the two

Column A	***Column B***
(a) $I \neq 0, V_1$ is proportional to I	(p)
(b) $I \neq 0, V_2 > V_1$	(q)
(c) $V_1 = 0, V_2 = V$	(r)
(d) $I \neq 0, V_2$ is proportional to I	(s)
	(t)

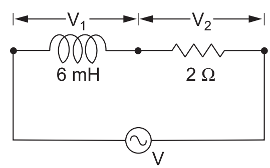
(p)



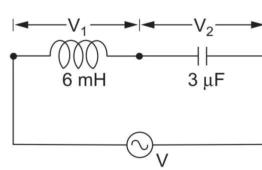
(q)



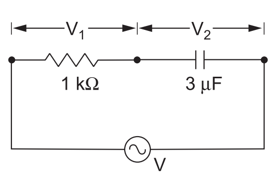
(r)



(s)



(t)



Answers

1. b	**2.** c	**3.** a	**4.** d	**5.** b
6. d	**7.** 8	**8.** 3	**9.** 6	**10.** 4
11. 2	**12.** c	**13.** a	**14.** b	**15.** d
16. b	**17.** c			

18.

	p	q	r	s	t
A	●	○	●	○	○
B	○	●	○	●	●
C	○	●	●	○	●
D	○	●	○	●	○

19.

	p	q	r	s	t
A	○	○	●	●	●
B	○	●	●	●	●
C	●	●	○	○	○
D	○	●	●	●	●

Hints and Solutions to Selected Questions

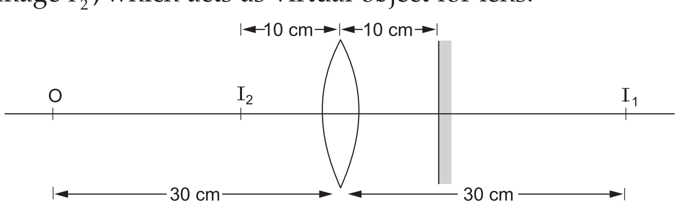
1. $\dfrac{V_{sound}}{2l_{pipe}} = 2\left[\dfrac{1}{2l_{string}}\sqrt{\dfrac{T}{m}}\right]$, mass of string $= ml_s = 10$ g.

2. Area under F–t curve $= 4.5$ kg m s $= p$, KE $= \dfrac{p^2}{2m} = 5.06$ J.

3. Only (A) has the dimension of force.

4. cf $E = \dfrac{4}{3}\pi r^3 \rho g = 6\pi\eta r v$

5. Image formed by lens (I_1) acts as object for mirror, creating image I_2, which acts as virtual object for lens.



6. $LC = 1\ MSD - 1\ VSD = 1\text{ mm} - \dfrac{16}{20}\text{ mm} = 0.2\text{ mm}.$

7. Slope of line $= -\dfrac{1}{2}$, $\quad \therefore\ \lambda = \dfrac{1}{2}\text{ yr}^{-1}$, $\quad t_{1/2} = \dfrac{0.693}{\lambda} = 1.386\text{ yr}$

$4.16\text{ yr} = 3t_{1/2}$, $\quad \therefore\ p = 2^3 = 8.$

8. $f = 10\text{ m}$ $\quad$ for $v = \dfrac{50}{7}\text{ m},\ \ u = -25\text{ m}$

for $v = \dfrac{25}{3}\text{ m},\ \ u = -50\text{ m},\ \ t = 30\text{ s}$

speed $= \left(\dfrac{25\text{ m}}{30\text{ s}}\right) = 3\text{ km/hr}$

9. $\sin C = \dfrac{1}{n} = \dfrac{3}{5} = \dfrac{R}{(R^2 + t^2)^{1/2}}$ $\quad t = 8\text{ cm} \quad R = 6\text{ cm}$

10. $TV^{\gamma-1} = \text{constant}$ $\quad T_i V^{(7/5-1)} = (aT_i)\left(\dfrac{V}{32}\right)^{(7/5-1)}$ $\quad a = 4$

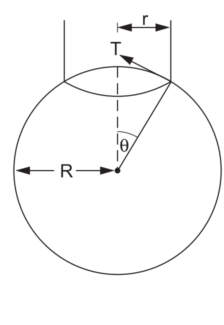
11. Time constant $\tau = RC = (10^6\ \Omega)(4\ \mu F) = 4\text{ s}$

$4\text{ V} = (10\text{ V})[1 - e^{-t/\tau}] \quad e^{-t/4} = 0.6 = 3/5$

$t/4 = \ln 5 - \ln 3 = 0.5 \quad t = 2\text{ s}$

12. Vertical upward force due to surface tension

$= (2\pi r)(T\sin\theta) = 2\pi r T\left(\dfrac{r}{R}\right) = 2\pi r^2 T/R.$



13. $2\pi r^2 T/R = mg = \left(\frac{4}{3}\pi R^3\right)\rho g.$

14. Surface energy $= 4\pi R^2 T.$

15. $\alpha = \frac{nh}{2\pi}$, Rotational KE $= \frac{\alpha^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I} = n^2\left(\frac{h^2}{8\pi^2 I}\right).$

16. $(4h/\pi)\times 10^{11} = \frac{(2^2 - 1^2)h^2}{8\pi^2 I}.$

17. $I = \left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2.$

❑