Class X

CCE Series
Mathematics

(Continuous and Comprehensive Evaluation)

Term 1: Summative Assessment - I

Formative Assessment - 1 & 2

Summative Assessment

- Model Question Papers
- Multiple Choice Questions
- Short Answer Questions
- Long Answer Questions

Formative Assessment

- Activity
- Seminar
- Project Work
- Rapid Fire Quiz
- Oral Questions
- Paper Pen Test
- Multiple Choice Questions

Xam idea

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Preface

The CCE Series seeks to provide a holistic profile to education. Focusing both on scholastic and non-scholastic facets of education, the Series stokes the positive (though dormant) attributes of the learner by way of his continuous and comprehensive evaluation. It is a complete package of the repository of knowledge, a comprehensive package of the art of learning and a continuous source of inspiration to the evolving minds.

The book has been incorporated keeping in mind the marking scheme provided by CBSE. It also comes with a purpose of providing answers to the most important questions that have been framed on a broad spectrum relating to every chapter.

Each chapter starts with basic concepts and results, thereby giving a glimpse of the chapter before the exercises begin. The aim of all the exercises, which appear in the form of Multiple Choice Questions, Short Answer Questions and Long Answer Questions is to permanently etch out the chapter and the various events constituting it in the minds of the learners. At the end of each chapter Formative Assessment has been given which appears with Activity, Project, Seminar, Oral Questions, Multiple Choice Questions, Match the Columns, Rapid fire Quiz, Class Worksheet and Paper Pen Test.

This is to make the learner self-sufficient and confident in his learning process. To make the learning process more stimulating, students also get the opportunity to experience real world problems through research works and projects. They are also encouraged to express or share their thoughts with their peers and teachers through group discussions and seminars.

To make the learning process even more fruitful and robust, one CBSE Sample Question Paper, Three Model Test Papers (Solved) and Ten Model Test Papers (Unsolved) are attached at the end of the book for learners to lay their hands on and thereby, assess their areas of weaknesses, strengths and lapses.

— Publishers

Mathematics

(April 2011 – September 2011)

Class X: Term–1

- As per CCE guidelines, the syllabus of Mathematics for class X has been divided into two terms.
- The units specified for each term shall be assessed through both formative and summative assessment.
- In each term there will be two Formative assessments and one Summative assessment.
- Listed Laboratory activities and projects will necessarily be assessed through Formative assessment.

Term one will include two Formative assessments and a term end Summative assessment. The weightages and time schedule will be as under:

Term–1

Types of Assessment	Weightage	Time Schedule
Formative Assessment–1	10%	April–May 2011
Formative Assessment–2	10%	July–August 2011
Summative Assessment–I	20%	September 2011
Total	**40%**	

Course Structure

First Term	Total Marks: 80
Units	**Marks**
I. Number Systems Real Numbers	10
II. Algebra Polynomials, Pair of Linear Equations in Two Variables	20
III. Geometry Triangles	15
IV. Trigonometry Introduction of Trigonometry	20
V. Statistics Statistics	15
Total	**80**

Unit I: Number System

1. Real Numbers **(15) Periods**

Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of results irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, decimal expansions of rational numbers in terms of terminating/non-terminating recurring decimals.

Unit II: Algebra

1. Polynomials **(7) Periods**

Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

2. Pair of Linear Equations in two Variables **(15) Periods**

Pair of linear equations in two variables and their graphical solution. Geometric representation of different possibilities of solutions/inconsistency.

Algebraic conditions for number of solutions. Solution of pair of linear equations in two variables algebraically- by substitution, by elimination and by cross multiplication. Simple situational problems must be included. Simple problems on equations reducible to linear equations may be included.

Unit III: Geometry

1. Triangles **(15) Periods**

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right triangle.

Unit IV: Trigonometry

1. Introduction to Trigonometry **(10) Periods**

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0° and 90°. Values (with proofs) of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. Trigonometric Identities **(15) Periods**

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.

Unit V: Statistics

1. Statistics **(18) Periods**

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

Continuous and Comprehensive Evaluation (CCE)

The CCE refers to a system of school based evaluation of students that covers all parameters of students' growth and development. The term 'continuous' in CCE refers to periodicity and regularity in assessment. Comprehensive on the other hand aims to cover both the scholastic and the co-scholastic aspects of a student's growth and development. The CCE intends to provide a holistic profile of the student through evaluation of both scholastic and co-scholastic areas spread over two terms during an academic year.

1. Evaluation of Scholastic Areas:

Evaluation of scholastic areas is done through two Formative assessments and one Summative assessment in each term of an academic year.

Formative Assessment

Formative assessment is a tool used by the teacher to continuously monitor student progress in a non-threatening and supportive environment. Some of the main features of the Formative assessment are:

- Encourages learning through employment of a variety of teaching aids and techniques.
- It is a diagnostic and remedial tool.
- Provides effective feedback to students so that they can act upon their problem areas.
- Allows active involvement of students in their own learning.
- Enables teachers to adjust teaching to take account of the result of assessment and to recognise the profound influence that assessment has on motivation and self-esteem of students.

If used effectively, formative assessment can improve student performance tremendously while raising the self-esteem of the child and reducing work load of the teacher.

Summative Assessment

The summative assessment is the terminal assessment of performance. It is taken by schools in the form of a pen-paper test. It 'sums-up' how much a student has learned from the course.

2. Evaluation of Co-Scholastic Areas:

Holistic education demands development of all aspects of an individual's personality including cognitive, affective and psychomotor domain. Therefore, in addition to scholastic areas (curricular or subject specific areas), co-scholastic areas like life skills, attitude and values, participation and achievement in activities involving Literary and Creative Skills, Scientific Skills, Aesthetic Skills and Performing Arts and Clubs, and Health and Physical Education should be evaluated.

Grading System

Scholastic A			
Marks Range	**Grade**	**Attributes**	**Grade Point**
91-100	A1	Exceptional	10.0
81-90	A2	Excellent	9.0
71-80	B1	Very Good	8.0
61-70	B2	Good	7.0
51-60	C1	Fair	6.0
41-50	C2	Average	5.0
33-40	D	Below Average	4.0
21-32	E1	Need to Improve	
00-20	E2	Unsatisfactory	

Scholastic B
Grade
A+
A
B+
B
C

Promotion is based on the day-to-day work of the students throughout the year and also on the performance in the terminal examination.

Chapter One

REAL NUMBERS

Basic Concepts and Results

- ***Euclid's Division Lemma*:** Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r, 0 \leq r < b$.
- ***Euclid's Division Algorithm***: This is based on Euclid's Division Lemma. According to this, the HCF of any two positive integers a and b, with $a > b$, is obtained as follows:

Step 1. Apply the division lemma to find q and r, where $a = bq + r, 0 \leq r < b$.

Step 2. If $r = 0$, the HCF is b. If $r \neq 0$, then apply Euclid's lemma to b and r.

Step 3. Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b). Also HCF (a, b) = HCF (b, r).

- ***The Fundamental Theorem of Arithmetic*:** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
- If p is a prime and p divides a^2, then p divides a, where a is a positive integer.
- If x is any rational number whose decimal expansion terminates, then we can express x in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers, then x has a decimal expansion which terminates.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers, then x has a decimal expansion which is non-terminating repeating (recurring).
- For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$
- For any three positive integers a, b and c

$$\text{LCM}(a, b, c) = \frac{a \cdot b \cdot c \ \text{HCF}(a, b, c)}{\text{HCF}(a, b) \cdot \text{HCF}(b, c) \cdot \text{HCF}(c, a)}$$

$$\text{HCF}(a, b, c) = \frac{a \cdot b \ c \cdot \text{LCM}(a, b, c)}{\text{LCM}(a, b) \cdot \text{LCM}(b, c) \cdot \text{LCM}(c, a)}$$

Summative Assessment

Multiple Choice Questions

Write the correct answer for each of the following:

1. $\sqrt{5}$ is

(*a*) an integer (*b*) an irrational number (*c*) a rational number (*d*) none of these

2. The decimal expansion of irrational number is

(*a*) terminating (*b*) non-terminating repeating

(*c*) non-terminating non-repeating (*d*) none of these

3. The decimal expansion of number $\frac{29}{2^2 \times 5 \times 7}$ is

(*a*) terminating (*b*) non-terminating repeating

(*c*) non-terminating non-repeating (*d*) none of these

4. If two positive integers a and b are written as $a = x^4 y^2$ and $b = x^2 y^3$, a, b are prime numbers, then HCF (a, b) is

(*a*) $x^4 y^3$ (*b*) xy (*c*) $x^2 y^3$ (*d*) $x^2 y^2$

5. If two positive integers a and b are written as $a = xy^2$ and $b = x^3 y$, a, b are prime numbers, then LCM (a, b) is

(*a*) $x^2 y^2$ (*b*) xy (*c*) $x^3 y^2$ (*d*) none of these

6. The product of LCM and HCF of two numbers m and n is

(*a*) $m + n$ (*b*) $m - n$ (*c*) $m \times n$ (*d*) none of these

7. The largest number which divides 615 and 963 leaving remainder 6 in each case is

(*a*) 82 (*b*) 95 (*c*) 87 (*d*) 93

8. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

(*a*) 4 (*b*) 2 (*c*) 11 (*d*) 3

9. The product of a non-zero rational and an irrational number is

(*a*) always rational (*b*) always irrational (*c*) one (*d*) rational or irrational

10. The product of two irrational numbers is

(*a*) always irrational (*b*) always rational (*c*) one (*d*) rational or irrational

11. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(*a*) 10 (*b*) 100 (*c*) 504 (*d*) 2520

12. For some integer m, every even integer is of the form

(*a*) m (*b*) $m + 1$ (*c*) $2\,m$ (*d*) $2m + 1$

13. For some integer q, every odd integer is of the form

(*a*) q (*b*) $q + 1$ (*c*) $2q$ (*d*) $2q + 1$

14. The product of two consecutive integers is divisible by

(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 7

15. The product of three consecutive integers is divisible by

(*a*) 5 (*b*) 6 (*c*) 7 (*d*) none of these

16. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number (*c*) an odd integer (*d*) an even integer

17. Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy

(*a*) $1 < r < b$ (*b*) $0 < r \le b$ (*c*) $0 \le r < b$ (*d*) $0 < r < b$

18. The decimal expansion of the rational number $\frac{47}{2^3 5^2}$ will terminate after

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) more than three decimal places

Short Answer Questions Type–I

1. The values of remainder r, when a positive integer a is divided by 3 are 0 and 1 only. Is this statement true or false? Justify your answer.

Sol. No. According to Euclid's division lemma

$$a = 3q + r, \text{ where } 0 \le r < 3$$

and r is an integer. Therefore, the values of r can be 0, 1 or 2.

2. The product of two consecutive integers is divisible by 2. Is this statement true or false? Give reason.

Sol. True, because $n(n+1)$ will always be even, as one out of the n or $(n+1)$ must be even.

3. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Sol. $3 \times 5 \times 7 + 7 = 7(3 \times 5 + 1) = 7 \times 16$, which has more than two factors.

4. Can the number 4^n, n being a natural number, end with the digit 0? Give reason.

Sol. If 4^n ends with 0, then it must have 5 as a factor. But, $(4)^n = (2^2)^n = 2^{2n}$ *i.e.*, the only prime factor of 4^n is 2. Also, we know from the fundamental theorem of arithmetic that the prime factorisation of each number is unique.

4^n can never end with 0.

5. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Sol. True, because $n(n+1)(n+2)$ will always be divisible by 6, as at least one of the factors will be divisible by 2 and at least one of the factors will be divisible by 3.

6. Write whether the square of any positive integer can be of the form $3m + 2$, where m is a natural number. Justify your answer.

Sol. No, because any positive integer can be written as $3q$, $3q+1$, $3q+2$, therefore, square will be $9q^2 = 3m$, $9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$, $9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3m + 1$.

7. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.

Sol. No, because HCF (18) does not divide LCM (380).

8. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form $\frac{p}{q}$? Give reason.

Sol. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^n$, where m, n are natural numbers.

9. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating repeating decimal expansion. Give reason for your answer.

Sol. Terminating decimal expansion, because $\frac{987}{10500} = \frac{47}{500}$ and $500 = 2^2 \times 5^3$.

Important Problems

Type A: Problems Based on Euclid's Division Algorithm

1. Use Euclid's division algorithm to find the HCF of:

(*i*) 960 and 432 (*ii*) 4052 and 12576. **[NCERT]**

Sol. (*i*) Since 960 > 432, we apply the division lemma to 960 and 432.

We have

$$960 = 432 \times 2 + 96$$

Since the remainder $96 \neq 0$, so we apply the division lemma to 432 and 96.

We have $432 = 96 \times 4 + 48$

Again remainder $48 \neq 0$, so we again apply division lemma to 96 and 48.

We have $96 = 48 \times 2 + 0$

The remainder has now become zero. So our procedure stops.

Since the divisor at this stage is 48.

Hence, HCF of 225 and 135 is 48.

i.e., HCF (960, 432) = 48

(*ii*) Since 12576 > 4052, we apply the division lemma to 12576 and 4052, to get

$$12576 = 4052 \times 3 + 420$$

Since the remainder $420 \neq 0$, we apply the division lemma to 4052 and 420, to get

$$4052 = 420 \times 9 + 272$$

We consider the new divisor 420 and the new remainder 272, and apply the division lemma to get

$$420 = 272 \times 1 + 148$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$272 = 148 \times 1 + 124$$

We consider the new divisor 148 and the new remainder 124, and apply the division lemma to get

$$148 = 124 \times 1 + 24$$

We consider the new divisor 124 and the new remainder 24, and apply the division lemma to get

$$124 = 24 \times 5 + 4$$

We consider the new divisor 24 and the new remainder 4, and apply the division lemma to get

$$24 = 4 \times 6 + 0$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 4, the HCF of 12576 and 4052 is 4.

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer. **[NCERT]**

Sol. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm, $a = 6q + r$, for some integer $q \geq 0$ and $0 \leq r < 6$.

i.e., the possible remainders are 0, 1, 2, 3, 4, 5.

Thus, a can be of the form $6q$, or $6q+1$, or $6q+2$, or $6q+3$, or $6q+4$, or $6q+5$, where q is some quotient.

Since a is odd integer, so a cannot be of the form $6q$, or $6q+2$, or $6q+4$, (since they are even).

Thus, a is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Hence, any odd positive integer is of the form $6q+1$ or $6q+3$ or $6q+5$, where q is some integer.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **[NCERT]**

Sol. For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since $616 > 32$, we apply division lemma to 616 and 32.

We have, $616 = 32 \times 19 + 8$

Here, remainder $8 \neq 0$. So, we again apply division lemma to 32 and 8.

We have, $32 = 8 \times 4 + 0$

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns is 8.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m+1$ for some integer m. **[NCERT]**

Sol. Let a be any positive integer, then it is of the form $3q$, $3q+1$ or $3q+2$. Now, we have to show that the square of these numbers can be rewritten in the form of $3m$ or $3m+1$.

Here, on squaring, we have

$$(3q)^2 = 9q^2 = 3(3q^2) = 3m, \quad \text{where } m = 3q^2$$

$$(3q+1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1, \quad \text{where } m = 3q^2 + 2q$$

and, $$(3q+2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1$$

$$= 3(3q^2 + 4q + 1) + 1 = 3m + 1, \quad \text{where } m = 3q^2 + 4q + 1.$$

Hence, square of any positive integer is either of the form $3m$ or $3m+1$.

5. Show that one and only one out of n, $n+2$, $n+4$ is divisible by 3, where n is any positive integer.

Sol. Let q be the quotient and r be the remainder when n is divided by 3.

Therefore, $n = 3q + r$, where $r = 0, 1, 2$

$n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$.

Case (*i*) if $n = 3q$, then n is divisible by 3.

Case (*ii*) if $n = 3q+1$ then $n+2 = 3q+3 = 3(q+1)$, which is divisible by 3 and $n+4 = 3q+5$, which is not divisible by 3.

So, only $(n+2)$ is divisible by 3.

Case (*iii*) if $n = 3q+2$, then $n+2 = 3q+4$, which is not divisible by 3 and $(n+4) = 3q+6 = 3(q+2)$, which is divisible by 3.

So, only $(n+4)$ is divisible by 3.

Hence one and only one out of n, $(n+2)$, $(n+4)$ is divisible by 3.

Type B: Problems Based on Prime Factorisation

1. Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method. **[NCERT]**

Sol. The prime factors of 12, 15 and 21 are

$$12 = 2^2 \times 3, 15 = 3 \times 5 \quad \text{and} \quad 21 = 3 \times 7$$

Therefore, the HCF of these integers is 3

$2^2, 3^1, 5^1$ and 7^1 are the greatest powers involved in the prime factors of 12, 15 and 21.

So, $\text{LCM}(12, 15, 21) = 2^2 \times 3^1 \times 5^1 \times 7^1 = 420.$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(*i*) 26 and 91 (*ii*) 198 and 144

Sol. (*i*) We have

$$26 = 2 \times 13$$
$$91 = 7 \times 13$$

Thus, $\text{LCM}(26, 91) = 2 \times 7 \times 13 = 182$

$\text{HCF}(26, 91) = 13$

Now, $\text{LCM}(26, 91) \times \text{HCF}(26, 91) = 182 \times 13 = 2366$

and Product of the two numbers $= 26 \times 91 = 2366$

Hence, LCM × HCF = Product of two numbers.

(*ii*)
$$144 = 2^4 \times 3^2$$
$$198 = 2 \times 3^2 \times 11$$

$\text{LCM}(198, 144) = 2^4 \times 3^2 \times 11 = 1584$

$\text{HCF}(198, 144) = 2 \times 3^2 = 18$

Now, $\text{LCM}(198, 144) \times \text{HCF}(198, 144) = 1584 \times 18 = 28512$

and product of 198 and 144 = 28512

Thus, product of LCM (198, 144) and HCF (198, 144) = Product of 198 and 144.

3. Using prime factorisation method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.

Sol. Given numbers = 30, 72, 432

$$30 = 2 \times 3 \times 5$$
$$72 = 2^3 \times 3^2$$
$$432 = 2^4 \times 3^3$$

Here, 2^1 and 3^1 are the smallest powers of the common factors 2 and 3 respectively.

So, $\text{HCF}(30, 72, 432) = 2^1 \times 3^1 = 2 \times 3 = 6$

Again, 2^4, 3^3 and 5^1 are the greatest powers of the prime factors 2, 3 and 5 respectively.

So, $\text{LCM}(30, 72, 432) = 2^4 \times 3^3 \times 5^1 = 2160$

$$\text{HCF} \times \text{LCM} = 6 \times 2160 = 12960$$

Product of numbers $= 30 \times 72 \times 432 = 933120$

Therefore, HCF × LCM ≠ Product of the numbers.

4. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point? **[NCERT]**

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

Sol. To find the time after which they meet again at the starting point, we have to find LCM of 18 and 12 minutes. We have

$$18 = 2 \times 3^2$$

and $$12 = 2^2 \times 3$$

Therefore, LCM of 18 and 12 $= 2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.

Type C: Problems Based on Decimal Expansion

1. Write down the decimal expansions of the following numbers:

(*i*) $\frac{35}{50}$ (*ii*) $\frac{15}{1600}$ **[NCERT]**

Sol. (*i*) We have, $\frac{35}{50} = \frac{35}{5^2 \times 2} = \frac{35 \times 2}{5^2 \times 2 \times 2} = \frac{70}{5^2 \times 2^2}$

$$= \frac{70}{10^2} = \frac{70}{100} = 0\ 70$$

(*ii*) We have, $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15}{2^4 \times 2^2 \times 5^2}$

$$= \frac{15}{2^4 \times (10)^2} = \frac{15 \times 5^4}{2^4 \times 5^4 \times 10^2} = \frac{15 \times 5^4}{(10)^4 \times (10)^2}$$

$$= \frac{15 \times 5^4}{10^6} = \frac{15 \times 625}{1000000} = \frac{9375}{1000000} = 0\ 009375$$

2. The decimal expansions of some real numbers are given below. In each case, decide whether they are rational or not. If they are rational, Write it in the form $\frac{p}{q}$, what can you say about the prime factors of q?

(*i*) 0.140140014000140000... (*ii*) $0.\overline{16}$

Sol. (*i*) We have, 0.140140014000140000... a non-terminating and non-repeating decimal expansion. So it is irrational. It cannot be written in the form of $\frac{p}{q}$.

(*ii*) We have, $0.\overline{16}$ a non-terminating but repeating decimal expansion. So it is rational.

Let $x = 0.\overline{16}$

Then, $x = 0.1616...$ ...(*i*)

$100x = 16.1616...$ ...(*ii*)

On subtracting (*i*) from (*ii*), we get

$$100x - x = 16.1616 - 0.1616$$

$$99x = 16 \qquad x = \frac{16}{99} = \frac{p}{q}$$

The denominator (q) has factors other than 2 or 5.

Type D: Problems Based on Rational and Irrational Numbers

1. Write a rational number between $\sqrt{3}$ and $\sqrt{5}$.

Sol. A rational number between $\sqrt{3}$ and $\sqrt{5}$ is

$$\sqrt{3\ 24} = 1\ 8 = \frac{18}{10} = \frac{9}{5}$$

2. Prove that $\sqrt{7}$ is irrational.

Sol. Let us assume, to the contrary, that $\sqrt{7}$ is rational.

Then, there exist co-prime positive integers a and b such that

$$\sqrt{7} = \frac{a}{b}, \qquad b \neq 0$$

So, $a = \sqrt{7}\, b$

Squaring on both sides, we have

$$a^2 = 7b^2 \qquad \ldots(i)$$

7 divides a^2 7 divides a

So, we can write

$$a = 7c, \qquad \text{(where } c \text{ is any integer)}$$

Putting the value of $a = 7c$ in (i), we have

$$49c^2 = 7b^2 \qquad 7c^2 = b^2$$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that $\sqrt{7}$ is rational is wrong.

Hence, we conclude that $\sqrt{7}$ is irrational.

3. Show that $5 - \sqrt{3}$ is an irrational number. **[NCERT]**

Sol. Let us assume that $5 - \sqrt{3}$ is rational.

So, $5 - \sqrt{3}$ may be written as

$5 - \sqrt{3} = \frac{p}{q}$, where p and q are integers, having no common factor except 1 and $q \neq 0$.

$$5 - \frac{p}{q} = \sqrt{3} \qquad \sqrt{3} = \frac{5q - p}{q}$$

Since $\frac{5q - p}{q}$ is a rational number as p and q are integers.

$\sqrt{3}$ is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, $5 - \sqrt{3}$ is an irrational number.

HOTS (Higher Order Thinking Skills)

1. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Sol. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required number. In other words, required number is a factor of 391.

Similarly, required positive integer is a factor of $436 - 11 = 425$ and $542 - 15 = 527$.

Clearly, required number is the HCF of 391, 425 and 527.

Using the factor tree, we get the prime factorisations of 391, 425 and 527 as follows :

$$391 = 17 \times 23, \qquad 425 = 5^2 \times 17 \qquad \text{and} \qquad 527 = 17 \times 31$$

HCF of 391, 425 and 527 is 17.

Hence, required number = 17.

2. Check whether 6^n can end with the digit 0 for any natural number n. **[NCERT]**

Sol. If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$ so the primes in factorisation of 6^n are 2 and 3. So the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes except 2 and 3 in the factorisation of 6^n. So there is no natural number n for which 6^n ends with digit zero.

3. Let a, b, c, k be rational numbers such that k is not a perfect cube.

If $a + bk^{\frac{1}{3}} + ck^{\frac{2}{3}} = 0$, then prove that $a = b = c = 0$.

Sol. Given,

$$a + bk^{\frac{1}{3}} + ck^{\frac{2}{3}} = 0 \qquad \dots (i)$$

Multiplying both sides by $k^{\frac{1}{3}}$, we have

$$ak^{\frac{1}{3}} + bk^{\frac{2}{3}} + ck = 0 \qquad \dots (ii)$$

Multiplying (i) by b and (ii) by c and then subtracting, we have

$$(ab + b^2k^{1/3} + bck^{2/3}) - (ack^{1/3} + bck^{2/3} + c^2k) = 0$$

$$(b^2 - ac)k^{1/3} + ab - c^2k = 0$$

$$b^2 - ac = 0 \quad \text{and} \quad ab - c^2k = 0 \qquad [\text{Since } k^{1/3} \text{ is irrational}]$$

$$b^2 = ac \quad \text{and} \quad ab = c^2k$$

$$b^2 = ac \quad \text{and} \quad a^2b^2 = c^4k^2$$

$$a^2(ac) = c^4k^2 \qquad [\text{By putting } b^2 = ac \text{ in } a^2b^2 = c^4k^2]$$

$$a^3c - k^2c^4 = 0 \qquad (a^3 - k^2c^3)c = 0$$

$$a^3 - k^2c^3 = 0, \text{ or } c = 0$$

Now, $a^3 - k^2c^3 = 0$`

$$k^2 = \frac{a^3}{c^3} \qquad (k^2)^{1/3} = \left(\frac{a^3}{c^3}\right)^{1/3} \qquad k^{2/3} = \frac{a}{c}$$

This is impossible as $k^{2/3}$ is irrational and $\frac{a}{c}$ is rational.

$$a^3 - k^2c^3 \neq 0$$

Hence, $c = 0$

Substituting $c = 0$ in $b^2 - ac = 0$, we get $b = 0$

Substituting $b = 0$ and $c = 0$ in $a + bk^{1/3} + ck^{2/3} = 0$, we get $a = 0$

Hence, $a = b = c = 0$.

Exercise

A. Multiple Choice Questions

Write the correct answer for each of the following:

1. $\sqrt{7}$ is

(*a*) an integer (*b*) an irrational number

(*c*) rational number (*d*) none of these

2. The decimal expansion of the rational number $\frac{33}{2^2 5}$ will terminate after

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) more than three decimal places

3. The largest number which exactly divides 70, 80, 105, 160 is

(*a*) 10 (*b*) 7 (*c*) 5 (*d*) none of these

4. The least number that is divisible by first five even numbers is

(*a*) 60 (*b*) 80 (*c*) 120 (*d*) 160

5. HCF of $(x^3 - 3x + 2)$ and $(x^2 - 4x + 3)$ is

(*a*) $(x-2)^3$ (*b*) $(x-1)(x+2)$ (*c*) $(x-1)$ (*d*) $(x-1)(x-3)$

6. LCM of $x^2 - 4$ and $x^4 - 16$ is

(*a*) $(x-2)(x+2)$ (*b*) $(x^2+4)(x-2)$ (*c*) $(x^2-4)(x+2)$ (*d*) $(x^2+4)(x^2-4)$

7. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible is

(*a*) 24 (*b*) 6 (*c*) 12 (*d*) 9

8. The largest number which divides 318 and 739 leaving remainder 3 and 4 respectively is

(*a*) 110 (*b*) 7 (*c*) 35 (*d*) 105

9. When 256 is divided by 17, remainder would be

(*a*) 16 (*b*) 1 (*c*) 14 (*d*) none of these

10. $6.\overline{6}$ is

(*a*) an integer (*b*) a rational number (*c*) an irrational number (*d*) none of these

B. Short Answer Questions Type–I

1. Write whether every positive integer can be of the form $4q + 2$, where q is an integer. Justify your answer.

2. A positive integer is of the form $3q + 1$, q being a natural number. Can you write its square in any form other than $3m + 1$ *i.e.*, 3m or $3m + 2$ for some integer m? Justify your answer.

3. Can the numbers 6^n, n being a natural number end with the digit 5? Give reasons.

4. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.

5. A rational number in its decimal expansion is 1.7112. What can you say about the prime factors of q, when this number is expressed in the form p/q?

6. What can you say about the prime factorisation of the denominators of the rational number 0.134?

C. Short Answer Questions Type–II

1. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.
2. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.
3. Prove that $2 + \sqrt{5}$ is irrational.
4. Show that the square of any odd integer is of the form $4q + 1$, for some integer q.
5. Show that $2\sqrt{3}$ is irrational.
6. Show that $\sqrt{3} + \sqrt{5}$ is irrational.
7. Show that $3 - \sqrt{5}$ is irrational.
8. Show that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.
9. Show that $\frac{1}{\sqrt{3}}$ is irrational.
10. Use Euclid's division algorithm to find the HCF of 4052 and 12576.
11. If the HCF (210, 55) is expressible in the form $210 \times 5 - 55y$, find y .
12. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.
13. Using prime factorisation method, find the LCM of 21, 28, 36, 45.
14. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.
15. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
16. Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

D. Long Answer Questions

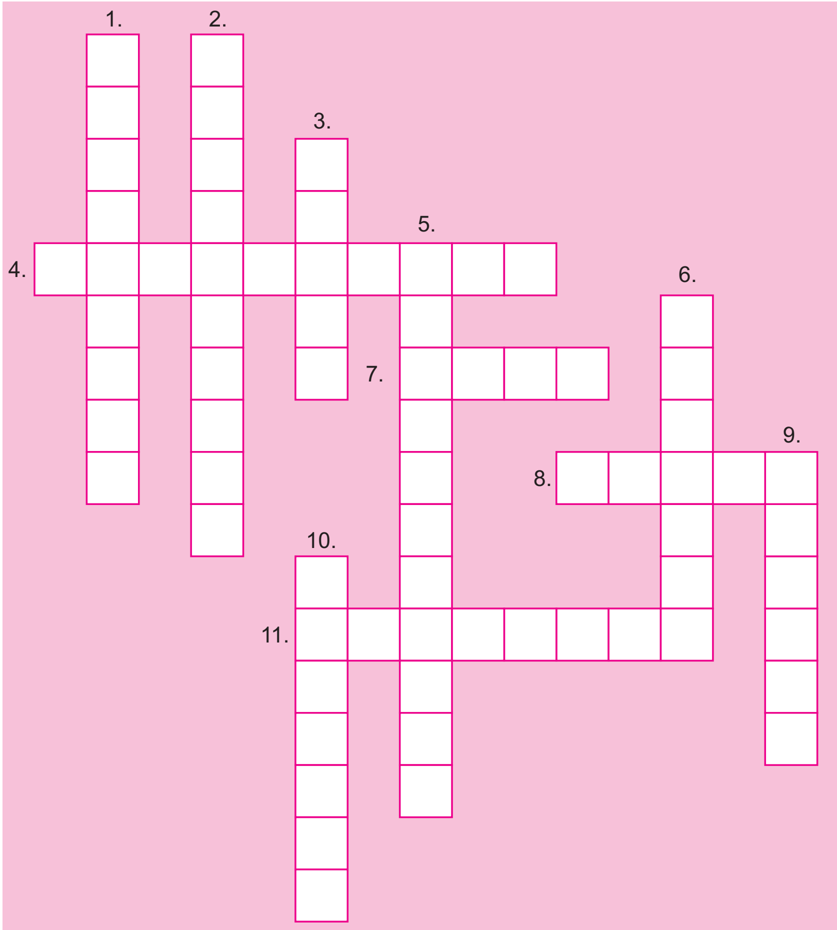
1. Show that one and only one out of n, $n + 2$, $n + 4$ is divisible by 3, where n is any positive integer.
2. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.
3. Show that cube of a positive integer of the form $6q + r$, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$.
4. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, where n is any positive integer.

   (**Hint:** Any positive integer can be written in the form of $5q$, $5q + 1$, $5q + 2$, $5q + 3$, $5q + 4$)

Formative Assessment

Activity: 1

- **Solve the following crossword puzzle, hints are given below:**



Across

4. The theorem that states that every composite number can be uniquely expressed as a product of primes, apart from the order of factors, is called fundamental theorem of __________.
7. The numbers that include rational and irrational number.
8. The number that has exactly two factors, one and the number itself.
11. The numbers that have either terminating or non-terminating repeating decimal expansion.

Down

1. A sequence of well defined steps to solve any problem, is called an ___________.
2. Numbers having non-terminating, non-repeating decimal expansion are known as ___________.
3. A proven statement used as a stepping stone towards the proof of another statement is known as ___________.
5. Decimal expansion of 7/35 is
6. The ___________ expansion of rational numbers is terminating if denominator has 2 and 5 as its only factors.
9. ____________ division algorithm is used to find the HCF of two positive numbers.
10. For any two numbers, HCF × LCM = ____________ of numbers.

Activity: 2

To build a birthday magic square of order four.

2	10	18	69

- An arrangement of different numbers in rows and columns is called a magic square if the total of the rows, the columns and the diagonals are same.

Steps of Constructing a birthday magic square of order four:

1. Draw a grid, containing four rows and four columns.
2. Write the four numbers corresponding to the birthday in first row as shown in the square grid for Mahatma Gandhi's birthday.
3. Find sum of two middle numbers of first row. Decompose this sum into two other numbers, say 12 and 16 to fill at the end cells of the corresponding fourth row.
4. Find the end sum of one diagonal. Decompose this sum into two numbers to fill in the middle cells of the other diagonal. Similarly, fill in the middle cells of the other diagonal.
5. Fill in the middle cells of fourth row, so that the sum of the numbers in 2nd and 3rd columns is same.
6. Get the sum of the end numbers of the first column. Decompose into two different numbers. Fill in the middle cells of the fourth column by these numbers.
7. Fill in the middle cells of the first column, so that the sum of the numbers in the 2nd and 3rd rows is equal. A magic square of the Mahatma Gandhi's birthday is built, which yields the same magic sum 99.

Drama

Divide your class into two groups. Ask one drama group to write and learn the properties of rational numbers, and the other to write about irrational numbers.

A drama can be played in the class, wherein two students can play the role of the King and the Prime Minister. The other two teams will present their respective properties and characteristics. The king and the prime minister will take decision on who won, on the basis of the number of properties described, variety in uses of their respective number, etc.

Role Play

- Consider yourself to be a rational number/irrational number.
- Write your properties.
- Write how you are different from other numbers.
- Write your similarities with other numbers.

Rapid Fire Quiz

State whether the following statements are true (T) or false (F).

1. Every composite number can be factorised as a product of primes and this factorisation is unique, apart from the order in which the prime factor occurs.
2. The decimal expansion of $\sqrt{5}$ is non-terminating recurring.
3. Prime factorisation of 300 is $2^2 \times 3 \times 5^2$
4. $\dfrac{\sqrt{72}}{\sqrt{50}}$ is an irrational number.
5. If $\dfrac{p}{q}$ is a rational number, such that the prime factorisation of q is of the form $2^n\ 5^m$ where n, m are non-negative integers, then $\dfrac{p}{q}$ has a decimal expansion which terminates.

6. Any positive odd integer is of the form $6p+1$ or $6p+3$ or $6p+5$, where p is some integer.
7. $\frac{7}{2^4 \times 5}$ has non-terminating decimal expansion.
8. The largest number which exactly divides 12 and 60 is 4.
9. The least number which is exactly divisible by 8 and 12 is 24.
10. If LCM and HCF of 18 and x are 36 and 6 respectively, then $x = 12$.
11. $\frac{17}{18}$ has terminating decimal expansion.

Match the Columns

Match the following columns I and II.

Column I	Column II
(*i*) $3-\sqrt{2}$ is	(*a*) a rational number
(*ii*) $\frac{\sqrt{50}}{\sqrt{18}}$ is	(*b*) an irrational number
(*iii*) 3 and 11	(*c*) non-terminating non-repeating
(*iv*) 6 and 28	(*d*) perfect numbers
(*v*) 2	(*e*) co-prime numbers
(*vi*) 1	(*f*) neither composite nor prime
(*vii*) The decimal expression of irrational numbers	(*g*) the only even prime number

Oral Questions

Answer the following in one line.

1. Define a composite number.
2. What is a prime number?
3. Is 1 a prime number? Justify your answer.
4. Can you write prime factorisation of a prime number? Justify your answer.
5. State fundamental theorem of arithmetic.
6. How will you find HCF by prime factorisation method?
7. How will you find LCM by prime factorisation method?
8. State Euclid's division lemma.
9. State Fundamental Theorem of Arithmetic.
10. What condition should be satisfied by q so that rational number $\frac{p}{q}$ has a terminating decimal expansion?
11. Is π a rational number?
12. Is $\frac{\sqrt{75}}{\sqrt{12}}$ a rational number?
13. Is there any prime number which is even?
14. Which number is neither prime nor composite?
15. Which two types of numbers constitute real numbers?

16. Is 1.203003000300003 .... a rational number? Give reason.

17. After how many decimal places the decimal expansion of the rational number $\frac{23}{2\times5^2}$ will terminate?

18. Give two irrational numbers whose product is rational.

19. What will be the HCF of two prime numbers?

20. State whether the product of two consecutive integers is even or odd.

Seminar

Study about irrational numbers from different sources: Make a presentation on inadequacy in the rational number system and then tell about the need of irrational numbers.

Multiple Choice Questions

Tick the correct answer for each of the following:

1. For some integer q, every even integer is of the form

(a) q (b) $q+1$ (c) $2q$ (d) $2q+1$

2. For some integer m, every odd integer is of the form

(a) m (b) $m+1$ (c) $2m$ (d) $2m+1$

3. The largest number which divides 85 and 77, leaving remainders 5 and 7 respectively is

(a) 5 (b) 20 (c) 35 (d) 10

4. n^2-1 is divisible by 8, if n is

(a) an integer (b) a natural number (c) an odd integer (d) an even integer

5. The least number that is divisible by all the numbers from 1 to 5 (both inclusive) is

(a) 20 (b) 30 (c) 60 (d) 120

6. The decimal expression of the rational number $\frac{44}{2^3\times5}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) more than three decimal places

7. If x and y are prime numbers, then HCF of x^3y^2 and x^2y is

(a) x^3y^2 (b) x^2y^2 (c) x^2y (d) xy

8. If $(-1)^n+(-1)^{4n}=0$, then n is

(a) any positive integer (b) any odd natural number

(c) any even natural number (d) any negative integer

9. Decimal expansion of a rational number is

(a) always terminating (b) always non-terminating

(c) either terminating or non-terminating recurring

(d) none of these

10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places (c) three decimal places (d) four decimal places

Project Work

Early History of Mathematics

Description: Outline of the major milestones in Mathematics from Euclid to Euler.

- Write your findings.

Students should mention all the sources they used to collect the information.

Class Worksheet

1.

Rational Number ($x = \frac{a}{b}$, $b \neq 0$, a and b are integers a and b are co-prime)	Decimal Expansion will terminate (Put ✓ or ✗) (If it terminates, then after how many decimal places will it terminate?	Decimal Expansion will not terminate (Put ✓ or ✗)
(*i*) $\frac{13}{1000}$		
(*ii*) $\frac{11}{122}$		
(*iii*) $\frac{37}{189}$		
(*iv*) $\frac{23}{2^3 5^2}$		
(*v*) $\frac{49}{2^7 5^2}$		

2. Tick the correct answer for each of the following:

(*i*) The decimal expansion of an irrational number is

(*a*) terminating (*b*) non-terminating recuring

(*c*) non-terminating non-recurring (*d*) none of these

(*ii*) If x and y are the prime numbers, then HCF of $x^5 y^3$ and $x^3 y^4$ is

(*a*) $x^5 y^3$ (*b*) $x^3 y^4$ (*c*) $x^5 y^4$ (*d*) $x^3 y^3$

(*iii*) The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(*a*) 10 (*b*) 100 (*c*) 504 (*d*) 2520

(*iv*) The number $3^{13} - 3^{10}$ is divisible by

(*a*) 3 and 5 (*b*) 3 and 10 (c) 2, 3 and 13 (*d*) 2, 3 and 10

(*v*) Which of the following is true?

(*a*) π is rational (*b*) 0 is natural number

(*c*) 1 is prime number (*d*) $\frac{\sqrt{48}}{\sqrt{12}}$ is rational number

(*vi*) The decimal expression of the rational number $\frac{44}{2^2 \times 5}$ will terminate after

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) more than three decimal places

3. State whether the following statements are true or false. Justify your answer.

(*i*) The product of three consecutive positive integers is divisible by 6.

(*ii*) The value of the remainder r, when a positive integer a is divided by 3 are 0 and 1 only.

4. (*i*) Show that $\sqrt{3}$ is irrational.

(*ii*) Using Euclid's division algorithm, find whether the numbers 847 and 2160 are co-prime.

5. (*i*) Using prime factorisation method, find the HCF and LCM of 336 and 54. Also show that HCF × LCM = Product of the two numbers.

(*ii*) Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Tick the correct answer for each of the following:

(*i*) The decimal expansion of the rational number $\frac{47}{2 \times 5^2}$ will terminate after

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) none of these 1

(*ii*) For some integer m, every odd integer is of the form

(*a*) m (*b*) $m + 1$ (*c*) $2m + 1$ (*d*) $2m$ 1

(*iii*) Euclid division Lemma states that if a and b are any two positive integers, then there exist unique integers q and r such that

(*a*) $a = bq + r$, $0 < r \leq b$ (*b*) $a = bq + r$, $0 \leq r < b$

(*c*) $a = bq + r^n$, $0 < q \leq b$ (*d*) $a = bq + rn$, $0 \leq r < b$ 1

(*iv*) The sum or difference of a rational and an irrational number is

(*a*) always irrational (*b*) always rational

(*c*) rational or irrational (*d*) none of these 1

(*v*) If two positive integers a and b can be expressed as $a = x^2 y^5$ and $b = x^3 y^2$; x, y being prime numbers, then L.C.M. (a, b) is

(*a*) $x^2 y^2$ (*b*) $x^3 y^3$ (*c*) $x^2 y^5$ (*d*) $x^3 y^5$ 1

(*vi*) The largest number which divides 71 and 97 leaving remainder 11 and 7 respectively is

(*a*) 15 (*b*) 20 (*c*) 60 (*d*) 30 2

2. State whether the following statements are true or false. Justify your answer.

(*i*) The product of two consecutive positive integers is divisible by 2.

(*ii*) $3 \times 5 \times 7 + 7$ is a composite number. $2 \times 2 = 4$

3. (*i*) Use Euclid's division algorithm to find the HCF of 81 and 237.

(*ii*) Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number. $3 \times 2 = 6$

4. (*i*) Prove that the product of three consecutive positive integers is divisible by 6.

(*ii*) Using prime factorisation method, find the HCF and LCM of 72, 120 and verify that LCM × HCF = product of the two numbers. $4 \times 2 = 8$

Chapter Two

POLYNOMIALS

Basic Concepts and Results

- ***Polynomial*:** An algebraic expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where $a_0, a_1, a_2, \ldots a_n$ are real numbers, n is a non-negative integer and $a_0 \neq 0$ is called a polynomial of degree n.
- ***Degree of polynomial*:** The highest power of x in a polynomial $p(x)$ is called the degree of polynomial.
- ***Types of polynomial*:**

  ***(i) Constant polynomial*:** A polynomial of degree zero is called a constant polynomial and it is of the form $p(x) = k$.

  ***(ii) Linear polynomial*:** A polynomial of degree one is called linear polynomial and it is of the form $p(x) = ax + b$, where a, b are real numbers and $a \neq 0$.

  ***(iii) Quadratic polynomial*:** A polynomial of degree two is called quadratic polynomial and it is of the form $p(x) = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

  ***(iv) Cubic polynomial*:** A polynomial of degree three is called cubic polynomial and it is of the form $p(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are real numbers and $a \neq 0$.

  ***(v) Bi-quadratic polynomial*:** A polynomial of degree four is called bi-quadratic polynomial and it is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$.
- ***Graph of polynomial*:**

  (i) Graph of a linear polynomial $p(x) = ax + b$ is a straight line.

  (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which open upwards like if $a > 0$.

  (iii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which open downwards like ∩ if $a < 0$.

  (iv) In general, a polynomial $p(x)$ of degree n crosses the x-axis at, at most n points.
- ***Zeroes of a polynomial*:** α is said to be zero of a polynomial $p(x)$ if $p(\alpha) = 0$.

  (i) Geometrically, the zeroes of a polynomial $p(x)$ are the x coordinates of the points, where the graph of $y = p(x)$ intersects the x-axis.

  (ii) A polynomial of degree 'n' can have at most n zeroes.

  That is, a quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

  (iii) 0 may be *a* zero of a polynomial.

  (iv) A non-zero constant polynomial has no zeroes.

- ***Discriminant of a quadratic polynomial*:** For polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$, the expression $b^2 - 4ac$ is known as its discriminant 'D'.

$$D = b^2 - 4ac$$

(*i*) If $D > 0$, graph of $p(x) = ax^2 + bx + c$ will intersect the x-axis at two distinct points.

The x coordinates of points of intersection with x-axis are known as 'zeroes' of $p(x)$.

(*ii*) If $D = 0$, graph of $p(x) = ax^2 + bx + c$ will touch the x-axis at exactly one point.

$p(x)$ will have only one 'zero'.

(*iii*) If $D < 0$, graph of $p(x) = ax^2 + bx + c$ will neither touch nor intersect the x-axis.

$p(x)$ will not have any real 'zero'.

- ***Relationship between the zeroes and the coefficients of a polynomial*:**

(*i*) If α, β are zeroes of $p(x) = ax^2 + bx + c$, then

Sum of zeroes $= \alpha + \beta = \dfrac{-b}{a} = \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \alpha\beta = \dfrac{c}{a} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

(*ii*) If α, β, γ are zeroes of $p(x) = ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3}$$

(*iii*) If α, β are roots of a quadratic polynomial $p(x)$, then

$$p(x) = x^2 - (\text{sum of zeroes})\, x + \text{product of zeroes}$$

$$p(x) = x^2 - (\alpha + \beta)\, x + \alpha\beta$$

(*iv*) If α, β, γ are the roots of a cubic polynomial $p(x)$, then

$$p(x) = x^3 - (\text{sum of zeroes})\, x^2 + (\text{sum of product of zeroes taken two at a time})\, x - \text{product of zeroes}$$

$$p(x) = x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\, x - \alpha\beta\gamma$$

- ***Division algorithm for polynomials*:** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = q(x) \times g(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

or Dividend = Quotient × Divisor + Remainder

***Step* 1.** Divide the highest degree term of the dividend by the highest degree term of the divisor and obtain the remainder.

***Step* 2.** If the remainder is 0 or degree of remainder is less than the divisor, then we cannot continue the division any further. If degree of remainder is equal to or more than divisor, then repeat step-1.

Summative Assessment

Multiple Choice Questions

Write the correct answer for each of the following:

1. The quadratic polynomial having zeroes –3 and 2 is

(*a*) $x^2 - x - 6$ (*b*) $x^2 + x - 6$ (*c*) $x^2 + x + 6$ (*d*) $x^2 - x + 6$

2. If $p(x) = ax^2 + bx + c$ has no real zero and $a + b + c < 0$, then

(*a*) $c = 0$ (*b*) $c < 0$ (*c*) $c > 0$ (*d*) none of these

3. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(*a*) $-\frac{c}{a}$ (*b*) $\frac{c}{a}$ (*c*) 0 (*d*) $-\frac{b}{a}$

4. A quadratic polynomial whose roots are –3 and 4 is

(*a*) $x^2 - x + 12$ (*b*) $x^2 + x + 12$ (*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (*d*) $2x^2 + 2x - 24$

5. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is –3, then the value of k is

(*a*) $\frac{4}{3}$ (*b*) $\frac{-4}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{-2}{3}$

6. If the product of two zeroes of the polynomial $p(x) = 2x^3 + 6x^2 - 4x + 9$ is 3, then its third zero is

(*a*) $\frac{-3}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{-9}{2}$ (*d*) $\frac{9}{2}$

7. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(*a*) c and a have opposite signs (*b*) c and b have opposite signs

(*c*) c and a have the same sign (*d*) c and b have the same sign

8. If one root of the polynomial $p(y) = 5y^2 + 13y + m$ is reciprocal of other, then the value of m is

(*a*) 6 (*b*) 0 (*c*) 5 (*d*) $\frac{1}{5}$

9. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(*a*) has no linear term and the constant term is negative.

(*b*) has no linear term and the constant term is positive.

(*c*) can have a linear term but the constant term is negative.

(*d*) can have a linear term but the constant term is positive.

10. If α and β are zeroes of $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to

(*a*) –1 (*b*) 1 (*c*) 2 (*d*) 0

11. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(*a*) both positive (*b*) both negative

(*c*) one positive and one negative (*d*) both equal

12. Which of the following is not the graph of a quadratic polynomial?

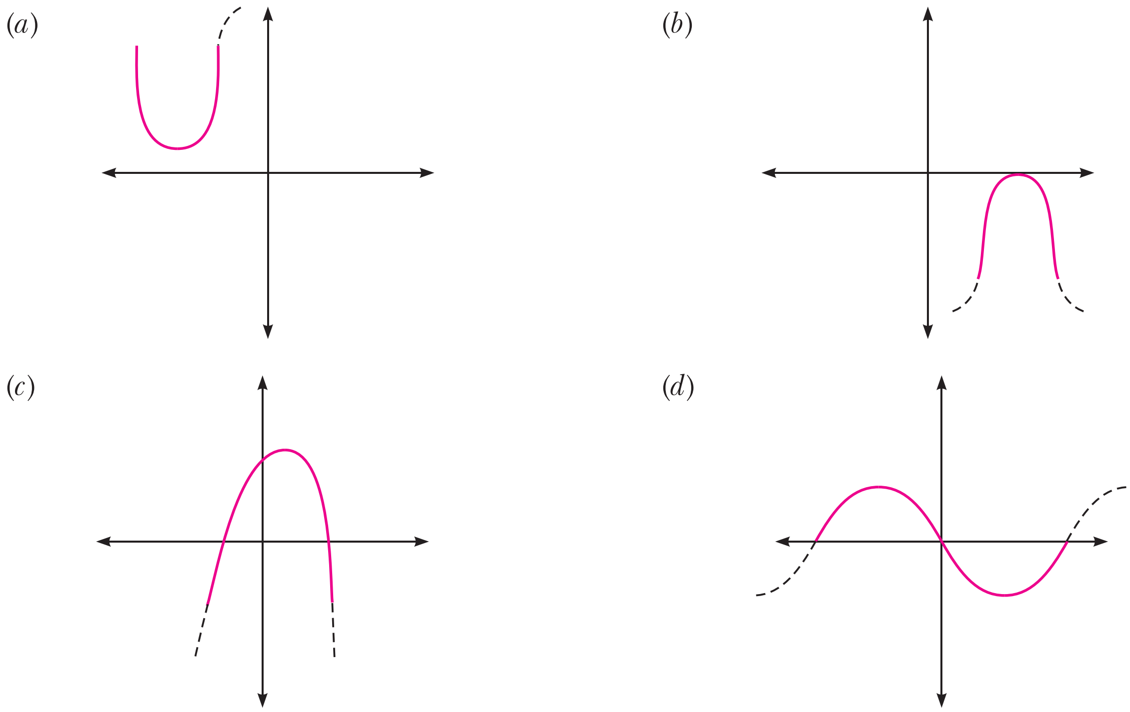


Fig. 2.1

13. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and –3, then

(*a*) $a = -7, b = -1$ (*b*) $a = 5, b = -1$ (*c*) $a = 2, b = -6$ (*d*) $a = 0, b = -6$

Short Answer Questions Type–I

The graphs of $y = p(x)$ for some polynomials (for questions 1 – 6) are given below. Find the number of zeroes in each case.

1.

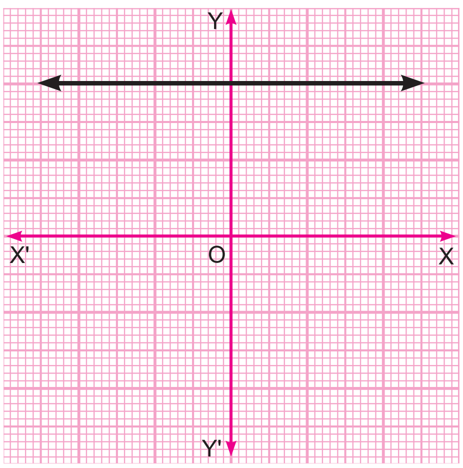


Fig. 2.2

2.

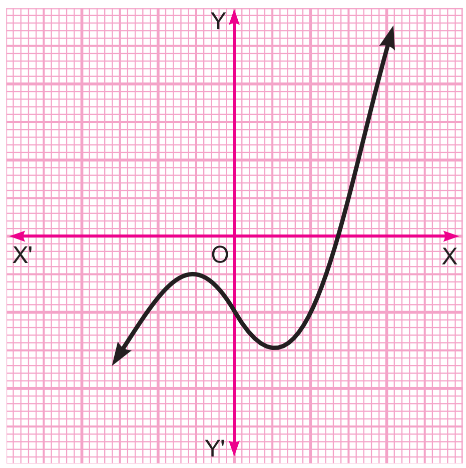


Fig. 2.3

3.

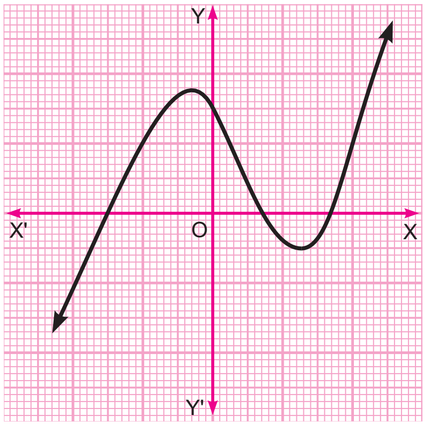


Fig. 2.4

4.

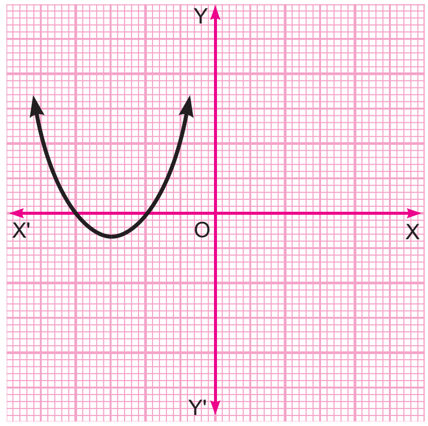


Fig. 2.5

5.

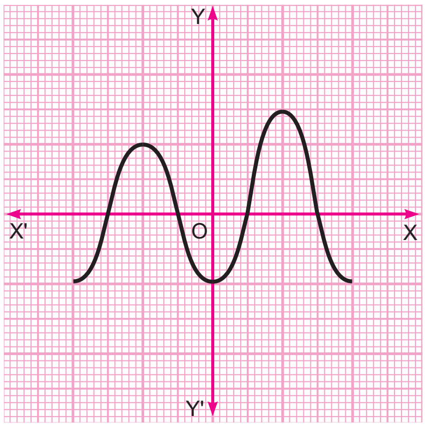


Fig. 2.6

6.

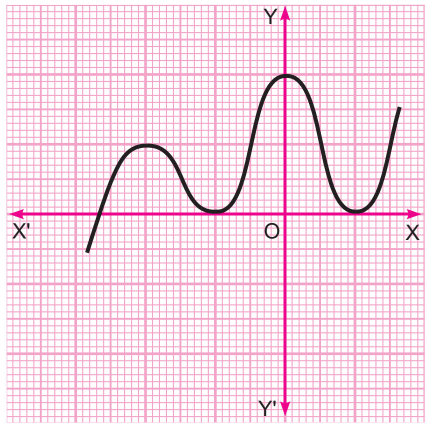


Fig. 2.7

Sol. **1.** There is no zero as the graph does not intersect the X-axis.

2. The number of zeroes is one as the graph intersects the X-axis at one point only.

3. The number of zeroes is three as the graph intersects the X-axis at three points.

4. The number of zeroes is two as the graph intersects the X-axis at two points.

5. The number of zeroes is four as the graph intersects the X-axis at four points.

6. The number of zeroes is three as the graph intersects the X-axis at three points.

Answer the following and justify:

7. Can $x-2$ be the remainder on division of a polynomial $p(x)$ by $x+3$?

Sol. No, as degree $(x-2)$ = degree $(x+3)$

8. What will the quotient and remainder be on division of ax^2+bx+c by px^3+qx^2+rx+5, $p \neq 0$?

Sol. $0,\ ax^2+bx+c$

9. Can a quadratic polynomial x^2+kx+k have equal zeroes for some odd integer $k>1$?

Sol. No, for equal zeroes, $k=0,4$

k is even

Are the following statements 'True' or 'False'? Justify your answer.

10. If the zeroes of a quadratic polynomial ax^2+bx+c are both negative, then a, b and c all have the same sign.

Sol. True, because $-\frac{b}{a}$ = sum of zeroes <0, so that $\frac{b}{a}>0$. Also the product of the zeroes $=\frac{c}{a}>0$.

11. If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

Sol. False, because every quadratic polynomial has at most two zeroes.

12. If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

Sol. True, $x^4 - 1$ is a polynomial intersecting the x-axis at exactly two points.

Important Problems

Type A: Problems Based on Zeroes and their Relationship with the Coefficients

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(*i*) $6x^2 - 3 - 7x$ (*ii*) $4u^2 + 8u$ (*iii*) $4s^2 - 4s + 1$ **[NCERT]**

Sol. (*i*) We have,

$$p(x) = 6x^2 - 3 - 7x$$

$$p(x) = 6x^2 - 7x - 3 \qquad \text{(In general form)}$$

$$= 6x^2 - 9x + 2x - 3$$

$$= 3x(2x - 3) + 1(2x - 3) = (2x - 3)(3x + 1)$$

The zeroes of polynomial $p(x)$ is given by

$$p(x) = 0$$

$$(2x - 3)(3x + 1) = 0 \qquad x = \frac{3}{2}, -\frac{1}{3}$$

Thus, the zeroes of $6x^2 - 7x - 3$ are $\alpha = \frac{3}{2}$ and $\beta = -\frac{1}{3}$

Now, sum of the zeroes $= \alpha + \beta = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6}$

and $\dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \dfrac{-(-7)}{6} = \dfrac{7}{6}$

Therefore, sum of the zeroes $= \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Again, product of zeroes $= \alpha . \beta = \frac{3}{2} \times \left(-\frac{1}{3}\right) = -\frac{1}{2}$

and $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-3}{6} = -\dfrac{1}{2}$

Therefore, product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

(*ii*) We have,

$$p(u) = 4u^2 + 8u \qquad p(u) = 4u(u + 2)$$

The zeroes of polynomial $p(u)$ is given by

$$p(u) = 0 \qquad 4u(u + 2) = 0$$

$$u = 0, -2$$

Thus, the zeroes of $4u^2 + 8u$ are $\alpha = 0$ and $\beta = -2$

Now, sum of the zeroes $= \alpha + \beta = 0 - 2 = -2$

and $\dfrac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2} = \dfrac{-8}{4} = -2$

Therefore, sum of the zeroes $= \dfrac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Again, product of the zeroes $= \alpha\beta = 0 \times (-2) = 0$

and $\dfrac{\text{Constant term}}{\text{Coefficient of } u^2} = \dfrac{0}{4} = 0$

Therefore, product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } u^2}$

(*iii*) We have,

$p(s) = 4s^2 - 4s + 1$

$p(s) = 4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1) = (2s - 1)(2s - 1)$

The zeroes of polynomial $p(s)$ is given by

$p(s) = 0$

$(2s - 1)(2s - 1) = 0$

$s = \dfrac{1}{2}, \dfrac{1}{2}$

Thus, the zeroes of $4s^2 - 4s + 1$ are

$\alpha = \dfrac{1}{2}$ and $\beta = \dfrac{1}{2}$

Now, sum of the zeroes $= \alpha + \beta = \dfrac{1}{2} + \dfrac{1}{2} = 1$

and $\dfrac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2} = \dfrac{-(-4)}{4} = 1$

Sum of the zeroes $= \dfrac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

Again, product of zeroes $= \alpha\,\beta = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

and $\dfrac{\text{Constant term}}{\text{Coefficient of } s^2} = \dfrac{1}{4}$

Product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } s^2}$

2. Verify that the numbers given alongside the cubic polynomial below are their zeroes. Also verify the relationship between the zeroes and the coefficients.

$$x^3 - 4x^2 + 5x - 2;\ \ 2, 1, 1$$

Sol. Let $p(x) = x^3 - 4x^2 + 5x - 2$

On comparing with general polynomial $p(x) = ax^3 + bx^2 + cx + d$, we get $a = 1$, $b = -4$, $c = 5$ and $d = -2$.

Given zeroes 2, 1, 1.

$$p(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$$

and $\quad p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0.$

Hence, 2, 1 and 1 are the zeroes of the given cubic polynomial.

Again, consider $\alpha = 2, \beta = 1, \gamma = 1$

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4$$

and $\quad \alpha + \beta + \gamma = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$$

and $\quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a} = \frac{5}{1} = 5$

$$\alpha\beta\gamma = (2)(1)(1) = 2$$

and $\quad \alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a} = \frac{-(-2)}{1} = 2.$

3. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(*i*) $-\frac{1}{4}, \frac{1}{4}$ (*ii*) $\sqrt{2}, \frac{1}{3}$

Sol. Let α, β be the zeroes of polynomial.

(*i*) We have, $\alpha + \beta = -\frac{1}{4}$ and $\alpha\beta = \frac{1}{4}$

Thus, polynomial is

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4} = x^2 + \frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(4x^2 + x + 1)$$

Quadratic polynomial $= 4x^2 + x + 1$

(*ii*) We have, $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

Thus, polynomial is $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - \sqrt{2}\,x + \frac{1}{3} = \frac{1}{3}(3x^2 - 3\sqrt{2}\,x + 1)$$

Quadratic polynomial $= 3x^2 - 3\sqrt{2}x + 1.$

4. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Sol. Since α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$.

$$\alpha + \beta = \frac{-(-5)}{2} = \frac{5}{2} \quad \text{and} \quad \alpha\beta = \frac{7}{2}$$

Let S and P denote respectively the sum and product of the zeroes of the required polynomial. Then,

$$S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$$

and $P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$$P = 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta = 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$$

$$P = 6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$$

Hence, the required polynomial $g(x)$ is given by

$$g(x) = k(x^2 - Sx + P)$$

or $g(x) = k\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

5. Find a cubic polynomial with the sum of the zeroes, sum of the products of its zeroes taken two at a time, and the product of its zeroes as 2, −7, −14 respectively.

Sol. Let the cubic polynomial be $p(x) = ax^3 + bx^2 + cx + d$. Then

Sum of zeroes $= \frac{-b}{a} = 2$

Sum of the products of zeroes taken two at a time $= \frac{c}{a} = -7$

and product of the zeroes $= \frac{-d}{a} = -14$

$$\frac{b}{a} = -2, \quad \frac{c}{a} = -7, \quad -\frac{d}{a} = -14 \quad \text{or} \quad \frac{d}{a} = 14$$

$$p(x) = ax^3 + bx^2 + cx + d \qquad p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$$

$$p(x) = a[x^3 + (-2)x^2 + (-7)x + 14]$$

$$p(x) = a[x^3 - 2x^2 - 7x + 14]$$

For real value of $a = 1$

$$p(x) = x^3 - 2x^2 - 7x + 14$$

6. Find the zeroes of the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$, if it is given that the product of its two zeroes is 12.

Sol. Let α, β and γ be the zeroes of polynomial $f(x)$ such that $\alpha\beta = 12$.

We have, $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-2}{1} = -2 \quad \text{and} \quad \alpha\beta\gamma = \frac{-d}{a} = \frac{-24}{1} = -24$$

Putting $\alpha\beta = 12$ in $\alpha\beta\gamma = -24$, we get

$$12\gamma = -24 \qquad \gamma = -\frac{24}{12} = -2$$

Now, $\alpha + \beta + \gamma = 5 \qquad \alpha + \beta - 2 = 5$

$$\alpha + \beta = 7 \qquad \alpha = 7 - \beta$$

$\because \alpha\beta = 12$

$$(7 - \beta)\beta = 12 \qquad 7\beta - \beta^2 = 12$$

$$\beta^2 - 7\beta + 12 = 0 \qquad \beta^2 - 3\beta - 4\beta + 12 = 0$$

$$\beta(\beta-3)-4(\beta-3)=0 \qquad (\beta-4)(\beta-3)=0$$
$$\beta=4 \quad \text{or} \quad \beta=3$$
$$\alpha=3 \quad \text{or} \quad \alpha=4$$

Type B: Problems Based on Division Algorithm for Polynomials

1. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(*i*) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$ (*ii*) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$ **[NCERT]**

Sol. (*i*) We have,

$$\begin{array}{r|l} & 3x^2-4x+2 \\ \hline x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\ & \underset{-}{3x^4}\ \underset{-}{+9x^3}\ \underset{-}{+3x^2} \\ \hline & -4x^3-10x^2+2x \\ & \underset{+}{-4x^3}\ \underset{+}{-12x^2}\ \underset{+}{-4x} \\ \hline & 2x^2+6x+2 \\ & \underset{-}{2x^2}\ \underset{-}{+6x}\ \underset{-}{+2} \\ \hline & 0 \end{array}$$

Clearly, remainder is zero, so x^2+3x+1 is a factor of polynomial $3x^4+5x^3-7x^2+2x+2$.

(*ii*) We have,

$$\begin{array}{r|l} & 2t^2+3t+4 \\ \hline t^2-3 & 2t^4+3t^3-2t^2-9t-12 \\ & \underset{-}{2t^4}\qquad \underset{+}{-6t^2} \\ \hline & 3t^3+4t^2-9t \\ & \underset{-}{3t^3}\qquad \underset{+}{-9t} \\ \hline & 4t^2-12 \\ & \underset{-}{4t^2}\ \underset{+}{-12} \\ \hline & 0 \end{array}$$

Clearly, remainder is zero, so t^2-3 is a factor of polynomial $2t^4+3t^3-2t^2-9t-12$.

2. What must be subtracted from $p(x) = 8x^4+14x^3-2x^2+7x-8$ so that the resulting polynomial is exactly divisible by $g(x) = 4x^2+3x-2$?

Sol. Let y be subtracted from polynomial $p(x)$

$$p(x)=8x^4+14x^3-2x^2+7x-8-y$$

Now,

$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 - y \\ & 8x^4 + 6x^3 - 4x^2 \\ & \underline{- \quad - \quad +} \\ & 8x^3 + 2x^2 + 7x - 8 - y \\ & 8x^3 + 6x^2 - 4x \\ & \underline{- \quad - \quad +} \\ & -4x^2 + 11x - 8 - y \\ & -4x^2 - 3x + 2 \\ & \underline{+ \quad + \quad -} \\ & 14x - 10 - y \end{array}$$

$\because$ Remainder should be 0.

$14x - 10 - y = 0$

or $14x - 10 = y$ or $y = 14x - 10$

$(14x - 10)$ should be subtracted from $p(x)$ so that it will be exactly divisible by $g(x)$.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, so $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of the given polynomial.

Now, we divide the given polynomial by $\left(x^2 - \frac{5}{3}\right)$ to obtain other zeroes.

$$\begin{array}{r|l} & 3x^2 + 6x + 3 \\ \hline x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \qquad\quad - 5x^2 \\ & \underline{- \qquad\qquad +} \\ & 6x^3 + 3x^2 - 10x \\ & 6x^3 \qquad\quad - 10x \\ & \underline{- \qquad\qquad +} \\ & 3x^2 - 5 \\ & 3x^2 - 5 \\ & \underline{- \qquad +} \\ & 0 \end{array}$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

Now, $3x^2 + 6x + 3 = 3\,(x^2 + 2x + 1) = 3(x+1)^2 = 3\,(x + 1)\,(x + 1)$

So its zeroes are $-1, -1$.

Thus, all the zeroes of given polynomial are $\sqrt{5/3}, -\sqrt{5/3}, -1$ and -1.

4. What must be added to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$?

Sol. By division algorithm, we have

$$f(x) = g(x) \times q(x) + r(x)$$

$$f(x) - r(x) = g(x) \times q(x) \qquad f(x) + \{-r(x)\} = g(x) \times q(x)$$

Clearly, RHS is divisible by $g(x)$. Therefore, LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$x^2 + 2x - 3 \overline{\big) 4x^4 + 2x^3 - 2x^2 + x - 1} \big(4x^2 - 6x + 22$$

$$\begin{array}{l} 4x^4 + 8x^3 - 12x^2 \\ - \quad - \quad + \\ \hline -6x^3 + 10x^2 + x - 1 \\ -6x^3 - 12x^2 + 18x \\ + \quad + \quad - \\ \hline 22x^2 - 17x - 1 \\ 22x^2 + 44x - 66 \\ - \quad - \quad + \\ \hline -61x + 65 \end{array}$$

$$r(x) = -61x + 65 \quad \text{or} \quad -r(x) = 61x - 65$$

Hence, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

HOTS (Higher Order Thinking Skills)

1. If α, β, γ be zeroes of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

Sol. $p(x) = 6x^3 + 3x^2 - 5x + 1$

$a = 6, b = 3, c = -5, d = 1$

$\because$ α, β and γ are zeroes of the polynomial $p(x)$.

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-3}{6} = \frac{-1}{2}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{-5}{6}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$$

Now $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-5/6}{-1/6} = 5$

2. Find the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeroes are in A.P.

Sol. If α, β, γ are in A.P., then,

$$\beta - \alpha = \gamma - \beta \qquad 2\beta = \alpha + \gamma \qquad \ldots(i)$$

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-12)}{1} = 12 \qquad \alpha + \gamma = 12 - \beta \qquad \ldots(ii)$$

From (*i*) and (*ii*)

$$2\beta = 12 - \beta \quad \text{or} \quad 3\beta = 12$$

or $\beta = 4$

Putting the value of β in (*i*), we have

$$8 = \alpha + \gamma \qquad \ldots(iii)$$

$$\alpha\beta\gamma = -\frac{d}{a} = \frac{-(-28)}{1} = 28$$

$$(\alpha\gamma)\,4 = 28 \quad \text{or} \quad \alpha\gamma = 7$$

or $\gamma = \dfrac{7}{\alpha}$ ...(*iv*)

Putting the value of $\gamma = \dfrac{7}{\alpha}$ in (*iii*), we get

$$8 = \alpha + \frac{7}{\alpha}$$

$$8\alpha = \alpha^2 + 7 \qquad \alpha^2 - 8\alpha + 7 = 0$$

$$\alpha^2 - 7\alpha - 1\alpha + 7 = 0 \qquad \alpha(\alpha - 7) - 1(\alpha - 7) = 0$$

$$(\alpha - 1)(\alpha - 7) = 0$$

$$\alpha = 1 \quad \text{or} \quad \alpha = 7$$

Putting $\alpha = 1$ in (*iv*), we get

$$\gamma = \frac{7}{1}$$

or $\gamma = 7$

and $\beta = 4$

Zeroes are 1, 7, 4.

Putting $\alpha = 7$ in (*iv*), we get

$$\gamma = \frac{7}{7}$$

or $\gamma = 1$

and $\beta = 4$

Zeroes are 7, 4, 1.

3. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$. Find k and a.

Sol. By division algorithm, we have

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

Dividend − Remainder is always divisible by the divisor.

When $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$ the remainder comes out to be $x + a$.

$$f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a)$$

$$= x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a$$

$$= x^4 - 6x^3 + 16x^2 - 26x + 10 - a$$

is exactly divisible by $x^2 - 2x + k$.

Let us now divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$.

$$x^2 - 2x + k \overline{)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a \,(}\; x^2 - 4x + (8 - k)$$

$$\underset{-\quad\;\; +\quad\;\; -}{x^4 - 2x^3 + kx^2}$$

$$-4x^3 + (16 - k)x^2 - 26x + 10 - a$$

$$\underset{+\qquad -\qquad\qquad +}{-4x^3 + 8x^2 \qquad - 4kx}$$

$$(8 - k)x^2 - (26 - 4k)x + 10 - a$$

$$\underset{-\qquad\qquad +\qquad\qquad -}{(8 - k)x^2 - (16 - 2k)x + (8k - k^2)}$$

$$(-10 + 2k)x + (10 - a - 8k + k^2)$$

For $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ to be exactly divisible by $x^2 - 2x + k$, we must have

$(-10 + 2k)x + (10 - a - 8k + k^2) = 0$ for all x

$-10 + 2k = 0$ and $10 - a - 8k + k^2 = 0$

$k = 5$ and $10 - a - 40 + 25 = 0$

$k = 5$ and $a = -5$.

Exercise

A. Multiple Choice Questions

Write the correct answer for each of the following:

1. If α, β are the zeroes of the polynomial $f(x) = x^2 - 3x + 2$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to:

(*a*) 3 (*b*) –1 (*c*) $\frac{3}{2}$ (*d*) $-\frac{3}{2}$

2. If $f(x) = ax^2 + bx + c$ has no real zeroes and $a + b + c < 0$, then:

(*a*) $c = 0$ (*b*) $c > 0$ (*c*) $c < 0$ (*d*) none of these

3. If α and $\frac{1}{\alpha}$ are the zeroes of polynomial $4x^2 - 2x + (k - 4)$, the value of k is:

(*a*) 4 (*b*) 8 (*c*) 0 (*d*) none of these

4. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is:

(*a*) 2 (*b*) 4 (*c*) –2 (*d*) –2

5. The zeroes of $\sqrt{3}x^2 + 10x + 7\sqrt{3}$ are:

(*a*) 7, 3 (*b*) $\sqrt{3}, 7\sqrt{3}$ (*c*) $-\sqrt{3}, \frac{-7}{\sqrt{3}}$ (*d*) none of these

6. If α, β are the zeroes of the polynomial $f(x) = ax^2 + bx + c$, then $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ equals to:

(*a*) $\frac{b^2 - 4ac}{a^2}$ (*b*) $\frac{b^2 - 2ac}{c^2}$ (*c*) $\frac{b^2 - 2ac}{a^2}$ (*d*) $\frac{b^2 + 2ac}{c^2}$

7. If the polynomial $f(x) = ax^3 + bx - c$ is divisible by the polynomial $g(x) = x^2 + bx + c$, then the value of ab is:

(*a*) $\frac{1}{c}$ (*b*) 1 (*c*) –1 (*d*) none of these

B. Short Answer Questions Type–I

Are the following statements 'True' or 'False' (1–4)? Justify your answers.

1. If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

2. The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$.

3. If all the zeroes of a cubic polynomial are negative, then all the coefficients and constant term of the polynomial have the same sign.

4. If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

Answer the following questions and justify:

5. Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?

6. If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

7. If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

C. Short Answer Questions Type–II

1. Find the zeroes of the following polynomials and verify the relationship between the zeroes and the coefficients of the polynomials.

(*i*) $3x^2 + 4x - 4$ (*ii*) $7y^2 - \frac{11}{3}y - \frac{2}{3}$ (*iii*) $4x^2 + 5\sqrt{2}x - 3$

(*iv*) $p^2 - 30$ (*v*) $\sqrt{3}x^2 - 11x + 6\sqrt{3}$ (*vi*) $a(x^2 + 1) - x(a^2 + 1)$

(*vii*) $6x^2 + x - 2$ (*viii*) $y^2 - \frac{1}{2}y + \frac{1}{16}$

2. Verify that the numbers given alongside the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients.

(*i*) $x^3 - 2x^2 - 5x + 6$ (*ii*) –2, 1, 3

(*iii*) $2x^3 + 7x^2 + 2x - 3$ (*iv*) $-3, -1, \frac{1}{2}$

3. Find a quadratic polynomial each with the given numbers as the sum and product of the zeroes respectively.

(*i*) $\frac{2}{3}, -\frac{1}{3}$ (*ii*) $-4\sqrt{3}$

(*iii*) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$ (*iv*) $\frac{21}{8}, \frac{5}{16}$

Also find the zeroes of those polynomials by factorisation.

4. Find the cubic polynomial with the sum, sum of the products of its zeroes taken two at a time, and the products of its zeroes as –3, –8 and 2 respectively.

5. Check whether $g(x)$ is a factor of $p(x)$ by dividing the first polynomial by the second polynomial:

(*i*) $p(x) = 4x^3 + 8x + 8x^2 + 7$, $g(x) = 2x^2 - x + 1$ (*ii*) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

(*iii*) $p(x) = 13x^3 - 19x^2 + 12x + 14$, $g(x) = 2 - 2x + x^2$

6. If $(x - 2)$ is a factor of $x^3 + ax^2 + bx + 16$ and $b = 4a$, find the values of a and b.

7. (*i*) Obtain all other zeroes of $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

(*ii*) Obtain all other zeroes of $2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$.

8. Give examples of polynomials $p(x), g(x), q(x)$ and $r(x)$, which satisfy the division algorithm and

(*i*) deg $p(x)$ = deg $q(x)$ (*ii*) deg $q(x) = 0$ (*iii*) deg $r(x) = 0$

9. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, then evaluate

(*i*) $\alpha^2 + \beta^2$ (*ii*) $\alpha^3 + \beta^3$ (*iii*) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

10. If the sum of the zeroes of the quadratic polynomial $f(x) = kx^2 + 2x + 3k$ is equal to their product, find the value of k.

11. If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - p(t + 1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$.

D. Long Answer Questions

1. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 7x - 6x$, find a polynomial whose zeroes are

(*i*) α^2 and β^2 (*ii*) $2\alpha + 3\beta$ and $3\alpha + 2\beta$

2. Given that $\sqrt{3}$ is a zero of the polynomial $x^3 + x^2 - 3x - 3$, find its other two zeroes.

3. On dividing the polynomial $f(x) = x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, the quotient and remainder are $x - 3$ and $-3x + 5$ respectively. Find the polynomial $g(x)$.

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

5. What must be subtracted from $x^3 - 6x^2 + 13x - 6$ so that the resulting polynomial is exactly divisible by $x^2 + x + 1$?

6. What must be added to $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$, so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$?

7. If α, β are zeroes of polynomial $6x^2 + x - 1$, then find the value of

(*i*) $\alpha^3\beta + \alpha\beta^3$ (*ii*) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

8. If the zeroes of the polynomial $f(x) = x^3 - 3x^2 - 6x + 8$ are of the form $a - b, a, a + b$, find all the zeroes.

9. If α and β are zeroes of polynomial $f(x) = 2x^2 + 11x + 5$, then find

(*i*) $\alpha^4 + \beta^4$ (*ii*) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

10. If α and β are the zeroes of the polynomial $f(x) = 4x^2 - 5x + 1$, find a quadratic polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Formative Assessment

Activity: 1

- **Search terms related to polynomials by the clues given below:**

I	R	E	M	A	I	N	D	E	R
P	O	L	Y	N	O	M	I	A	L
D	I	V	I	D	E	N	D	X	I
E	V	A	R	I	A	B	L	E	N
G	U	R	A	S	Z	E	R	O	E
R	R	O	F	A	C	T	O	R	A
E	C	O	N	S	T	A	N	T	R
E	O	T	C	U	B	I	C	E	B
R	E	A	L	E	V	X	T	R	A
I	D	E	N	T	I	T	Y	M	S

1. The number that remains when the division is not exact.
2. An algebraic expression in which the variable has non-negative integral exponents only.
3. In division, the number being divided into.
4. A quantity that can vary in value.
5. Numbers which when multiplied together give the original number.
6. A polynomial of degree zero.
7. A collection of rational and irrational numbers.
8. Polynomial of degree three.
9. A real number at which the value of the polynomial is zero is called ____________ of the polynomial.
10. A quantity which when substituted for the unknown quantity in an equation satisfies the equation.
11. An equation which is valid for all values of its variables.
12. The highest power of a variable in a polynomial is called _____________ of the polynomial.
13. A polynomial of degree one.

Activity: 2

Geometrical method for finding zeroes of a polynomial.

Material required

A graph sheet or grid sheet.

Method

Name the values of a polynomial as y for different values of the variable in the polynomial $p(x)$, we can write $y = p(x)$.

Now, draw the graph of the polynomial $y = p(x)$ by taking some points.

x	...	...	...	...	...
y	...	...	...	...	...

Join the points to get a smooth curve.

The points of intersection of the curve with x-axis, will give the zeroes of the polynomial.

Think Discuss and Write

Justify the following statements with examples:

1. We can have a trinomial having degree 7.
2. The degree of a binomial cannot be more than two.
3. There is only one term of degree one in a monomial.
4. A cubic polynomial always has degree three.

Oral Questions

Answer the following in one line.

1. A linear polynomial can have atmost one zero. State true or false.
2. A quadratic polynomial has at least one zero. State true or false.
3. Can $(x-2)$ be the remainder of a polynomial when divided by $p(x) = 3x+4$? Justify.
4. If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
5. What will be the degree of quotient and remainder on division of x^3+3x-5 by x^2+1? Justify.
6. If the graph of a polynomial intersects the x-axis at only one point can it be a quadratic polynomial?
7. If the graph of a polynomial intersects the x-axis exactly at two points, it may not be quadratic polynomial. State true or false. Give reason.
8. If two of the zeroes of a cubic polynomial are zero, then does it have linear and constant terms? Give reason.
9. If all the zeroes of cubic polynomial are negative, what can you say about the signs of all the coefficient and the constant term? Give reason.
10. The only value of k for which the quadratic polynomial kx^2+x+k has equal zeroes is $\frac{1}{2}$, state true or false.
11. The degree of a cubic polynomial is at least 3. State true or false. Give reason.

Group Discussion

Divide the whole class into groups of 2-3 students each and ask them to discuss the examples of the following polynomials.

- Linear polynomial having no zero.
- Linear polynomial having one zero.

- Quadratic polynomial having no zero, one zero, two zeroes.
- Cubic polynomial having no zero, one zero, two zeroes, three zeroes.

Multiple Choice Questions

Tick the correct answer for each of the following:

1. If 5 is a zero of the quadratic polynomial $x^2 - kx - 15$, then the value of k is

(*a*) 2 (*b*) –2 (*c*) 4 (*d*) – 4

2. A quadratic polynomial with 3 and 2 as the sum and product of its zeroes respectively is

(*a*) $x^2 + 3x - 2$ (*b*) $x^2 - 3x + 2$ (*c*) $x^2 - 2x + 3$ (*d*) $x^2 - 2x - 3$

3. A quadratic polynomial, whose zeroes are 5 and –8 is

(*a*) $x^2 + 13x - 40$ (*b*) $x^2 + 4x - 3$ (*c*) $x^2 - 3x + 40$ (*d*) $x^2 + 3x - 40$

4. The number of polynomials having exactly two zeroes 1 and –2 is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) infinitely many

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(*a*) $\frac{-c}{a}$ (*b*) $\frac{c}{a}$ (*c*) 0 (*d*) $\frac{-b}{a}$

6. Given that two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the value of c is

(*a*) less than 0 (*b*) greater than 0 (*c*) equal to 0 (*d*) can't say

7. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(*a*) c and a have opposite signs (*b*) c and a have the same sign
(*c*) c and b have opposite signs (*d*) c and b have the same sign

8. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$

(*a*) cannot both be positive (*b*) cannot both be negative
(*c*) are always equal (*d*) are always unequal

9. The zeroes of the quadratic polynomial $x^2 + ax + b$ $a, b > 0$ are

(*a*) both positive (*b*) both negative (*c*) one positive one negative (*d*) can't say

10. The degree of the remainder $r(x)$ when $p(x) = bx^3 + cx + d$ is divided by a polynomial of degree 4 is

(*a*) less than 4 (*b*) less than 3
(*c*) equal to 3 (*d*) less than or equal to 3

11. If the graph of a polynomial intersects the x-axis at exactly two points, then it

(*a*) cannot be a linear or a cubic polynomial (*b*) can be a quadratic polynomial only
(*c*) can be a cubic or a quadratic polynomial (*d*) can be a linear or a quadratic polynomial

12. Which of the following is not the graph of a quadratic polynomial?

(*a*) (*b*) (*c*) (*d*)

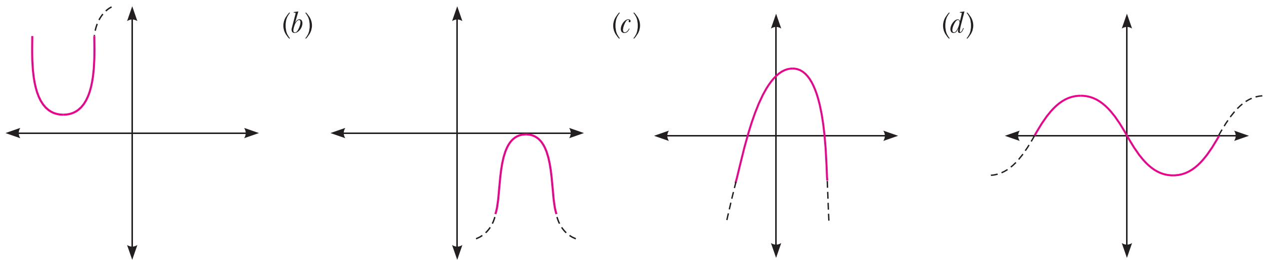


Fig. 2.8

13. If $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$, then its other two zeroes are:

(*a*) –1, –1 (*b*) 1, –1 (*c*) 1, 1 (*d*) 3, –3

14. Which of the following is a polynomial:

(*a*) $x^2 + \frac{1}{x}$ (*b*) $2x^2 - 3\sqrt{x} + 1$ (*c*) $3x^2 - 3x + 1$ (*d*) $x^2 + x^{-2} + 7$

15. The product and sum of zeroes of the quadratic polynomial $ax^2 + bx + c$ are respectively.

(*a*) $\frac{b}{a}, \frac{c}{a}$ (*b*) $\frac{c}{a}, \frac{b}{a}$ (*c*) $\frac{c}{b}, 1$ (*d*) $\frac{c}{a}, \frac{-b}{a}$

Match the Columns

Match the following columns I and II.

Column I	Column II
(*i*) Degree of a linear polynomial	(*a*) 3
(*ii*) Degree of a cubic polynomial	(*b*) less than 1
(*iii*) Degree of quotient when a cubic polynomial is divided by a linear polynomial.	(*c*) 2
(*iv*) Degree of remainder when $p(x) = x^2 + kx + k$ is divided by $q(x) = x^2 + 1$.	(*d*) 1
(*v*) Degree of $g(x)$ when $p(x) = x^3 + 1$ is divided by $g(x)$ and quotient is zero.	(*e*) less than or equal to 3
(*vi*) Degree of $g(x)$ when $p(x) = x^3 + 1$ is divided by $g(x)$ and remainder is a constant.	(*f*) greater than 3

Class Worksheet

Rapid Fire Quiz

Divide your class into two groups and each group would be given two minutes to answer as many questions.

1. State whether the following statements are true (T) or false (F).

(*i*) A polynomial having two variables is called a quadratic polynomial.

(*ii*) A cubic polynomial has at least one zero.

(*iii*) A quadratic polynomial can have atmost two zeroes.

(*iv*) If $r(x)$ is the remainder and $p(x)$ is the divisor, then deg $r(x) \le$ deg $p(x)$.

(*v*) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both negative, then *a*, *b* and *c* all have the same sign.

(*vi*) The quadratic polynomial $x^2 + kx + k$ can have equal zeroes for some odd integer $k > 1$.

(*vii*) If the graph of a polynomial intersects the *x*-axis at exactly two points, it can be a cubic polynomial.

(*viii*) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of *a*, *b* and *c* is non-negative.

(*ix*) The degree of a quadratic polynomial is less than or equal to 2.

(*x*) The degree of a constant polynomial is not defined.

(*xi*) The degree of a zero polynomial is not defined.

2. Tick the correct answer for each of the following:

(*i*) If one of the zeroes of the quadratic polynomial $(k-1)x^2+kx+1$ is –3, then the value of k is

(*a*) $\frac{4}{3}$ (*b*) $-\frac{4}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{-2}{3}$

(*ii*) A quadratic polynomial, whose zeroes are 4 and –6, is

(*a*) $x^2-2x-24$ (*b*) x^2-4x+6 (*c*) $x^2+2x-24$ (*d*) $x^2-2x+24$

(*iii*) Given that two of the zeroes of the cubic polynomial ax^3+bx^2+cx+d are 0, the third zero is

(*a*) $\frac{-b}{a}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{c}{a}$ (*d*) $\frac{-d}{a}$

(*iv*) The zeroes of the quadratic polynomial $x^2-34x+288$ are

(*a*) both positive (*b*) both negative

(*c*) both equal (*d*) one positive and one negative

(*v*) If a polynomial of degree 5 is divided by a polynomial of degree 3, then the degree of the remainder is

(*a*) less than 5 (*b*) less than 3

(*c*) less than or equal to 3 (*d*) less than 2

(*vi*) The graph of a quadratic polynomial intersects the x-axis at

(*a*) exactly two points (*b*) at least one point

(*c*) atmost two points (*d*) less than two points

3. State true or false for the following statements and justify your answer.

(*i*) If the graph of a polynomial intersects the x-axis at only one point, it is necessarily a linear polynomial.

(*ii*) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, then degree of $g(x) \le$ degree of $p(x)$.

4. (*i*) Find the zeroes of the polynomial $x^2+\frac{1}{6}x-2$ and verify the relation between the coefficients and the zeroes of the polynomial.

(*ii*) Divide the polynomial $p(x) = 4x^4-11x^2+3x-7$ by the polynomial $g(x) = 4-x^2$ and find the quotient and remainder.

5. (*i*) Find a quadratic polynomial, the sum and product of whose zeroes are $-2\sqrt{3}$ and –9, respectively. Also find its zeroes by factorisation.

(*ii*) Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$, find its other two zeroes.

6. Find the mistake in the following factorisation:

(*i*)
$$\begin{aligned} & 3x^2-4-4x \\ &= 3x^2-4x-4 \\ &= 3x^2+6x-2x-4 \\ &= 3x(x+2)-2(x+2) \\ &= (x+2)(3x-2) \end{aligned}$$

(*ii*)
$$\begin{aligned} & 3x^2-4-4x \\ &= 3x^2-4x-4 \\ &= 3x^2-6x-2x-4 \\ &= 3x(x-2)-2(x-2) \\ &= (x-2)(3x-2) \end{aligned}$$

7. Complete the solution by filling the blanks.

Step-1: Using splitting the middle term method, factorise $p(x) = 5x^2 - 4 - 8x$

$$p(x) = 5x^2 - 4 - 8x$$
$$= 5x^2 - \square\, x + \square\, x - 4$$
$$= 5x(x - \square) + 2(x - \square)$$
$$= (5x + 2)(\square - \square)$$

Step-2: To get zeroes $p(x) = 0$

____

____

zeroes are ____________, ____________

Sum of zeroes = ____________ + ____________ = $\square$ ...(*i*)

$$\frac{-(\text{Cofficient of } x)}{(\text{Coefficent of } x^2)} = -\frac{\square}{\square} \quad ...(ii)$$

Compare (*i*) and (*ii*)

Are they equal?

Product of zeroes = ____________ × ____________ = $\square$ ...(*iii*)

$$\frac{(\text{Constant term})}{(\text{Coefficent of } x^2)} = \frac{\square}{\square} \quad ...(iv)$$

Compare (*iii*) and (*iv*)

Are they equal?

Project Work

The graph of a quadratic equation has one of the two shapes either open upwards like ∪ or open downwards like ∩ depending on whether $a > 0$ or $a < 0$. Such curves are called parabolas.

Draw graphs of some quadratic polynomials with the leading coefficient a as +ve and –ve. Observe the graphs and answer the following questions:

1. What type of polynomials are represented by parabolas?

2. How many real zeroes does a quadratic polynomial have?

3. Find the number of real zeroes of the polynomials represented by each of the following parabolas.

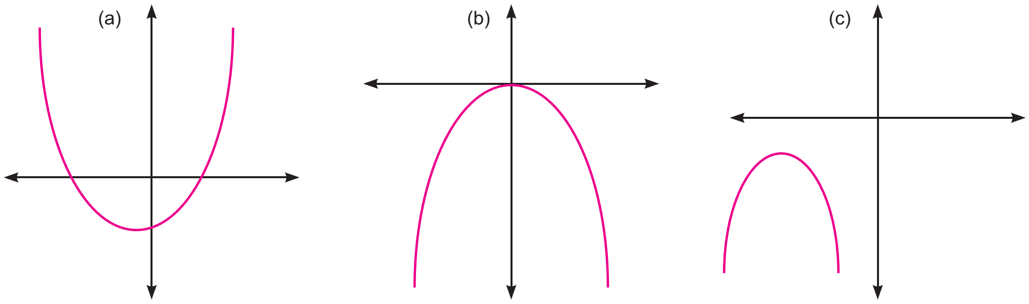


Fig. 2.9

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Write the correct answer for each of the following:

(*i*) If one zero of the quadratic polynomial $x^2 - 5x + k$ is –4 ,then the value of k is 1

(*a*) 36 (*b*) –36 (*c*) 18 (*d*) –18

(*ii*) If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and –3, then 1

(*a*) $a = -7, b = -1$ (*b*) $a = 5, b = -1$ (*c*) $a = 2, b = -6$ (*d*) $a = 0, b = -6$

(*iii*) If a polynomial of degree 6 is divided by a polynomial of degree 2, then the degree of the quotient is 1

(*a*) less than 4 (*b*) less than 2 (*c*) equal to 2 (*d*) equal to 4

(*iv*) If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is negative of the other, then it 1

(*a*) has no linear term and the constant term is negative

(*b*) has no linear term and the constant term is positive

(*c*) can have a linear term but the constant term is positive

(*d*) can have a linear term but the constant term is negative

(*v*) A quadratic polynomial with sum and product of its zeroes as 8 and –9 respectively is 1

(*a*) $x^2 - 8x + 9$ (*b*) $x^2 - 8x - 9$ (*c*) $x^2 + 8x - 9$ (*d*) $x^2 + 8x + 9$

(*vi*) If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is 2

(*a*) $a - b - 1$ (*b*) $b - a - 1$ (*c*) $b - a + 1$ (*d*) $a - b + 1$

2. State whether the following statements are true or false. Justify your answer.

(*i*) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(*ii*) The quotient and remainder on division of $2x^2 + 3x + 4$ by $x^3 + 1$ are 0 and $2x^2 + 3x + 4$ respectively. $2 \times 2 = 4$

3. (*i*) Find the zeroes of the polynomial $2x^2 + (1 + 2\sqrt{2})x + \sqrt{2}$ and verify the relation between the coefficients and the zeroes of the polynomial.

(*ii*) On dividing $8x^3 + 2x^2 - 14x + 9$ by a polynomial $g(x)$, the quotient and remainder were $(-2x + 1)$ and $(x + 3)$ respectively. Find $g(x)$. $3 \times 2 = 6$

4. (*i*) If the remainder on division of $x^3 - 2x^2 + kx + 5$ by $x - 2$ is 11, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 - 2x^2 + kx - 6$.

(*ii*) Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a + b, a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial. $4 \times 2 = 8$

Chapter Three

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Basic Concepts and Results

- ***Algebraic expression***: A combination of constants and variables, connected by four fundamental arithmetical operations of +, −, × and ÷ is called an algebraic expression.

  For example, $3x^3 + 4xy - 5y^2$ is an algebraic expression.

- ***Equation***: An algebraic expression with equal to sign (=) is called the equation. Without an equal to sign, it is an expression only.

  For example, $3x + 9 = 0$ is an equation, but only $3x + 9$ is an expression.

- ***Linear equation***: If the greatest exponent of the variable(s) in an equation is one, then equation is said to be a linear equation.

- If the number of variables used in linear equation is one, then equation is said to be linear equation in one variable.

  For example, $3x + 4 = 0$, $3y + 15 = 0$; $2t + 15 = 0$; and so on.

- If the number of variables used in linear equation is two, then equation is said to be linear equation in two variables.

  For example, $3x + 2y = 12$; $4x + 6z = 24$, $3y + 4t = 15$, etc.

  Thus, equations of the form $ax + by + c = 0$, where a, b are non-zero real numbers (*i.e.*, $a, b \neq 0$) are called linear equations in two variables.

- ***Solution***: Solution(s) is/are the value/values for the variable(s) used in equation which make(s) the two sides of the equation equal.

- Two linear equations of the form $ax + by + c = 0$, taken together form a system of linear equations, and pair of values of x and y satisfying each one of the given equation is *called a solution of the system.*

- To get the solution of simultaneous linear equations, two methods are used :

  (*i*) Graphical method (*ii*) Algebraic method

- ***Graphical Method***

  (*a*) If two or more pairs of values for x and y which satisfy the given equation are joined on paper, we get the *graph of the given equation.*

  (*b*) Every solution $x = a$, $y = b$ (where a and b are real numbers), of the given equation determines a point (a, b) which lies on the graph of line.

  (*c*) Every point (c, d) lying on the line determines a solution $x = c$, $y = d$ of the given equation. Thus, line is known as the graph of the given equation.

  (*d*) When $a \neq 0$, $b = 0$ and $c \neq 0$, then the equation $ax + by + c = 0$ becomes $ax + c = 0$ or $x = -\frac{c}{a}$. Then the graph of this equation is a *straight line parallel to y-axis* and passing through a point $\left(-\frac{c}{a}, 0\right)$.

(*e*) When $a = 0$, $b \neq 0$ and $c \neq 0$, then the equation $ax + by + c = 0$ becomes $by + c = 0$ or $y = -\frac{c}{b}$,

Then the graph of the equation is a *straight line parallel to x-axis* and passing through the point $\left(0, -\frac{c}{b}\right)$.

(*f*) When $a \neq 0$, $b = 0$ and $c = 0$, then the equation becomes $ax = 0$ or $x = 0$. *Then the graph is y-axis itself.*

(*g*) When $a = 0$, $b \neq 0$, and $c = 0$, then equation becomes $by = 0$ or $y = 0$. Then the graph *of this equation is x-axis itself.*

(*h*) When only $c = 0$, then the equation becomes $ax + by = 0$. Then the graph of this *equation is a line passing through the origin.*

(*i*) The graph of $\boldsymbol{x}$ = ***constant*** is a line parallel to the y-axis.

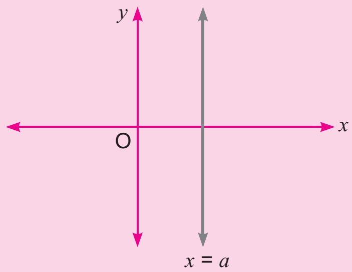


Fig. 3.1

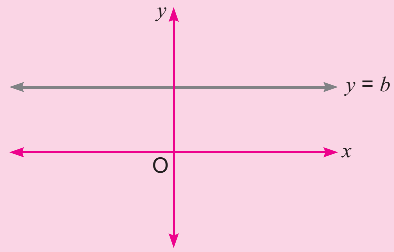


Fig. 3.2

(*j*) The graph of $\boldsymbol{y}$ = ***constant*** is a line parallel to the x-axis.

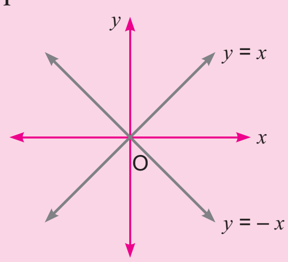


Fig. 3.2

(*k*) The graph of $y = \pm x$ is a line passing through the origin.

(*l*) The graph of a pair of linear equations in two variables is represented by two lines.

(*i*) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is *consistent*.

(*ii*) If the lines coincide, then there are infinitely many solutions—each point on the line being a solution. In this case, the pair of equations is dependent (*consistent*).

(*iii*) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is *inconsistent*.

■ ***Algebraic Method***

(*a*) Substitution Method

(*b*) Method of Elimination

(*c*) Cross-multiplication method.

Suppose $a_1x + b_1y + c_1 = 0$ ... (*i*)

$a_2x + b_2y + c_2 = 0$ ... (*ii*)

be a system of simultaneous linear equations in two variables x and y such that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, that is, $a_1b_2 - a_2b_1 \neq 0$. Then the system has a unique solution given by

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

- ***Conditions for solvability (or consistency)***
- If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise:

  (*i*) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

  In this case, the pair of linear equations has a unique solution (consistent pair of equations)

  (*ii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

  In this case, the pair of linear equations has no solution (inconsistent pair of equations)

  (*iii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

  In this case, the pair of linear equations has infinitely many solutions [dependent (consistent) pair of equations].

Summative Assessment

Multiple Choice Questions

Write the correct answer for each of the following:

1. The pair of equations $6x - 7y = 1$ and $3x - 4y = 5$ has

(*a*) a unique solution (*b*) two solutions
(*c*) infinitely many solutions (*d*) no solution

2. The number of solutions of the pair of equations $2x + 5y = 10$ and $6x + 15y - 30 = 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) infinite

3. The value of k for which the system of equations $x + 3y - 4 = 0$ and $2x + ky = 7$ is inconsistent is

(*a*) $\frac{21}{4}$ (*b*) $\frac{1}{6}$ (*c*) 6 (*d*) $\frac{4}{21}$

4. The value of k for which the system of equations $kx - y = 2$, $6x - 2y = 3$ has a unique solution is

(*a*) $= 0$ (*b*) $= 3$ (*c*) $\neq 0$ (*d*) $\neq 3$

5. If the system of equations

$$2x + 3y = 7$$
$$(a + b)x + (2a - b)y = 21$$

has infinitely many solutions, then

(*a*) $a = 1, b = 5$ (*b*) $a = -1, b = 5$ (*c*) $a = 5, b = 1$ (*d*) $a = 5, b = -1$

6. If $am \neq bl$, then the system of equations

$$ax + by = c, \quad lx + my = n$$

(*a*) has a unique solution (*b*) has no solution
(*c*) has infinitely many solutions (*d*) may or may not have a solution

7. If $2x-3y=7$ and $(a+b)x-(a+b-3)y=4a+b$ represent coincident lines, then a and b satisfy the equation

(a) $a+5b=0$ (b) $5a+b=0$ (c) $a-5b=0$ (d) $5a-b=0$

8. The pair of equations $x=a$ and $y=b$ graphically represent lines which are

(a) parallel (b) intersecting at (b,a) (c) coincident (d) intersecting at (a,b)

9. If the lines given by $3x+2ky=2$ and $2x+5y+1=0$ are parallel, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{15}{4}$ (d) $\frac{3}{2}$

10. A pair of linear equations which has a unique solution $x=3, y=-2$ is

(a) $\begin{matrix} x+y=-1 \\ 2x-3y=12 \end{matrix}$ (b) $\begin{matrix} 2x+5y+4=0 \\ 4x+10y+8=0 \end{matrix}$ (c) $\begin{matrix} 2x-y=1 \\ 3x+2y=0 \end{matrix}$ (d) $\begin{matrix} x-4y=14 \\ 5x-y=13 \end{matrix}$

11. Gunjan has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1, and ₹ 2 coins are respectively

(a) 25 and 25 (b) 15 and 35 (c) 35 and 15 (d) 35 and 20

12. The sum of the digits of a two digit number is 12. If 18 is subtracted from it, the digits of the number get reversed. The number is

(a) 57 (b) 75 (c) 84 (d) 48

Short Answer Questions Type–I

1. Does the following pair of equations represent a pair of coincident lines? Justify your answer.

$$\frac{x}{2}+y+\frac{2}{5}=0, \qquad 4x+8y+\frac{5}{16}=0$$

Sol. No. Here, $a_1=\frac{1}{2},\ b_1=1,\ c_1=\frac{2}{5}$ and $a_2=4,\ b_2=8,\ c_2=\frac{5}{16}$

$$\frac{a_1}{a_2}=\frac{\frac{1}{2}}{4}=\frac{1}{8}, \quad \frac{b_1}{b_2}=\frac{1}{8}, \quad \frac{c_1}{c_2}=\frac{\frac{2}{5}}{\frac{5}{16}}=\frac{32}{25}$$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

The given system represents parallel lines.

2. Does the following pair of linear equations have no solution? Justify your answer.

$$x=2y, \quad y=2x$$

Sol. Here, $\frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{-2}{-1}=2$

$\because \quad \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

The given system has a unique solution.

3. Is the following pair of linear equations consistent? Justify your answer.

$$2ax+by=a, \quad 4ax+2by-2a=0; \quad a,b\neq 0$$

Sol. Yes,

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The given system of equations is consistent.

4. For all real values of c, the pair of equations

$$x - 2y = 8, \quad 5x + 10y = c$$

have a unique solution. Justify whether it is true or false.

Sol. Here, $\frac{a_1}{a_2} = \frac{1}{5}$, $\frac{b_1}{b_2} = \frac{-2}{+10} = \frac{-1}{5}$, $\frac{c_1}{c_2} = \frac{8}{c}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, for all real values of c, the given pair of equations have a unique solution.

The given statement is true.

5. Write the number of solutions of the following pair of linear equations:

$$x + 2y - 8 = 0, \quad 2x + 4y = 16$$

Sol. Here, $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The given pair of linear equations has infinitely many solutions.

Important Problems

Type A: Solution of System of Linear Equations Using Different Methods (Graphical or Algebraic)

1. Form the pair of linear equations in this problem, and find their solutions graphically : 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz. **[NCERT]**

Sol. Let x be the number of girls and y be the number of boys.

According to question, we have

$$x = y + 4$$

$$x - y = 4 \quad \ldots(i)$$

Again, total number of students = 10

Therefore, $x + y = 10 \quad \ldots(ii)$

Hence, we have following system of equations

$$x - y = 4$$

$$x + y = 10$$

From equation (i), we have the following table:

x	0	4	7
y	– 4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have

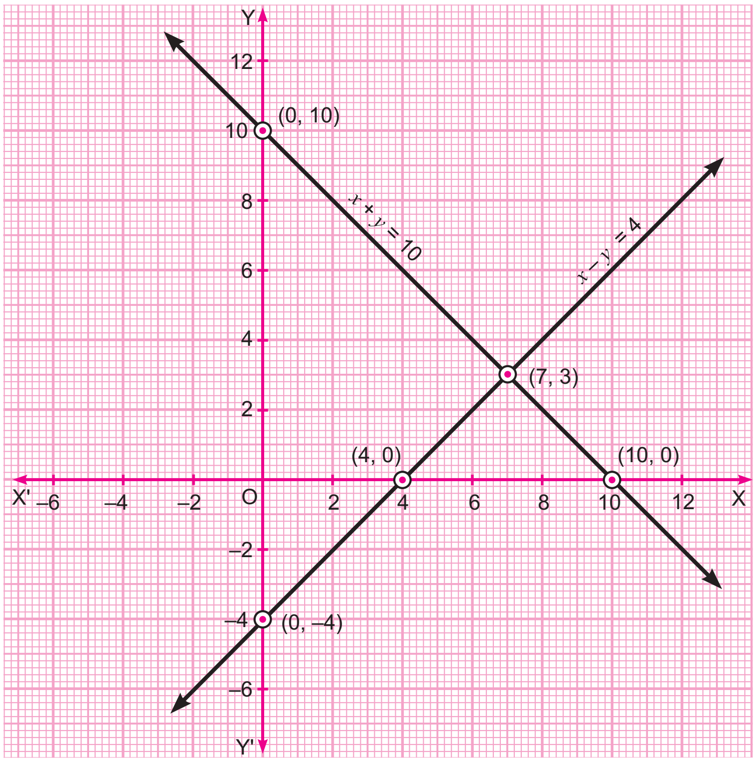


Fig. 3.4

Here, two lines intersect at point (7, 3) *i.e.*, $x = 7$, $y = 3$.

So, the number of girls = 7

and number of boys = 3.

2. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region. **[NCERT]**

Sol. We have, $x - y + 1 = 0$

and $3x + 2y - 12 = 0$

Thus, $x - y = -1 \quad x = y - 1$ ...(i)

$3x + 2y = 12 \quad x = \dfrac{12 - 2y}{3}$ ...(ii)

From equation (i), we have

x	−1	0	2
y	0	1	3

From equation (*ii*), we have

x	0	4	2
y	6	0	3

Plotting this, we have

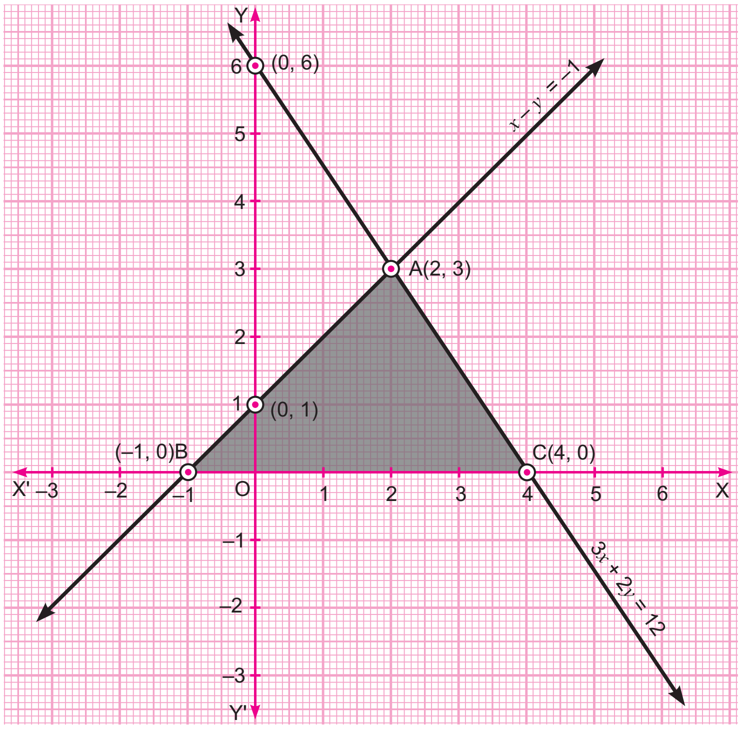


Fig. 3.5

ABC is the required (shaded) region.

Point of intersection is (2, 3).

The vertices of the triangle are $(-1, 0), (4, 0), (2, 3)$.

3. Show graphically the given system of equations

$$2x + 4y = 10$$
$$3x + 6y = 12$$

has no solution.

Sol. We have, $2x + 4y = 10 \qquad 4y = 10 - 2x \qquad y = \frac{5 - x}{2}$

When $x = 1$, we have $y = \frac{5 - 1}{2} = 2$

When $x = 3$, we have $y = \frac{5 - 3}{2} = 1$

When $x = 5$, we have $y = \frac{5 - 5}{2} = 0$

Thus, we have the following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5, 0) on the graph paper. Join A, B and C and extend it on both sides to obtain the graph of the equation $2x + 4y = 10$.

We have, $3x + 6y = 12$ $\quad 6y = 12 - 3x$ $\quad y = \dfrac{4 - x}{2}$

When $x = 2$, we have $y = \dfrac{4-2}{2} = 1$

When $x = 0$, we have $y = \dfrac{4-0}{2} = 2$

When $x = 4$, we have $y = \dfrac{4-4}{2} = 0$

Thus, we have the following table :

x	2	0	4
y	1	2	0

Plot the points D(2, 1), E (0, 2) and F (4, 0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation $3x + 6y = 12$.

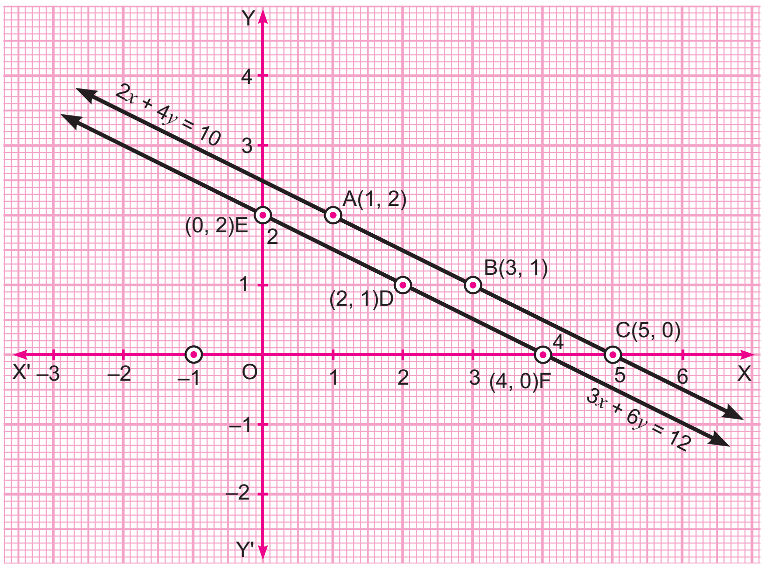


Fig. 3.6

We find that the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

4. Solve the following pairs of linear equations by the elimination method and the substitution method:

(*i*) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ $\qquad$ (*ii*) $\dfrac{x}{2} + \dfrac{2y}{3} = -1$ and $x - \dfrac{y}{3} = 3$ **[NCERT]**

Sol. (*i*) We have, $3x - 5y - 4 = 0$

$3x - 5y = 4$ ...(*i*)

Again, $9x = 2y + 7$

$9x - 2y = 7$ ...(*ii*)

By Elimination Method:

Multiplying equation (*i*) by 3, we get

$9x - 15y = 12$ ...(*iii*)

Subtracting (*ii*) from (*iii*), we get

$$\begin{array}{r} 9x - 15y = 12 \\ \underline{\_9x \mp 2y = \_7} \\ -13y = 5 \end{array}$$

$$y = -\frac{5}{13}$$

Putting the value of y in equation (*ii*), we have

$$9x - 2\left(-\frac{5}{13}\right) = 7$$

$$9x + \frac{10}{13} = 7 \qquad 9x = 7 - \frac{10}{13}$$

$$9x = \frac{91 - 10}{13} \qquad 9x = \frac{81}{13}$$

$$x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$.

By Substitution Method:

Expressing x in terms of y from equation (*i*), we have

$$x = \frac{4 + 5y}{3}$$

Substituting the value of x in equation (*ii*), we have

$$9 \times \left(\frac{4 + 5y}{3}\right) - 2y = 7$$

$$3 \times (4 + 5y) - 2y = 7 \qquad 12 + 15y - 2y = 7$$

$$13y = 7 - 12$$

$$y = -\frac{5}{13}$$

Putting the value of y in equation (*i*), we have

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \qquad 3x + \frac{25}{13} = 4$$

$$3x = 4 - \frac{25}{13} \qquad 3x = \frac{27}{13}$$

$$x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$.

(*ii*) We have

$$\frac{x}{2} + \frac{2y}{3} = -1 \qquad \frac{3x + 4y}{6} = -1$$

$$3x + 4y = -6$$

and $x - \frac{y}{3} = 3$ $\frac{3x - y}{3} = 3$

$3x - y = 9$

Thus, we have system of linear equations

$$3x + 4y = -6 \quad ...(i)$$

and $$3x - y = 9 \quad ...(ii)$$

By Elimination Method:

Subtracting (*ii*) from (*i*), we have

$$5y = -15$$

$$y = -\frac{15}{5} = -3$$

Putting the value of y in equation (*i*), we have

$$3x + 4 \times (-3) = -6$$

$$3x - 12 = -6$$

$$3x = -6 + 12 \qquad 3x = 6$$

$$x = \frac{6}{3} = 2$$

Hence, solution is $x = 2$, $y = -3$.

By Substitution Method:

Expressing x in terms of y from equation (*i*), we have

$$x = \frac{-6 - 4y}{3}$$

Substituting the value of x in equation (*ii*), we have

$$3 \times \left(\frac{-6 - 4y}{3}\right) - y = 9$$

$$-6 - 4y - y = 9$$

$$-6 - 5y = 9$$

$$-5y = 9 + 6 = 15$$

$$y = \frac{15}{-5} = -3$$

Putting the value of y in equation (*i*), we have

$$3x + 4 \times (-3) = -6 \qquad 3x - 12 = -6$$

$$3x = 12 - 6 = 6$$

$$x = \frac{6}{3} = 2$$

Hence, the required solution is $x = 2$, $y = -3$.

5. Solve: $ax + by = a - b$

$bx - ay = a + b$

Sol. The given system of equations may be written as

$$ax + by - (a - b) = 0$$

$$bx - ay - (a + b) = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{matrix} b & -(a-b) \\ -a & -(a+b) \end{matrix}} = \frac{-y}{\begin{matrix} a & -(a-b) \\ b & -(a+b) \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & -a \end{matrix}}$$

$$\frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a-b)} = \frac{1}{-a^2 - b^2}$$

$$\frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2 + b^2)}$$

$$\frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-(a^2 + b^2)} \qquad \frac{x}{-(a^2 + b^2)} = \frac{y}{(a^2 + b^2)} = \frac{1}{-(a^2 + b^2)}$$

$$x = -\frac{(a^2 + b^2)}{-(a^2 + b^2)} = 1 \quad \text{and} \quad y = \frac{(a^2 + b^2)}{-(a^2 + b^2)} = -1$$

Hence, the solution of the given system of equations is $x = 1$, $y = -1$.

6. Solve the following pairs of equations by reducing them to a pair of linear equations:

(*i*) $\dfrac{7x - 2y}{xy} = 5$

$\dfrac{8x + 7y}{xy} = 15$

(*ii*) $\dfrac{1}{3x + y} + \dfrac{1}{3x - y} = \dfrac{3}{4}$

$\dfrac{1}{2(3x + y)} - \dfrac{1}{2(3x - y)} = \dfrac{-1}{8}$ **[NCERT]**

Sol. (*i*) We have

$$\frac{7x - 2y}{xy} = 5 \qquad \frac{7x}{xy} - \frac{2y}{xy} = 5 \qquad \frac{7}{y} - \frac{2}{x} = 5$$

And,
$$\frac{8x + 7y}{xy} = 15 \qquad \frac{8x}{xy} + \frac{7y}{xy} = 15 \qquad \frac{8}{y} + \frac{7}{x} = 15$$

Let $\dfrac{1}{y} = u$ and $\dfrac{1}{x} = v$

$$7u - 2v = 5 \qquad \ldots(i)$$

$$8u + 7v = 15 \qquad \ldots(ii)$$

Multiplying (*i*) by 7 and (*ii*) by 2 and adding, we have

$$49u - 14v = 35$$
$$16u + 14v = 30$$
$$65u = 65$$
$$u = \frac{65}{65} = 1$$

Putting the value of u in equation (*i*), we have

$$7 \times 1 - 2v = 5$$
$$-2v = 5 - 7 = -2$$
$$-2v = -2$$
$$v = \frac{-2}{-2} = 1$$

Here $u = 1$ $\quad \frac{1}{y} = 1$ $\quad y = 1$

and $v = 1$ $\quad \frac{1}{x} = 1$ $\quad x = 1$

Hence, the solution of given system of equations is $x = 1$, $y = 1$.

(*ii*) We have

$$\frac{1}{3x + y} + \frac{1}{3x - y} = \frac{3}{4}$$

$$\frac{1}{2(3x + y)} - \frac{1}{2(3x - y)} = -\frac{1}{8}$$

Let $\frac{1}{3x + y} = u$ and $\frac{1}{3x - y} = v$

We have, $$u + v = \frac{3}{4} \quad \ldots(i)$$

$$\frac{u}{2} - \frac{v}{2} = -\frac{1}{8}$$

$$\frac{u - v}{2} = -\frac{1}{8}$$

$$u - v = -\frac{2}{8} = -\frac{1}{4}$$

$$u - v = -\frac{1}{4} \quad \ldots(ii)$$

Adding (*i*) and (*ii*), we have

$$u + v = \frac{3}{4}$$

$$u - v = -\frac{1}{4}$$

$$2u = \frac{3}{4} - \frac{1}{4} = \frac{3 - 1}{4} = \frac{2}{4}$$

$$u = \frac{2}{4 \times 2} = \frac{1}{4} \qquad u = \frac{1}{4}$$

Now putting the value of u in equation (*i*), we have

$$\frac{1}{4} + v = \frac{3}{4} \qquad v = \frac{3}{4} - \frac{1}{4} = \frac{3 - 1}{4} = \frac{2}{4} = \frac{1}{2} \qquad v = \frac{1}{2}$$

Here, $u = \frac{1}{4}$

$$\frac{1}{3x + y} = \frac{1}{4} \qquad 3x + y = 4 \quad \ldots(iii)$$

and $v = \frac{1}{2}$

$$\frac{1}{3x - y} = \frac{1}{2} \qquad 3x - y = 2 \quad \ldots(iv)$$

Now, adding (*iii*) and (*iv*), we have

$$3x + y = 4$$
$$3x - y = 2$$
$$6x = 6$$
$$x = \frac{6}{6} = 1$$

Putting the value of x in equation (*iii*), we have

$$3 \times 1 + y = 4 \qquad y = 4 - 3 = 1$$

Hence, the solution of given system of equations is $x = 1$, $y = 1$.

7. Solve the following linear equations:

$$152x - 378y = -74$$
$$-378x + 152y = -604$$

[NCERT]

Sol. We have, $152x - 378y = -74$ ... (*i*)

$$-378x + 152y = -604 \quad \text{... (ii)}$$

Adding equation (*i*) and (*ii*), we get

$$152x - 378y = -74$$
$$-378x + 152y = -604$$
$$-226x - 226y = -678$$

$$-226(x + y) = -678 \qquad x + y = \frac{-678}{-226}$$

$$x + y = 3 \quad \text{... (iii)}$$

Subtracting equation (*ii*) from (*i*), we get

$$152x - 378y = -74$$
$$\mp 378x \pm 152y = \mp 604$$
$$530x - 530y = 530$$
$$x - y = 1 \quad \text{... (iv)}$$

Adding equation (*iii*) and (*iv*), we get

$$x + y = 3$$
$$x - y = 1$$
$$2x = 4$$
$$x = 2$$

Putting the value of x in (*iii*), we get

$$2 + y = 3 \qquad y = 1$$

Hence, the solution of given system of equations is $x = 2$, $y = 1$.

8. Solve for x and y

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; \;\; x + y = 2ab$$

Sol. We have,

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2 \quad \text{...(i)}$$
$$x + y = 2ab \quad \text{...(ii)}$$

Multiplying (*ii*) by b / a, we get

$$\frac{b}{a}x + \frac{b}{a}y = 2b^2 \qquad \ldots(iii)$$

Subtracting (*iii*) from (*i*), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right)y = a^2 + b^2 - 2b^2$$

$$\left(\frac{a^2 - b^2}{ab}\right)y = (a^2 - b^2)$$

$$y = (a^2 - b^2) \times \frac{ab}{(a^2 - b^2)} \qquad y = ab$$

Putting the value of y in (*ii*), we get

$$x + ab = 2ab \qquad x = 2ab - ab$$

$$x = ab$$

$$x = ab,\ y = ab$$

Type B: Problems Based on Consistency or Inconsistency of Pair of Linear Equations

1. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent or inconsistent.

(*i*) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

(*ii*) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$ **[NCERT]**

Sol. (*i*) We have,

$$\frac{3}{2}x + \frac{5}{3}y = 7 \qquad \ldots(i)$$

$$9x - 10y = 14 \qquad \ldots(ii)$$

Here $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = 7$

$a_2 = 9, b_2 = -10, c_2 = 14$

Thus, $\frac{a_1}{a_2} = \frac{3}{2 \times 9} = \frac{1}{6}$, $\frac{b_1}{b_2} = \frac{5}{3 \times (-10)} = -\frac{1}{6}$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ So it has a unique solution and it is consistent.

(*ii*) We have,

$$\frac{4}{3}x + 2y = 8 \qquad \ldots(i)$$

$$2x + 3y = 12 \qquad \ldots(ii)$$

Here $a_1 = \frac{4}{3}, b_1 = 2, c_1 = 8$ and $a_2 = 2, b_2 = 3, c_2 = 12$

Thus, $\frac{a_1}{a_2} = \frac{4}{3 \times 2} = \frac{2}{3}$; $\frac{b_1}{b_2} = \frac{2}{3}$; $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so equations (*i*) and (*ii*) represent coincident lines.

Hence, the pair of linear equations is consistent with infinitely many solutions.

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:

(*i*) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

(*ii*) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

(*iii*) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

[NCERT]

Sol. (*i*) We have,

$$5x - 4y + 8 = 0 \quad \ldots(i)$$
$$7x + 6y - 9 = 0 \quad \ldots(ii)$$

Here, $a_1 = 5, b_1 = -4, c_1 = 8$

and, $a_2 = 7, b_2 = 6, c_2 = -9$

Here, $\frac{a_1}{a_2} = \frac{5}{7}$ and $\frac{b_1}{b_2} = -\frac{4}{6} = -\frac{2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ So, equations (*i*) and (*ii*) represent intersecting lines.

(*ii*) We have,

$$9x + 3y + 12 = 0 \quad \ldots(i)$$
$$18x + 6y + 24 = 0 \quad \ldots(ii)$$

Here, $a_1 = 9, b_1 = 3, c_1 = 12$

and $a_2 = 18, b_2 = 6, c_2 = 24$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so equations (*i*) and (*ii*) represent coincident lines.

(*iii*) We have,

$$6x - 3y + 10 = 0 \quad \ldots(i)$$
$$2x - y + 9 = 0 \quad \ldots(ii)$$

Here, $a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$
$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$

and $\frac{a_1}{a_2} = \frac{6}{2} = 3, \quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \quad \frac{c_1}{c_2} = \frac{10}{9}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, equations (*i*) and (*ii*) represent parallel lines.

3. (*i*) For which values of *a* and *b* does the following pair of linear equations have an infinite number of solutions ?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

(*ii*) For which value of *k* will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$ **[NCERT]**

Sol. (*i*) We have,

$$2x + 3y = 7 \quad \ldots(i)$$

$$(a - b)x + (a + b)y = 3a + b - 2 \quad \ldots(ii)$$

Here, $a_1 = 2, \quad b_1 = 3, \quad c_1 = 7$

and $a_2 = a - b, \quad b_2 = a + b, \quad c_2 = 3a + b - 2$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\dfrac{2}{a-b} = \dfrac{3}{a+b}$

$$2a + 2b = 3a - 3b \qquad 2a - 3a = -3b - 2b$$

$$-a = -5b$$

$$a = 5b \quad \ldots(iii)$$

Again, we have

$$\frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$9a + 3b - 6 = 7a + 7b \qquad 9a - 7a + 3b - 7b - 6 = 0$$

$$2a - 4b - 6 = 0 \qquad 2a - 4b = 6$$

$$a - 2b = 3 \quad \ldots(iv)$$

Putting $a = 5b$ in equation (*iv*), we get

$$5b - 2b = 3 \quad \text{or} \quad 3b = 3 \qquad i.e., \quad b = \frac{3}{3} = 1$$

Putting the value of b in equation (*iii*), we get

$$a = 5(1) = 5$$

Hence, the given system of equations will have an infinite number of solutions for $a = 5$ and $b = 1$.

(*ii*) We have,

$$3x + y = 1 \qquad 3x + y - 1 = 0 \quad \ldots(i)$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

$$(2k - 1)x + (k - 1)y - (2k + 1) = 0 \quad \ldots(ii)$$

Here, $a_1 = 3, b_1 = 1, c_1 = -1$

$a_2 = 2k - 1, b_2 = k - 1, c_2 = -(2k + 1)$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \qquad \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

Now, $\dfrac{3}{2k-1} = \dfrac{1}{k-1} \qquad 3k - 3 = 2k - 1$

$$3k - 2k = 3 - 1 \qquad k = 2$$

Hence, the given system of equations will have no solutions for $k = 2$.

4. For what value of k, will the system of equations

$$x + 2y = 5$$
$$3x + ky - 15 = 0$$

have (*i*) a unique solution ? (*ii*) no solution ?

Sol. The given system of equations can be written as

$$x + 2y = 5$$
$$3x + ky = 15$$

Here, $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{2}{k}$, $\frac{c_1}{c_2} = \frac{5}{15}$

(*i*) The given system of equations will have a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{1}{3} \neq \frac{2}{k}$ $\quad k \neq 6$

Hence, the given system of equations will have a unique solution, if $k \neq 6$.

(*ii*) The given system of equations will have no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{1}{3} = \frac{2}{k} \neq \frac{5}{15}$

$\frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{1}{3}$

$k = 6$ and $k \neq 6$, which is not possible.

Hence, there is no value of k for which the given system of equations has no solution.

Type C: Problems Based on Application of System of Linear Equations

1. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method: **[NCERT]**

(*i*) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(*ii*) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(*iii*) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deduced for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

(*iv*) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cards ?

(*v*) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol. (*i*) Let the fixed charge be ₹ x and the cost of food per day be ₹ y.

Therefore, according to question,

$$x + 20y = 1000 \qquad \ldots(i)$$
$$x + 26y = 1180 \qquad \ldots(ii)$$

Now, subtracting equation (*ii*) from (*i*), we have

$$\begin{array}{r} x + 20y = 1000 \\ \underline{-x +- 26y = -1180} \\ -6y = -180 \end{array}$$

$$y = \frac{-180}{-6} = 30$$

Putting the value of y in equation (*i*), we have

$$x + 20 \times 30 = 1000$$

$$x + 600 = 1000 \qquad x = 1000 - 600$$

$$x = 400$$

Hence, fixed charge is ₹ 400

and cost of food per day is ₹ 30.

(*ii*) Let the numerator be x and denominator be y.

$$\text{Fraction} = \frac{x}{y}$$

Now, according to question,

$$\frac{x-1}{y} = \frac{1}{3} \qquad 3x - 3 = y$$

$$3x - y = 3 \qquad \ldots(i)$$

and $$\frac{x}{y+8} = \frac{1}{4}$$

$$4x = y + 8$$

$$4x - y = 8 \qquad \ldots(ii)$$

Now, subtracting equation (*ii*) from (*i*), we have

$$\begin{array}{r} 3x - y = 3 \\ \underline{-4x \mp y = -8} \\ -x = -5 \\ x = 5 \end{array}$$

Putting the value of x in equation (*i*), we have

$$3 \times 5 - y = 3$$

$$15 - y = 3 \qquad 15 - 3 = y$$

$$y = 12$$

Hence, the required fraction is $\frac{5}{12}$

(*iii*) Let x be the number of questions of right answer and y be the number of questions of wrong answer.

According to question,

$$3x - y = 40 \qquad \ldots (i)$$

and $$4x - 2y = 50$$

or $$2x - y = 25 \qquad \ldots (ii)$$

Subtracting (*ii*) from (*i*), we have

$$\begin{array}{r} 3x - y = 40 \\ \underset{-}{2x} \underset{+}{-} y = \underset{-}{25} \\ \hline x = 15 \end{array}$$

Putting the value of x in equation (*i*), we have

$3 \times 15 - y = 40$ $\qquad$ $45 - y = 40$

$y = 45 - 40 = 5$

Hence, total number of questions is $x + y$ *i.e.,* $5 + 15 = 20$.

(*iv*) Let the speed of two cars be x km/h and y km/h respectively.

Case I: When two cars move in the same direction, they will meet each other at P after 5 hours.

Fig. 3.7

The distance covered by car from $A = 5x$ $\qquad$ (Distance = Speed × Time)

and $\quad$ distance covered by the car from $B = 5y$

$5x - 5y = AB = 100$ $\qquad$ $x - y = \dfrac{100}{5}$

$x - y = 20$ $\qquad$ ...(*i*)

Case II: When two cars move in opposite direction, they will meet each other at Q after one hour.

Fig. 3.8

The distance covered by the car from $A = x$

The distance covered by the car from $B = y$

$x + y = AB = 100$

$x + y = 100$ $\qquad$ ...(*ii*)

Now, adding equations (*i*) and (*ii*), we have

$2x = 120$ $\qquad$ $x = \dfrac{120}{2} = 60$

Putting the value of x in equation (*i*), we get

$60 - y = 20$ $\qquad$ $-y = -40$ $\qquad$ $y = 40$

Hence, the speeds of two cars are 60 km/h and 40 km/h respectively.

(*v*) Let the length and breadth of a rectangle be x and y respectively.

Then area of the rectangle = xy

According to question, we have

$(x - 5)(y + 3) = xy - 9$

$xy + 3x - 5y - 15 = xy - 9$

$3x - 5y = 15 - 9 = 6$

$3x - 5y = 6$ $\qquad$ ...(*i*)

Again, we have

$$(x+3)(y+2) = xy + 67$$
$$xy + 2x + 3y + 6 = xy + 67$$
$$2x + 3y = 67 - 6 = 61$$
$$2x + 3y = 61 \qquad \ldots(ii)$$

Now, from equation (*i*), we express the value of x in terms of y.

$$x = \frac{6+5y}{3}$$

Substituting the value of x in equation (*ii*), we have

$$2 \times \left(\frac{6+5y}{3}\right) + 3y = 61$$
$$\frac{12+10y}{3} + 3y = 61 \qquad \frac{12+10y+9y}{3} = 61$$
$$19y + 12 = 61 \times 3 = 183 \qquad 19y = 183 - 12 = 171$$
$$y = \frac{171}{19} = 9$$

Putting the value of y in equation (*i*), we have

$$3x - 5 \times 9 = 6 \qquad 3x = 6 + 45 = 51$$
$$x = \frac{51}{3} = 17$$

Hence, the length of rectangle = 17 units

and breadth of rectangle = 9 units.

2. Formulate the following problems as a pair of equations, and hence find their solutions:

(*i*) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(*ii*) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by bus and the remaining by train. If she travels 100 km by bus and the remaining by train, she takes 10 minutes longer. Find the speed of the train and the bus separately. **[NCERT]**

Sol. (*i*) Let her speed of rowing in still water be x km/h and the speed of the current be y km/h.

Case I: When Ritu rows downstream

Her speed (downstream) = $(x + y)$ km/h

Now, We have speed $= \dfrac{\text{distance}}{\text{time}}$

$$(x+y) = \frac{20}{2} = 10$$
$$x + y = 10 \qquad \ldots(i)$$

Case II: When Ritu rows upstream

Her speed (upstream) = $(x - y)$ km/h

Again, Speed $= \dfrac{\text{distance}}{\text{time}}$

$$x - y = \frac{4}{2} = 2$$
$$x - y = 2 \qquad \ldots(ii)$$

Now, adding (*i*) and (*ii*), we have

$$2x = 12 \qquad x = \frac{12}{2} = 6$$

Putting the value of x in equation (*i*), we have

$$6 + y = 10$$

$$y = 10 - 6 = 4$$

Hence, speed of Ritu in still water = 6 km/h.

and speed of current = 4 km/h.

(*ii*) Let the speed of the bus be x km/h and speed of the train be y km/h.

According to question, we have

$$\frac{60}{x} + \frac{240}{y} = 4$$

And $\quad \frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60} = 4 + \frac{1}{6} = \frac{25}{6} \qquad \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$

Now, let $\frac{1}{x} = u$ and $\frac{1}{y} = v$,

$$60u + 240v = 4 \qquad \ldots(i)$$

$$100u + 200v = \frac{25}{6} \qquad \ldots(ii)$$

Multiplying equation (*i*) by 5 and (*ii*) by 6 and subtracting, we have

$$\begin{array}{rcl} 300u + 1200v &=& 20 \\ \underset{-}{\ } 600u \ \underset{-}{+}\ 1200v &=& \underset{-}{\ } 25 \\ \hline -300u &=& -5 \end{array}$$

$$u = \frac{-5}{-300} = \frac{1}{60}$$

Putting the value of u in equation (*i*), we have

$$60 \times \frac{1}{60} + 240v = 4 \qquad 240v = 4 - 1 = 3$$

$$v = \frac{3}{240} = \frac{1}{80}$$

Now, $\quad u = \frac{1}{60} \qquad \frac{1}{x} = \frac{1}{60} \qquad x = 60$

and $\quad v = \frac{1}{80} \qquad \frac{1}{y} = \frac{1}{80} \qquad y = 80$

Hence, speed of the bus is 60 km/h and speed of the train is 80 km/h.

3. The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

Sol. Let the digits at unit and tens places be x and y respectively.

Then, Number $= 10y + x \qquad \ldots(i)$

Number formed by interchanging the digits $= 10x + y$

According to the given condition, we have

$$(10y + x) + (10x + y) = 110$$
$$11x + 11y = 110$$
$$x + y - 10 = 0$$

Again, according to question, we have

$$(10y + x) - 10 = 5(x + y) + 4$$
$$10y + x - 10 = 5x + 5y + 4$$
$$10y + x - 5x - 5y = 4 + 10$$
$$5y - 4x = 14$$

or $$4x - 5y + 14 = 0$$

By using cross-multiplication, we have

$$\frac{x}{1 \times 14 - (-5) \times (-10)} = \frac{-y}{1 \times 14 - 4 \times (-10)} = \frac{1}{1 \times (-5) - 1 \times 4}$$

$$\frac{x}{14 - 50} = \frac{-y}{14 + 40} = \frac{1}{-5 - 4} \qquad \frac{x}{-36} = \frac{-y}{54} = \frac{1}{-9}$$

$$x = \frac{-36}{-9} \quad \text{and} \quad y = \frac{-54}{-9}$$

$$x = 4 \quad \text{and} \quad y = 6$$

Putting the values of x and y in equation (i), we get

Number $= 10 \times 6 + 4 = 64$.

HOTS (Higher Order Thinking Skills)

1. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Sol. Let one man alone can finish the work in x days and one boy alone can finish the work in y days. Then,

One day work of one man $= \frac{1}{x}$

One day work of one boy $= \frac{1}{y}$

One day work of 8 men $= \frac{8}{x}$

One day work of 12 boys $= \frac{12}{y}$

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x} + \frac{12}{y}\right) = 1 \qquad \frac{80}{x} + \frac{120}{y} = 1 \qquad \ldots(i)$$

Again, 6 men and 8 boys can finish the work in 14 days

$$14\left(\frac{6}{x} + \frac{8}{y}\right) = 1 \qquad \frac{84}{x} + \frac{112}{y} = 1 \qquad \ldots(ii)$$

Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (*i*) and (*ii*), we get

$$80u + 120v - 1 = 0$$
$$84u + 112v - 1 = 0$$

By using cross-multiplication, we have

$$\frac{u}{120 \times -1 - 112 \times -1} = \frac{-v}{80 \times -1 - 84 \times -1} = \frac{1}{80 \times 112 - 84 \times 120}$$

$$\frac{u}{-120 + 112} = \frac{-v}{-80 + 84} = \frac{1}{80 \times 112 - 84 \times 120}$$

$$\frac{u}{-8} = \frac{-v}{4} = \frac{1}{-1120}$$

$$u = \frac{-8}{-1120} = \frac{1}{140} \quad \text{and} \quad v = \frac{-4}{-1120} = \frac{1}{280}$$

We have, $u = \frac{1}{140}$ $\quad \frac{1}{x} = \frac{1}{140}$ $\quad x = 140$

and $v = \frac{1}{280}$ $\quad \frac{1}{y} = \frac{1}{280}$ $\quad y = 280.$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

2. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Sol. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream $= (x - y)$ km/h

Speed downstream $= (x + y)$ km/h

Now, time taken to cover 25 km upstream $= \frac{25}{x - y}$ hours

Time taken to cover 44 km downstream $= \frac{44}{x + y}$ hours

The total time of journey is 9 hours

$$\frac{25}{x - y} + \frac{44}{x + y} = 9 \qquad \ldots(i)$$

Time taken to cover 15 km upstream $= \frac{15}{x - y}$

Time taken to cover 22 km downstream $= \frac{22}{x + y}$

In this case, total time of journey is 5 hours.

$$\frac{15}{x - y} + \frac{22}{x + y} = 5 \qquad \ldots(ii)$$

Put $\frac{1}{x - y} = u$ and $\frac{1}{x + y} = v$ in equations (*i*) and (*ii*), we get

$$25u + 44v = 9 \qquad 25u + 44v - 9 = 0 \qquad \ldots(iii)$$

$15u + 22v = 5$ $\qquad\qquad$ $15u + 22v - 5 = 0$ $\qquad$ ...(*iv*)

By cross-multiplication, we have

$$\frac{u}{44 \times (-5) - 22 \times (-9)} = \frac{-v}{25 \times (-5) - 15 \times (-9)} = \frac{1}{25 \times 22 - 15 \times 44}$$

$$\frac{u}{-220 + 198} = \frac{-v}{-125 + 135} = \frac{1}{550 - 660}$$

$$\frac{u}{-22} = \frac{-v}{10} = \frac{1}{-110} \qquad \frac{u}{22} = \frac{v}{10} = \frac{1}{110}$$

$$\frac{u}{22} = \frac{1}{110} \quad \text{and} \quad \frac{v}{10} = \frac{1}{110}$$

$$u = \frac{22}{110} = \frac{1}{5} \quad \text{and} \quad v = \frac{1}{11}$$

We have, $\quad u = \frac{1}{5} \qquad \frac{1}{x - y} = \frac{1}{5} \qquad x - y = 5$ $\qquad$ ...(*v*)

and $\quad v = \frac{1}{11} \qquad \frac{1}{x + y} = \frac{1}{11} \qquad x + y = 11$ $\qquad$ ...(*vi*)

Solving equations (*v*) and (*vi*), we get $x = 8$ and $y = 3$.

Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

3. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Sol. Let total number of rows be y

and total number of students in each row be x.

Total number of students $= xy$

Case I: If one student is extra in a row, there would be two rows less.

Now, number of rows $= (y - 2)$

Number of students in each row $= (x + 1)$

Total number of students $=$ Number of rows $\times$ Number of students in each row

$$xy = (y - 2)(x + 1)$$

$$xy = xy + y - 2x - 2$$

$$xy - xy - y + 2x = -2$$

$$2x - y = -2 \qquad \text{...}(i)$$

Case II: If one student is less in a row, there would be 3 rows more.

Now, number of rows $= (y + 3)$

and number of students in each row $= (x - 1)$

Total number of students $=$ Number of rows $\times$ Number of students in each row

$$xy = (y + 3)(x - 1)$$

$$xy = xy - y + 3x - 3$$

$$xy - xy + y - 3x = -3$$

$$-3x + y = -3 \qquad \text{...}(ii)$$

On adding equations (*i*) and (*ii*), we have

$$\begin{array}{r} 2x - y = -2 \\ -3x + y = -3 \\ \hline -x = -5 \end{array}$$

or $x = 5$

Putting the value of x in equation (*i*), we get

$$2(5) - y = -2$$
$$10 - y = -2$$
$$-y = -2 - 10 \qquad -y = -12$$

or $y = 12$

Total number of students in the class $= 5 \times 12 = 60$.

4. Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.

Sol. We have, $2x + y = 6$

$$y = 6 - 2x$$

When $x = 0$, we have $y = 6 - 2 \times 0 = 6$

When $x = 3$, we have $y = 6 - 2 \times 3 = 0$

When $x = 2$, we have $y = 6 - 2 \times 2 = 2$

Thus, we get the following table:

x	0	3	2
y	6	0	2

Now, we plot the points $A(0, 6)$, $B(3, 0)$ and $C(2, 2)$ on the graph paper. We join A, B and C and extend it on both sides to obtain the graph of the equation $2x + y = 6$.

We have, $2x - y + 2 = 0$

$$y = 2x + 2$$

When $x = 0$, we have $y = 2 \times 0 + 2 = 2$

When $x = -1$, we have $y = 2 \times (-1) + 2 = 0$

When $x = 1$, we have $y = 2 \times 1 + 2 = 4$

Thus, we have the following table:

x	0	-1	1
y	2	0	4

Now, we plot the points $D(0, 2)$, $E(-1, 0)$ and $F(1, 4)$ on the same graph paper. We join D, E and F and extend it on both sides to obtain the graph of the equation $2x - y + 2 = 0$.

It is evident from the graph that the two lines intersect at point $F(1, 4)$. The area enclosed by the given lines and x-axis is shown in Fig. 3.9.

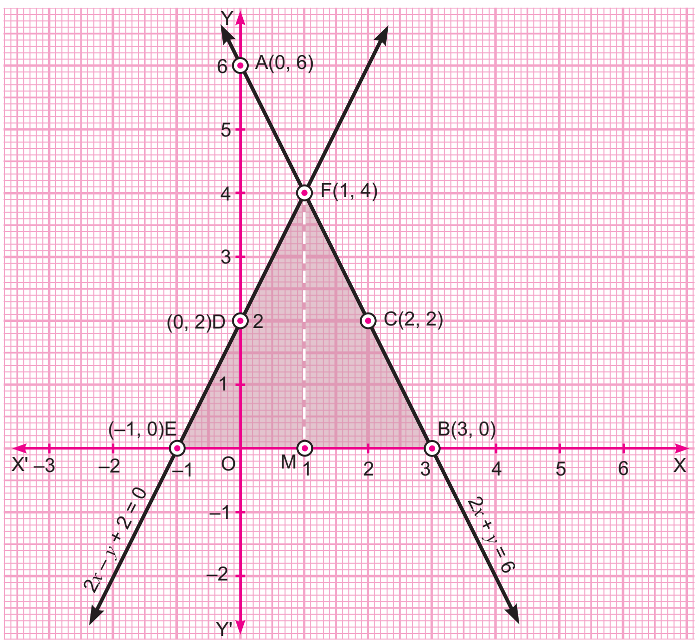


Fig. 3.9

Thus, $x = 1$, $y = 4$ is the solution of the given system of equations. Draw FM perpendicular from F on x-axis.

Clearly, we have

$FM = y$-coordinate of point $F\,(1, 4) = 4$ and $BE = 4$

Area of the shaded region = Area of ΔFBE

$$\text{Area of the shaded region} = \frac{1}{2}(\text{Base} \times \text{Height})$$

$$= \frac{1}{2}(BE \times FM)$$

$$= \left(\frac{1}{2} \times 4 \times 4\right)\text{sq. units} = 8 \text{ sq. units.}$$

5. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol. Let the ages of Ani and Biju be x and y years respectively. Then

$$x - y = \pm 3$$

Age of Dharam = $2x$ years

Age of Cathy = $\frac{y}{2}$ years

Clearly, Dharam is older than Cathy.

$$2x - \frac{y}{2} = 30$$

$$\frac{4x - y}{2} = 30 \qquad 4x - y = 60$$

Thus, we have following two systems of linear equations

$$x - y = 3 \qquad \ldots (i)$$

$$4x - y = 60 \qquad \ldots (ii)$$

and

$$x - y = -3 \qquad \ldots (iii)$$

$$4x - y = 60 \qquad \ldots (iv)$$

Subtracting equation (*i*) from (*ii*), we get

$$\begin{array}{r} 4x - y = 60 \\ \underline{\underset{-}{x} \underset{+}{-} y = \underset{-}{3}} \\ 3x = 57 \\ x = 19 \end{array}$$

Putting $x = 19$ in equation (*i*), we get

$$19 - y = 3 \qquad y = 16$$

Now, subtracting equation (*iii*) from (*iv*)

$$\begin{array}{r} 4x - y = 60 \\ \underline{\underset{-}{x} \underset{+}{-} y = \underset{+}{-}3} \\ 3x = 63 \\ x = 21 \end{array}$$

Putting $x = 21$ in equation (*iii*), we get

$$21 - y = -3$$

$$y = 24$$

Hence, age of Ani = 19 years

age of Biju = 16 years

or

age of Ani = 21 years

age of Biju = 24 years

6. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h it would have taken 3 hours more than the scheduled time. Find distance covered by the train.

Sol. Let actual speed of the train be x km/h and actual time taken be y hours.

Then, distance covered = Speed × time

$$= xy \text{ km} \quad \ldots (i)$$

Case I: When speed is $(x + 10)$ km/h, then

time taken is $(y - 2)$ hours

Distance covered = $(x + 10)(y - 2)$

$$xy = (x + 10)(y - 2) \quad \text{[from } (i)\text{]}$$

$$xy = xy - 2x + 10y - 20$$

$$2x - 10y = -20$$

$$x - 5y = -10 \quad \ldots (ii)$$

Case II: When speed is $(x - 10)$ km/h, then time taken is $(y + 3)$ hours.

Distance covered = $(x - 10)(y + 3)$

$$xy = (x - 10)(y + 3) \quad \text{[from } (i)\text{]}$$

$$xy = xy + 3x - 10y - 30$$

$$3x - 10y = 30 \quad \ldots (iii)$$

Multiplying equation (*ii*) by 2 and subtracting it from (*iii*), we get

$$3x - 10y = 30$$

$$\underline{\underset{-}{}2x \underset{+}{-} 10y = \underset{+}{-} 20}$$

$$x = 50$$

Putting $x = 50$ in equation (*ii*), we get

$$50 - 5y = -10$$

$$50 + 10 = 5y$$

$$y = 12$$

Distance covered by the train = xy km = 50 × 12 km = 600 km

Exercise

A. Multiple Choice Questions

Write the correct answer for each of the following:

1. The number of solutions of the pair of linear equations $x+3y-4=0$ and $2x+6y=7$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) infinite

2. A pair of linear equations which has $x=0,\ y=-5$ as a solution is

(*a*) $x+y+5=0$, $2x+3y=10$ (*b*) $x+y=3$, $2x-y=5$ (*c*) $2x+y+5=0$, $3y=x-15$ (*d*) $3x+4y=-20$, $4x-3y=-15$

3. The value of k for which the lines $(k+1)x+3ky+15=0$ and $5x+ky+5=0$ are coincident is

(*a*) 14 (*b*) 2 (*c*) –14 (*d*) –2

4. The value of γ for which the system of equations $5\gamma x-2y=1$ and $10x+y=3$ has a unique solution is

(*a*) $= 4$ (*b*) $\neq 4$ (*c*) $=-4$ (*d*) $\neq -4$

5. The value of k for which the system of equations $2x+y-3=0$ and $5x+ky+7=0$ has no solution is

(*a*) 2 (*b*) 5 (*c*) $\frac{5}{2}$ (*d*) $\frac{3}{7}$

6. If the system of equations $4x+y=3$ and $(2k-1)x+(k-1)y=2k+1$ is inconsistent, then $k=$

(*a*) $\frac{2}{3}$ (*b*) $\frac{-2}{3}$ (*c*) $\frac{-3}{2}$ (*d*) $\frac{3}{2}$

7. If the system of equations

$$4x+3y=9$$
$$2ax+(a+b)y=18$$

has infinitely many solutions, then

(*a*) $b=2a$ (*b*) $a=2b$ (*c*) $a+2b=0$ (*d*) $2a-b=0$

8. The value of k for which the system of equation $2x+3y=7$ and $8x+(k+4)y-28=0$ has infinitely many solution is

(*a*) –8 (*b*) 8 (*c*) 3 (*d*) –3

9. If $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are respectively

(*a*) 3 and 5 (*b*) 5 and 3 (*c*) 3 and 1 (*d*) –1 and –3

10. *A*'s age is six times *B*'s age. Four years hence, the age of *A* will be four times *B*'s age. The present ages, in years, of *A* and *B* are, respectively

(*a*) 3 and 24 (*b*) 36 and 6 (*c*) 6 and 36 (*d*) 4 and 24

11. The sum of the digits of a two digit number is 14. If 18 is added to the number, the digits get reversed. The number is

(*a*) 95 (*b*) 59 (*c*) 68 (*d*) 86

12. Two numbers are in the ratio 1 : 3. If 5 is added to both the numbers, the ratio becomes 1 : 2. The numbers are

(*a*) 4 and 12 (*b*) 5 and 15 (*c*) 6 and 18 (*d*) 7 and 21

B. Short Answer Questions Type–I

1. For the pair of equations $\lambda x + 3y = -7$, $2x - 6y = 14$ to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reason.
2. Is the pair of equations $3x - 5y = 6$ and $4x - 6y = 7$ consistent? Justify your answer.
3. Do the equations $5x + 7y = 8$ and $10x + 14y = 4$ represent a pair of coincident lines? Justify your answer.
4. Is it true to say that the pair of equations $-2x + y + 3 = 0$ and $\frac{1}{3}x + 2y - 1 = 0$ has a unique solution? Justify your answer.
5. Write the number of solutions of the following pair of linear equations:

   $3x - 7y = 1$ and $6x - 14y - 3 = 0$
6. How many solutions does the pair of equations.

   $x + 2y = 3$ and $\frac{1}{2}x + y - \frac{3}{2} = 0$ have?
7. Is the pair of equations $x - y = 5$ and $2y - x = 10$ inconsistent? Justify your answer.

C. Short Answer Questions Type–II

1. Given the linear equations $3x - 2y + 7 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is

   (*i*) intersecting lines (*ii*) parallel lines (*iii*) coincident lines
2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent or inconsistent.

   (*i*) $4x - 5y = 8$, $3x - \frac{15}{4}y = 6$ (*ii*) $x - 5y = 7$, $-3x + 15y = 8$
3. For which value (s) of k will the pair of equations $kx + 3y = k - 3$, $12x + ky = k$ have no solution?
4. Find the values of a and b for which the following pair of equations have infinitely many solutions:

   (*i*) $2x + 3y = 7$ and $2ax + ay = 28 - by$

   (*ii*) $2x + 3y = 7$, $(a - b)x + (a + b)y = 3a + b - 2$

   (*iii*) $2x - (2a + 5)y = 5$, $(2b + 1)x - 9y = 15$
5. Write a pair of linear equations which has the unique solution $x = 2, y = -3$. How many such pairs can you write?
6. If $3x + 7y = -1$ and $4y - 5x + 14 = 0$, find the values of $3x - 8y$ and $\frac{y}{x} - 2$.
7. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$. Hence, find λ, if $y = \lambda x + 5$.
8. Draw the graph of the pair of equations $x - 2y = 4$ and $3x + 5y = 1$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.
9. If $x + 1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$.
10. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.
11. The angles of a cyclic quadrilateral $ABCD$ are $A = (2x + 4)°$, $B = (y + 3)°$, $C = (2y + 10)°$, $D = (4x - 5)°$. Find x and y and hence the values of the four angles.

12. Solve the following pairs of equations:

(i) $\frac{x}{3}+\frac{y}{4}=4$, $\frac{5x}{6}-\frac{y}{8}=4$

(ii) $0.2x+0.3y=1.3$, $0.4x+0.5y=2.3$

(iii) $\frac{x}{a}+\frac{y}{b}=a+b$, $\frac{x}{a^2}+\frac{y}{b^2}=2, a, b\neq 0$

(iv) $2x+3y+5=0$, $3x-2y-12=0$

(v) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$, $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(vi) $\frac{2xy}{x+y}=\frac{3}{2}$, $\frac{xy}{2x-y}=\frac{-3}{10}, x+y\neq 0, 2x-y\neq 0$

(vii) $\frac{44}{x+y}+\frac{30}{x-y}=10$, $\frac{55}{x+y}+\frac{40}{x-y}=13, \quad x\neq y$

(viii) $\frac{4}{x}+5y=7$, $\frac{3}{x}+4y=5, \quad x\neq 0$

(ix) $7(y+3)-2(x+2)=14$, $4(y-2)+3(x-3)=2$

(x) $3x-\frac{y+7}{11}-8=0$, $2y+\frac{x+11}{7}=10$

(xi) $\frac{x+y}{xy}=2$, $\frac{x-y}{xy}=6, \quad x\neq 0, y\neq 0$

(xii) $\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$, $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}, \quad x\neq 0, y\neq 0$

(xiii) $2(3u-v)=5uv$, $2(u+3v)=5uv$

(xiv) $\frac{5}{x-1}+\frac{1}{y-2}=2$, $\frac{6}{x-1}-\frac{3}{y-2}=1$

13. Find whether the following pairs of equations are consistent or not by graphical method. If consistent, solve them.

(i) $x-2y=6$, $3x-6y=0$

(ii) $5x+3y=1$, $x+5y+13=0$

(iii) $4x+7y=-11$, $5x-y+4=0$

14. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

15. There are some students in the two examination halls *A* and *B*. To make the number of students equal in each hall, 10 students are sent from *A* and *B*. But if 20 students are sent from *B* to A, the number of students in *A* becomes double the number of students in *B*. Find the number of students in the two halls.

16. Half the perimeter of a rectangular garden, whose length is 4m more than its width is 36 m. Find the dimensions of the garden.

17. The larger of two supplementary angles exceeds thrice the smaller by 20 degrees. Find them.

18. Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of y.

(i) $x+2y-7=0$, $2x-y-4=0$

(ii) $3x+2y=12$, $5x-2y=4$

19. Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x.

(i) $x+2y=5$, $2x-3y=-4$

(ii) $2x+3y=8$, $x-2y=-3$

20. Solve each of the following systems of equations by the method of cross-multiplication:

(*i*) $ax + by = a - b$
$bx - ay = a + b$

(*ii*) $2(ax - by) + a + 4b = 0$
$2(bx + ay) + b - 4a = 0$

(*iii*) $mx - ny = m^2 + n^2$
$x + y = 2m$

(*iv*) $\frac{57}{x+y} + \frac{6}{x-y} = 5$
$\frac{38}{x+y} + \frac{21}{x-y} = 9$

21. A two digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Represent this situation algebraically and geometrically.

D. Long Answer Questions

1. Determine graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$, $x + y = 8$.

2. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

3. Draw the graphs of the equations $y = -1$, $y = 3$ and $4x - y = 5$. Also, find the area of the quadrilateral formed by the lines and the y-axis.

4. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

5. The sum of a two digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.

6. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

7. Solve the following system of linear equations graphically and shade the region between the two lines and x-axis.

(*i*) $3x + 2y - 4 = 0$
$2x - 3y - 7 = 0$

(*ii*) $3x + 2y - 11 = 0$
$2x - 3y + 10 = 0$

8. Solve graphically the system of linear equations:

$4x - 3y + 4 = 0$
$4x + 3y - 20 = 0$

Find the area bounded by these lines and x-axis.

9. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investment in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

10. A two digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

11. The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

12. Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

13. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

14. A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

15. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of two cars?

16. The car hire charges in a city comprise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is ₹ 89 and for a journey of 20 km, the charge paid is ₹ 145. What will a person have to pay for travelling a distance of 30 km?

17. A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 15 days, he has to pay ₹ 1200 as hostel charges whereas a student B, who takes food for 24 days, pays ₹ 1560 as hostel charges. Find the fixed charge and the cost of food per day.

18. 2 women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone, and that taken by 1 man alone to finish the embroidery.

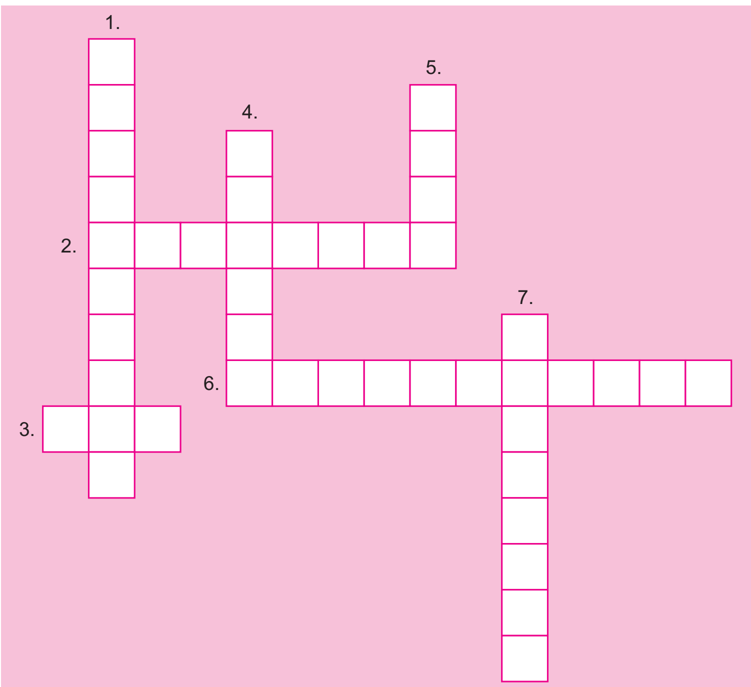
19. Yash scored 35 marks in a test, getting 2 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

20. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Formative Assessment

Activity: 1

■ **Solve the following crossword puzzle, hints are given alongside:**



Across

2. Number of solutions given by two coincident lines.
3. The degree of variables in a linear equation.
6. Method of solving a pair of linear equations in which one variable is eliminated by making its coefficient equal in the two equations.

Down

1. A pair of linear equations in two variables having a solution.
4. Type of solutions of pair of linear equations represented by two intersecting lines.
5. Graphical representation of a linear equation in two variables.
7. A pair of lines representing a pair of linear equations in two variables having no solution.

Oral Questions

1. Define consistent system of linear equations.
2. What does a linear equation in two variables represent geometrically?
3. When is a system of linear equations called inconsistent?
4. Do the equations $x+2y-7=0$ and $2x+4y+5=0$ represent a pair of parallel lines?
5. Is it true to say that the pair of equations $x+2y-3=0$ and $3x+6y-9=0$ are dependent?
6. If lines corresponding to two given linear equations are coincident, what can you say about the solution of the system of given equations?
7. If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then what does the system of linear equations, represent graphically?

Activity: 2 Hands on Activity (Math Lab Activity)

Objective

- To obtain the conditions for consistency of a system of linear equations in two variables by graphical method.

Materials Required

- 3 graph papers, pencil, ruler.

Procedure

1. Take the first pair of linear equations in two variables of the form

$$a_1x+b_1y+c_1=0$$
$$a_2x+b_2y+c_2=0$$

2. Obtain a table of ordered pairs (x, y), which satisfy the given equations.
   Find at least three such pairs for each equation.
3. Plot the graph for the two equations on the graph paper.
4. Observe if the lines are intersecting, parallel or coincident and note the following:

   $\frac{a_1}{a_2}=$ $\qquad$ $\frac{b_1}{b_2}=$ $\qquad$ $\frac{c_1}{c_2}=$

5. Take the second pair of linear equations in two variables and repeat steps 2 to 4.
6. Take the third pair of linear equations in two variables and repeat steps 2 to 4.
7. Fill in the following observations table:

Type of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	**Conclusion**
Intersecting				
Parallel				
Coincident				

8. Obtain the conditions for two lines to be intersecting, parallel or coincident from the observations table by comparing the values of $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$.

Observations

- You will observe that for intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, for parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and for coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Remarks

When a system of linear equations has solution (whether unique or not), the system is said to be **consistent** (dependent); when the system of linear equations has no solution, it is said to be **inconsistent**.

After activity 2, answer the following questions.

1. Write the condition for having a unique solution in the following pair of linear equations in two variables $lx + my = p$ and $tx + ny = r$.
2. Without actually drawing graph, can you comment on the type of graph of a given pair of linear equations in two variables? Justify your answer.
3. Coment on the type of solution and type of graph of following pair of linear equations:

$$2x - 5y = 9$$
$$5x + 6y = 8$$

4. For what value of k does the pair of equations $x - 2y = 3$, $3x + ky + 7 = 0$ have a unique solution?
5. Comment on the consistency or inconsistency of a pair of linear equations in two variables having intersecting lines on graph.
6. Find the value of k for which the pair of equations $x + 2y = 3$, $5x + ky + 7 = 0$ has a unique solution.

Activity 3: Analysis of Graph

Aim:

Given alongside is a graph representing pair of linear equations in two variables.

$$x - y = 2$$
$$x + y = 4$$

Observe the graph carefully.

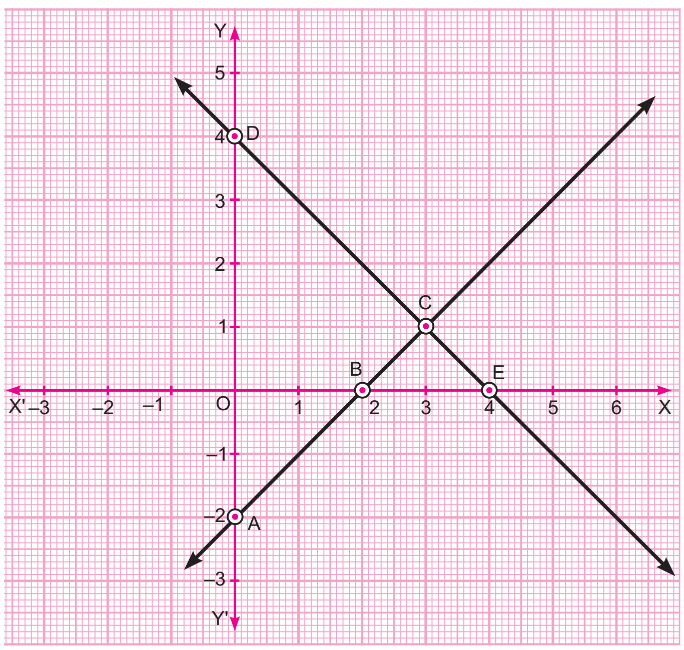


Fig. 3.10

Answer the following questions.

1. What are the coordinates of points where two lines representing the given equations meet x-axis?
2. What are the coordinates of points where two lines representing the given equations meet y-axis?

3. What is the solution of given pair of equations? Read from graph.
4. What is the area of triangle formed by given lines and x-axis?
5. What is the area of triangle formed by given lines and y-axis?

Suggested Math Lab Activities

1. Given a pair of linear equations:

   $4x + 5y = 28, \qquad 7x - 3y = 2$

   Formulate a word problem for the given system of equations and solve it graphically.
2. To find the condition for consistency and inconsistency for a given set of system of Linear Equations in two variables.

   Given a pair of linear equations:

   Set I: $x + 2y - 4 = 0, \qquad x + 2y - 6 = 0$

   Set II: $2x + 4y = 10, \qquad 3x + 6y = 12$
3. Find whether the following pair of equations are consistent or not by the graphical method. If consistent, solve them.

   (*a*) $x + 2y = 3, \qquad 4x + 3y = 2$

   (*b*) $2x + 3y = 9, \qquad 4x + 6y = 18$

   (*c*) $x + 2y - 4 = 0, \qquad x + 2y - 6 = 0$

Group Discussion

Divide the whole class into small groups and ask them to discuss some examples, from daily life where we use the concept of the pair of linear equations in two variables to solve the problems.

The students should write the problems and their corresponding equations.

Multiple Choice Questions

Tick the correct answer for each of the following.

1. A pair of linear equations in two variables cannot have

   (*a*) a unique solution (*b*) no solution

   (*c*) infinitely many solutions (*d*) exactly two solutions
2. The pair of equations $3x - 2y = 5$ and $6x - y = 3$ have

   (*a*) no solution (*b*) a unique solution

   (*c*) two solutions (*d*) infinitely many solutions
3. If a pair of linear equations is inconsistent, then the lines representing them will be

   (*a*) parallel (*b*) always coincident

   (*c*) intersecting or coincident (*d*) always intersecting
4. If a pair of linear equations has infinitely many solutions, then the lines representing them will be

   (*a*) parallel (*b*) intersecting or coincident

   (*c*) always intersecting (*d*) always coincident

5. The pair of equations $4x - 3y + 5 = 0$ and $8x - 6y - 10 = 0$ graphically represents two lines which are
   (*a*) coincident (*b*) parallel
   (*c*) intersecting at exactly one point (*d*) intersecting at exactly two points
6. The pair of equations $y = a$ and $y = b$ graphically represents lines which are
   (*a*) intersecting at (a, b) (*b*) intersecting at (b, a) (*c*) parallel (*d*) coincident
7. The pair of equations $x = 2$ and $y = 3$ has
   (*a*) one solution (*b*) two solutions (*c*) many solutions (*d*) no solution
8. The value of k for which the pair of equations $kx + y = 3$ and $3x + 6y = 5$ has a unique solution is
   (*a*) $-\frac{1}{2}$ (*b*) 2 (*c*) –2 (*d*) all the above
9. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is
   (*a*) $\frac{3}{2}$ (*b*) $\frac{15}{4}$ (*c*) $\frac{2}{5}$ (*d*) $-\frac{5}{4}$
10. One equation of a pair of dependent linear equations is $3x - 4y = 7$. The second equation can be
    (*a*) $-6x + 8y = 14$ (*b*) $-6x + 8y + 14 = 0$ (*c*) $6x + 8y = 14$ (*d*) $-6x - 8y - 14 = 0$
11. If $x = a$ and $y = b$ is the solution of the equations $x + y = 5$ and $x - y = 7$, then values of a and b are respectively
    (*a*) 1 and 4 (*b*) 6 and –1 (*c*) – 6 and 1 (*d*) –1 and –6
12. A pair of linear equations which has a unique solution $x = -1$, $y = -2$ is
    (*a*) $x - y = 1$; $2x + 3y = 5$ (*b*) $2x - 3y = 4$; $x - 5y = 9$
    (*c*) $x + y - 3 = 0$; $x - y = 1$ (*d*) $x + y + 3 = 0$; $2x - 3y + 5 = 0$
13. Sanya's age is three times her sister's age. Five years hence, her age will be twice her sister's age. The present ages (in years) of Sanya and her sister are respectively
    (*a*) 12 and 4 (*b*) 15 and 5 (*c*) 5 and 15 (*d*) 4 and 12
14. The sum of the digits of a two digit number is 8. If 18 is added to it, the digits of the number get reversed. The number is
    (*a*) 53 (*b*) 35 (*c*) 62 (*d*) 26
15. Divya has only ₹ 2 and ₹ 5 coins with her. If the total number of coins that she has is 25 and the amount of money with her is ₹ 80, then the number of ₹ 2 and ₹ 5 coins are , respectively
    (*a*) 15 and 10 (*b*) 10 and 15 (*c*) 12 and 10 (*d*) 13 and 12

Rapid Fire Quiz

State whether the following statements are true (T) or false (F).

1. A linear equation in two variables always has infinitely many solutions.
2. A pair of linear equations in two variables is said to be consistent if it has no solution.
3. A pair of intersecting lines represent a pair of linear equations in two variables having a unique solution.
4. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers is called a linear equation in two variables.
5. A pair of linear equations in two variables may not have infinitely many solutions.
6. The pair of equations $4x - 5y = 8$ and $8x - 10y = 3$ has a unique solution.
7. A pair of linear equations cannot have exactly two solutions.
8. If two lines are parallel, then they represent a pair of inconsistent linear equations.

Match the Columns

Match the following columns I and II.

Column I	Column II
(*i*) $x + y + 5 = 0$ $5x + 2y = -13$	(*a*) infinitely many solutions
(*ii*) $2x + y + 7 = 0$ $y - x = 8$	(*b*) no solution
(*iii*) $3x - 4y + 7 = 0$ $8y - 6x - 14 = 0$	(*c*) $x = 2,\ y = 3$
(*iv*) $x + y + 1 = 0$ $3x - 2y = 22$	(*d*) $x = -1,\ y = -4$
(*v*) $y = 5;\quad y = -3$	(*e*) $x = -5,\ y = 3$
(*vi*) $3x - 2y = 0$ $5x + y = 13$	(*f*) $x = 4,\ y = -5$

Class Worksheet

1. Tick the correct answer for each of the following:

(*i*) The pair of equations $6x - 4y + 9 = 0$ and $3x - 2y + 10 = 0$ has

(*a*) a unique solution (*b*) no solution

(*c*) exactly two solutions (*d*) infinitely many solutions

(*ii*) The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(*a*) coincident (*b*) parallel (*c*) intersecting at (a, b) (*d*) intersecting at (b, a)

(*iii*) If the lines given by $2x - 5y + 10 = 0$ and $kx + 15y - 30 = 0$ are coincident, then the value of k is

(*a*) –6 (*b*) 6 (*c*) $\frac{1}{3}$ (*d*) $\frac{-1}{3}$

(*iv*) If $x = a$, $y = b$ is the solution of the equation $x + y = 3$ and $x - y = 5$, then the values of a and b are, respectively

(*a*) 4 and –1 (*b*) 1 and 2 (*c*) –1 and 4 (*d*) 2 and 3

(*v*) If we add 1 to the numerator and denominator of a fraction, it becomes $\frac{1}{2}$. It becomes $\frac{1}{3}$ if we only add 1 to the denominator. The fraction is

(*a*) $\frac{3}{4}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{1}{4}$

2. State whether the following statements are true or false. Justify your answer.

(*i*) The pair of equations $3x - 4y = 1$ and $4x + 3y = 1$ has a unique solution.

(*ii*) For the pair of equations $4x + \lambda y = -3$ and $6x + 9y + 4 = 0$ to have no solution, the value of λ should not be 6.

3. (*i*) If $2x + y = 23$ and $4x - y = 19$, find the values of $3y - 4x$ and $\frac{y}{x} + 3$.

(*ii*) The angles of a cyclic quadrilateral *ABCD* are

$A = (6x + 10)°$, $B = (5x)°$, $C = (x + y)°$, $D = (3y - 10)°$

Find x and y, and hence the value of the four angles.

4. (*i*) Draw the graphs of the equations $y = 3$, $y = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the y-axis.

(*ii*) A motorboat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Tick the correct answer for each of the following:

(*i*) If a pair of linear equations is consistent, then the lines will be

(*a*) always intersecting (*b*) always coincident

(*c*) intersecting or coincident (*d*) parallel 1

(*ii*) The pair of equations $x + 2y - 3 = 0$ and $4x + 5y = 8$ has

(*a*) no solution (*b*) infinitely many solutions

(*c*) a unique solution (*d*) exactly two solutions 1

(*iii*) The value of c for which the pair of equations $4x - 5y + 7 = 0$ and $2cx - 10y + 8 = 0$ has no solution is

(*a*) 8 (*b*) – 8 (*c*) 4 (*d*) – 4 1

(*iv*) A pair of linear equations which has a unique solution $x = 1, y = -3$ is

(*a*) $x - y = 4;\quad 2x + 3y = 5$ (*b*) $2x - y = -5;\quad 5x - 2y = 11$

(*c*) $3x + y = 0;\quad x + 2y = -5$ (*d*) $x + y = -2;\quad 4x + 3y = 5$ 2

(*v*) Anmol's age is six times his son's age. Four years hence, the age of Anmol will be four times his son's age. The present age in years, of the father and the son are respectively

(*a*) 24 and 4 (*b*) 30 and 5 (*c*) 36 and 6 (*d*) 24 and 3 2

2. State whether the following statements are true or false. Justify your answer.

(*i*) The equations $\frac{x}{2} + y + \frac{1}{5} = 0$ and $4x + 8y + \frac{8}{5} = 0$ represent a pair of coincident lines.

(*ii*) For all real values of k, except –6, the pair of equations $kx - 3y = 5$ and $2x + y = 7$ has a unique solution. $2 \times 2 = 4$

3. (*i*) For what values of a and b, will the following pair of linear equations have infinitely many solutions?

$$x + 2y = 1;\quad (a - b)x + (a + b)y = a + b - 2$$

(*ii*) Solve for x and y

$$\frac{x}{a} + \frac{y}{b} = a + b,\ \frac{x}{a^2} + \frac{y}{b^2} = 2,\ a, b \neq 0$$ $3 \times 2 = 6$

4. (*i*) Graphically solve the pair of equations: $2x + y = 6,\ 2x - y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

(*ii*) Saksham travels 360 km to his home partly by train and partly by bus. He takes four and a half hours if he travels 90 km by bus and the remaining by train. If he travels 120 km by bus and remaining by train, he takes 10 minutes longer. Find the speed of the train and the bus separately. $4 \times 2 = 8$

TRIANGLES

Basic Concepts and Results

- Three or more points are said to be collinear if there is a line which contains all of them.
- Two figures having the same shape but not necessarily the same size are called similar figures.
- All congruent figures are similar but the converse is not true.
- Two polygons with same number of sides are similar, if (*i*) their corresponding angles are equal and (*ii*) their corresponding sides are in the same ratio (*i.e.*, proportion).
- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio (*Basic Proportionality Theorem or Thales Theorem*).
- If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
- If in two triangles, corresponding angles are equal, then the two triangles are similar (*AAA* similarity criterion).
- If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
- If in two triangles, corresponding sides are in the same ratio, then the two triangles are similar (SSS similarity criterion).
- If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the two triangles are similar (SAS similarity criterion).
- The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
- In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras Theorem).
- If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Summative Assessment

Multiple Choice Questions

Write the correct answer for each of the following:

1. In ΔABC, D and E are points on sides AB and AC respectively such that $DE \parallel BC$ and $AD : DB = 2 : 3$. If $EA = 6$ cm, then AC is equal to

(*a*) 9 cm (*b*) 15 cm (*c*) 4 cm (*d*) 10 cm

2. AD is the bisector of BAC in ΔABC. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then BD is equal to

(*a*) 5 cm (*b*) 6.5 cm (*c*) 7.5 cm (*d*) 5.6 cm

3. D and E are respectively the points on the sides AB and AC of a triangle ABC such that $AE = 5$ cm, $AC = 7.5$ cm, $DE = 4.2$ cm and $DE || BC$. Then length of BC is equal to

(*a*) 10.5 cm (*b*) 2.1 cm (*c*) 8.4 cm (*d*) 6.3 cm

4. The area of two similar triangles are respectively 25 cm^2 and 81 cm^2. The ratio of their corresponding sides is

(*a*) 5 : 9 (*b*) 5 : 4 (*c*) 9 : 5 (*d*) 10 : 9

5. If ΔABC and ΔDEF are similar such that $2AB = DE$ and $BC = 8$ cm, then EF is equal to

(*a*) 16 cm (*b*) 12 cm (*c*) 8 cm (*d*) 4 cm

6. The lengths of the diagonals of a rhombus are 18 cm and 24 cm. Then the length of the side of the rhombus is

(*a*) 26 cm (*b*) 15 cm (*c*) 30 cm (*d*) 28 cm

7. XY is drawn parallel to the base BC of a ΔABC cutting AB at X and AC at Y. If $AB = 4BX$ and $YC = 2$cm, then AY is equal to

(*a*) 2 cm (*b*) 4 cm (*c*) 6 cm (*d*) 8 cm

8. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If distance between their foot is 12m, the distance between their tops is

(*a*) 12 m (*b*) 13 m (*c*) 14 m (*d*) 11 m

9. In a ΔABC right-angled at A, $AB = 5$ cm and $AC = 12$ cm. If AD BC, then AD is equal to

(*a*) $\frac{13}{2}$ cm (*b*) $\frac{60}{13}$ cm (*c*) $\frac{13}{60}$ cm (*d*) $\frac{2\sqrt{15}}{13}$ cm

10. If ABC is an equilateral triangle such that AD BC, then AD^2 is equal to

(*a*) $\frac{3}{2}DC^2$ (*b*) $2DC^2$ (*c*) $3CD^2$ (*d*) $4DC^2$

11. $ABCD$ is a trapezium such that $BC || AD$ and $AB = 4$ cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then DC is equal to

(*a*) 7 cm (*b*) 8 cm (*c*) 9 cm (*d*) 6 cm

12. If ABC is a triangle right-angled at B and M, N are the mid-points of AB and BC respectively, then 4 $(AN^2 + CM^2)$ is equal to

(*a*) $4AC^2$ (*b*) $5AC^2$ (*c*) $\frac{5}{4}AC^2$ (*d*) $6AC^2$

Short Answer Questions Type – I

1. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.

Sol. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 18^2$

The given triangle is not a right triangle.

2. In triangle PQR and MST, $P = 55°$, $Q = 25°$, $M = 100°$ and $S = 25°$. Is $\Delta QPR \sim \Delta TSM$? Why?

Sol. Since, $R = 180° - (P + Q) = 180° - (55° + 25°) = 100° = M$

$Q = S = 25°$

$\Delta QPR \sim \Delta STM$

i.e., ΔQPR is not similar to ΔTSM.

3. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Sol. Since the perimeters and two sides are proportional

the third side is proportional to the third side.

i.e., the two triangles will be similar by SSS criterion.

4. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that $PQ = 12.5$ cm, $PA = 5$ cm, $BR = 6$ cm and $PB = 4$ cm. Is $AB || QR$? Give reason.

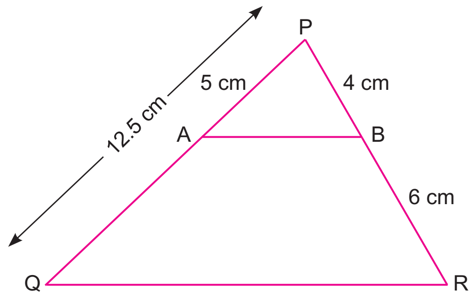


Fig. 4.1

Sol. $\frac{PA}{AQ} = \frac{5}{12.5-5} = \frac{5}{7.5} = \frac{2}{3}$

$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$

Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$

$AB || QR$

5. If ABC and DEF are similar triangles such that $A = 47°$ and $E = 63°$, then the measures of $C = 70°$. Is it true? Give reason.

Sol. Since $\Delta ABC \sim \Delta DEF$

$A = D = 47°$, $B = E = 63°$

$C = 180° - (A + B) = 180 - (47 + 63) = 70°$

Given statement is true.

Important Problems

Type A: Problems Based on Basic Proportionality Theorem and its Converse.

1. Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Using the above result, do the following:

In Fig. 4.2 $DE || BC$ and $BD = CE$. Prove that ΔABC is an isosceles triangle.

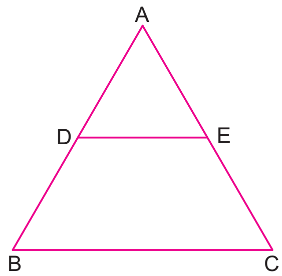


Fig. 4.2

Sol. **Given:** A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and then draw DM AC and EN AB.

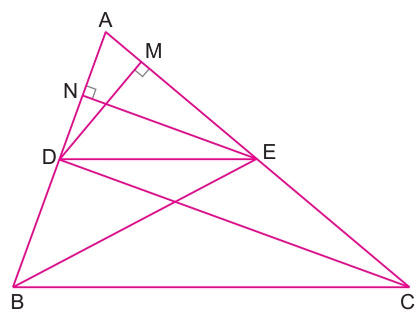


Fig. 4.2 (a)

Proof: Area of $\Delta ADE = \left(\frac{1}{2}\text{ base} \times \text{height}\right)$

So, $\text{ar}(ADE) = \frac{1}{2} AD \times EN$

and $\quad ar(\Delta BDE) = \frac{1}{2} DB \times EN.$

Similarly, $\quad ar(\Delta ADE) = \frac{1}{2} AE \times DM$

and $\quad ar(\Delta DEC) = \frac{1}{2} EC \times DM$

Therefore, $\quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ...(*i*)

and $\quad \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ...(*ii*)

Now, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

So, $\quad ar(\Delta BDE) = ar(\Delta DEC)$ ...(*iii*)

Therefore, from (*i*), (*ii*) and (*iii*) we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Second Part

As $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\frac{AD + DB}{DB} = \frac{AE + EC}{EC} \qquad \frac{AB}{DB} = \frac{AC}{EC}$$

$$AB = AC \qquad (\text{As } DB = EC)$$

ΔABC is an isosceles triangle.

2. In Fig.4.3, $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x.

Sol. In ΔABC, we have

$$DE \parallel BC,$$

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[By Basic Proportionality Theorem]}$$

$$\frac{x}{x-2} = \frac{x+2}{x-1} \qquad x(x-1) = (x-2)(x+2)$$

$$x^2 - x = x^2 - 4 \qquad x = 4$$

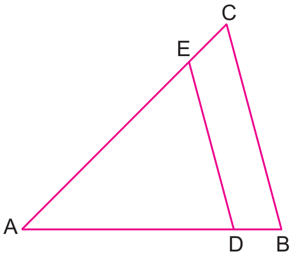


Fig. 4.3

3. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether $EF \parallel QR$.

(*i*) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(*ii*) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm **[NCERT]**

Sol. (*i*) We have, $PE = 4$ cm, $QE = 4.5$ cm

$$PF = 8 \text{ cm}, RF = 9 \text{ cm}$$

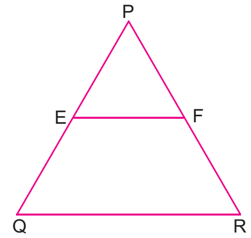


Fig. 4.4

Now, $\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9}$

And $\frac{PF}{RF} = \frac{8}{9}$

Thus, $\frac{PE}{QE} = \frac{PF}{RF}$,

Therefore, $EF \parallel QR$. [By the converse of Basic Proportionality Theorem]

(*ii*) We have,

$PQ = 1.28$ cm, $PR = 2.56$ cm

$PE = 0.18$ cm, $PF = 0.36$ cm

Now, $QE = PQ - PE = 1.28 - 0.18 = 1.10$ cm

and $FR = PR - PF = 2.56 - 0.36 = 2.20$ cm

Now, $\frac{PE}{QE} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

and, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\qquad \frac{PE}{QE} = \frac{PF}{FR}$

Therefore, $EF \parallel QR$ [By the converse of Basic Proportionality Theorem]

4. In Fig.4.5, if $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$. **[NCERT]**

Sol. Firstly, in ΔABC, we have

$LM \parallel CB$ (Given)

Therefore, by Basic Proportionality Theorem, we have

$\frac{AM}{AB} = \frac{AL}{AC}$ ...(*i*)

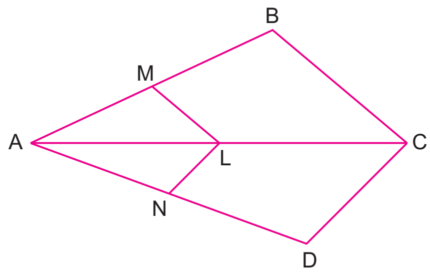


Fig. 4.5

Again, in ΔACD, we have

$LN \parallel CD$ (Given)

By Basic Proportionality Theorem, we have

$\frac{AN}{AD} = \frac{AL}{AC}$ ...(*ii*)

Now, from (*i*) and (*ii*), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

5. In Fig.4.6, $DE \parallel OQ$ and $DF \parallel OR$, Show that $EF \parallel QR$. **[NCERT]**

Sol. In ΔPOQ, we have

$DE \parallel OQ$ (Given)

By Basic Proportionality Theorem, we have

$\frac{PE}{EQ} = \frac{PD}{DO}$ ...(*i*)

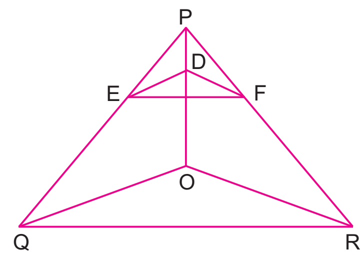


Fig. 4.6

Again, in ΔPOR, we have

$DF \parallel OR$ (Given)

By Basic Proportionality Theorem, we have

$$\frac{PD}{DO} = \frac{PF}{FR} \qquad \ldots(ii)$$

Now, from (*i*) and (*ii*), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \qquad EF \parallel QR$$

[Applying the converse of Basic Proportionality Theorem in ΔPQR]

6. In Fig.4.7, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. **[NCERT]**

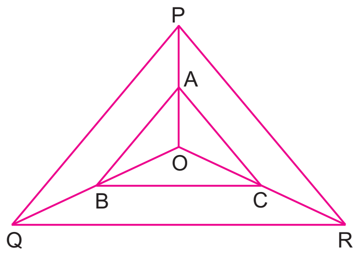


Fig. 4.7

Sol. In ΔOPQ, we have

$$AB \parallel PQ \qquad \text{(Given)}$$

By Basic Proportionality Theorem, we have

$$\frac{OA}{AP} = \frac{OB}{BQ} \qquad \ldots(i)$$

Now, in ΔOPR, we have

$$AC \parallel PR \qquad \text{(Given)}$$

By Basic Proportionality Theorem, we have

$$\frac{OA}{AP} = \frac{OC}{CR} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore, $BC \parallel QR$ (Applying the converse of Basic Proportionality Theorem in ΔOQR)

7. Using Basic Proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. **[NCERT]**

Sol. **Given:** A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: $AE = EC$

Proof: In ΔABC, $DE \parallel BC$

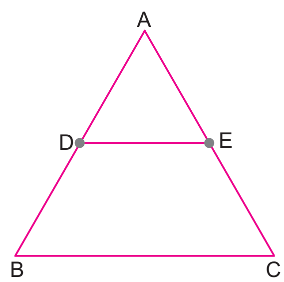


Fig. 4.8

By Basic Proportionality Theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \ldots(i)$$

Now, since D is the mid-point of AB

$$AD = BD \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$\frac{BD}{BD} = \frac{AE}{EC} \qquad 1 = \frac{AE}{EC}$$

$$AE = EC$$

Hence, E is the mid-point of AC.

8. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. **[NCERT]**

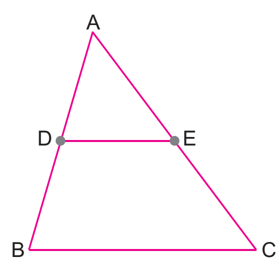


Fig. 4.9

Sol. **Given:** ΔABC in which D and E are the mid-points of sides AB and AC respectively.

To prove: $DE \parallel BC$

Proof: Since, D and E are the mid-points of AB and AC respectively

$$AD = DB \quad \text{and} \quad AE = EC$$

$$\frac{AD}{DB} = 1 \quad \text{and} \quad \frac{AE}{EC} = 1$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, $DE \parallel BC$ (By the converse of Basic Proportionality Theorem)

9. $ABCD$ is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$. **[NCERT]**

Sol. **Given:** $ABCD$ is a trapezium, in which $AB \parallel DC$ and its diagonals intersect each other at the point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

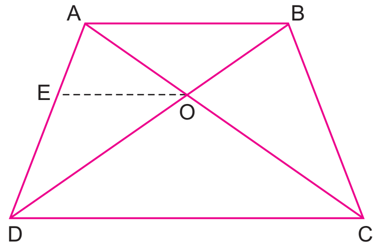


Fig. 4.10

Construction: Through O, draw $OE \parallel AB$ *i.e.*, $OE \parallel DC$.

Proof: In ΔADC, we have $OE \parallel DC$ (Construction)

By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \quad ...(i)$$

Now, in ΔABD, we have $OE \parallel AB$ (Construction)

By Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \qquad \frac{AE}{ED} = \frac{BO}{DO} \quad ...(ii)$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \qquad \frac{AO}{BO} = \frac{CO}{DO}$$

Type B: Problems Based on Similarity of Triangles

1. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form. **[NCERT]**

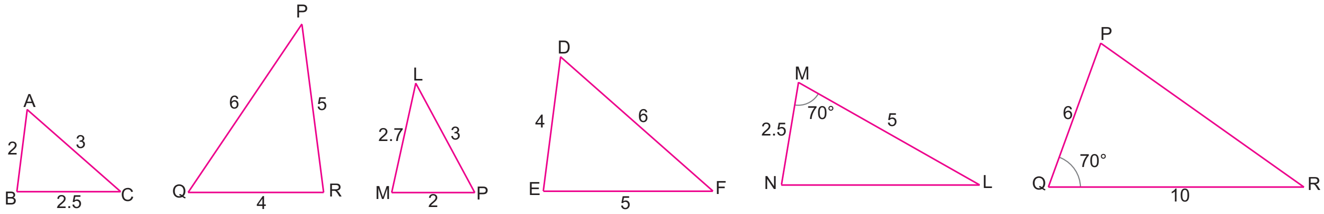


Fig. 4.11

Sol. (i) In ΔABC and ΔPQR, we have

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{BC}{PR} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{PQ} = \frac{BC}{PR}$

$\Delta ABC \sim \Delta QRP$ by SSS criterion of similarity.

(*ii*) In ΔLMP and ΔDEF, we have

$$\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LM}{EF} = \frac{2.7}{5}$$

Hence, $\frac{LP}{DF} = \frac{MP}{DE} \neq \frac{LM}{EF}$

ΔLMP is not similar to ΔDEF.

(*iii*) In ΔNML and ΔPQR, we have

$M = Q = 70°$

Now, $\frac{MN}{PQ} = \frac{2.5}{6} = \frac{5}{12}$

And $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence $\frac{MN}{PQ} \neq \frac{ML}{QR}$

ΔNML is not similar to ΔPQR because they do not satisfy SAS criterion of similarity.

2. In Fig. 4.12, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 5$ cm. Find the value of DC.

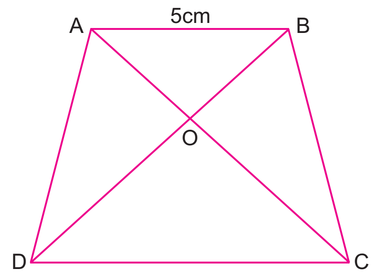


Fig. 4.12

Sol. In ΔAOB and ΔCOD, we have

$AOB = COD$ [Vertically opposite angles]

$\frac{AO}{OC} = \frac{BO}{OD}$ [Given]

So, by *SAS* criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

$\frac{1}{2} = \frac{5}{DC}$ [$\because AB = 5$ cm]

$DC = 10$ cm

3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower. **[NCERT]**

Sol. Let AB be a vertical pole of length 6m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

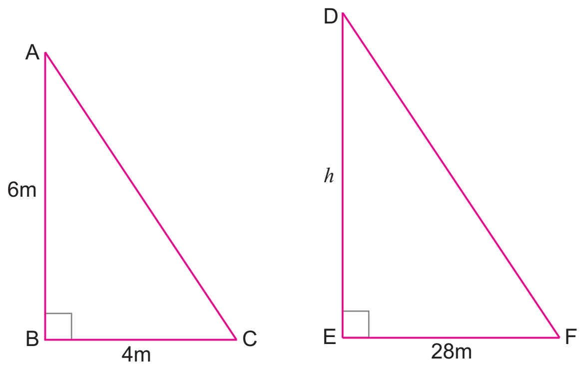


Fig. 4.13

Now, in ΔABC and ΔDEF, we have

$B = E = 90°$

$C = F$ (Angle of elevation of the Sun)

$\Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{6}{h} = \frac{4}{28}$ (Let $DE = h$)

$\frac{6}{h} = \frac{1}{7}$ $h = 42$

Hence, height of tower, $DE = 42$ m

4. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$. **[NCERT]**

Sol. **Given:** $ABCD$ is a trapezium in which $AB \parallel DC$.

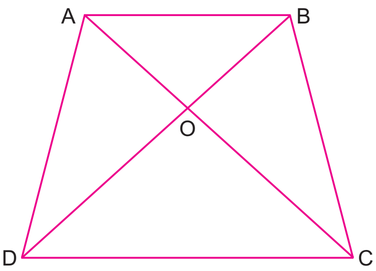


Fig. 4.14

To prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In ΔOAB and ΔODC, we have

$OAB = OCD$ (Alternate angles)

$AOB = DOC$ (Vertically opposite angles)

$ABO = ODC$ (Alternate angles)

$\Delta OAB \sim \Delta OCD$ (By AA criterion of similarity)

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

5. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$. **[NCERT]**

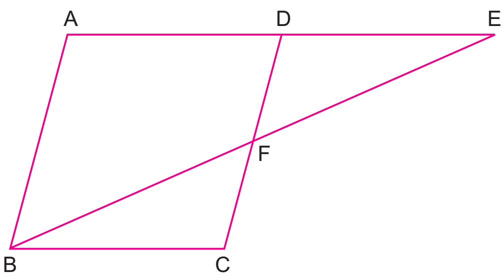


Fig. 4.15

Sol. In ΔABE and ΔCFB, we have

$AEB = CBF$ (Alternate angles)

$A = C$ (Opposite angles of a parallelogram)

$\Delta ABE \sim \Delta CFB$ (By AA criterion of similarity)

6. S and T are points on sides PR and QR of ΔPQR such that $P = RTS$. Show that $\Delta RPQ \sim \Delta RTS$. **[NCERT]**

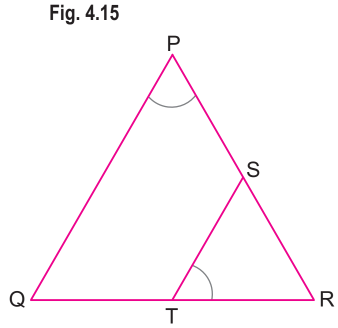


Fig. 4.16

Sol. In ΔRPQ and ΔRTS, we have

$RPQ = RTS$ (Given)

$PRQ = TRS = R$ (Common)

$\Delta RPQ \sim \Delta RTS$ (By AA criterion of similarity.)

7. In Fig. 4.17, ABC and AMP are two right triangles right-angled at B and M respectively. Prove that:

(*i*) $\Delta ABC \sim \Delta AMP$ (*ii*) $\frac{CA}{PA} = \frac{BC}{MP}$ **[NCERT]**

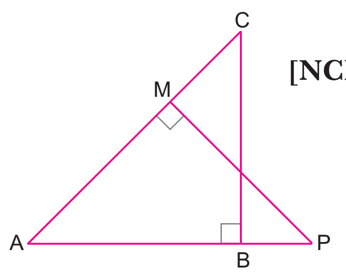


Fig. 4.17

Sol. (*i*) In ΔABC and ΔAMP, we have

$ABC = AMP = 90°$ (Given)

And, $BAC = MAP$ (Common angle)

$\Delta ABC \sim \Delta AMP$ (By AA criterion of similarity)

(*ii*) As $\Delta ABC \sim \Delta AMP$ (Proved above)

$\frac{CA}{PA} = \frac{BC}{MP}$ (Sides of similar triangles are proportional)

8. In Fig.4.18, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If AD BC and EF AC, prove that $\Delta ABD \sim \Delta ECF$. **[NCERT]**

Sol. We have, $B = C$ [$\because$ ABC is an isosceles triangle with $AB = AC$]

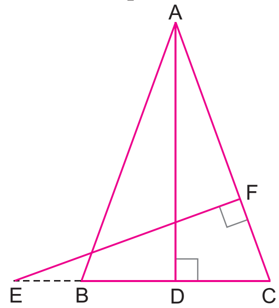


Fig. 4.18

Now, in ΔABD and ΔECF

$ABD = ECF$ [$\because$ $B = C$]

$ADB = EFC = 90°$ [$\because$ AD BC and EF AC]

$\Delta ABD \sim \Delta ECF$ (By AA criterion of similarity)

9. D is a point on the side BC of a triangle ABC such that $ADC = BAC$. Show that $CA^2 = CB.CD$. **[NCERT]**

Sol. In ΔABC and ΔDAC, we have

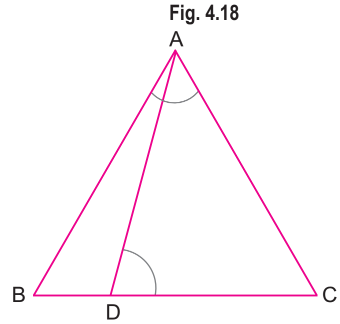


Fig. 4.19

$BAC = ADC$ (Given)

and $C = C$ (Common)

$\Delta ABC \sim \Delta DAC$ (By AA criterion of similarity)

$$\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

$$\frac{CB}{CA} = \frac{CA}{CD}$$

$$CA^2 = CB \times CD$$

10. If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. **[NCERT]**

Sol. In ΔABD and ΔPQM, we have

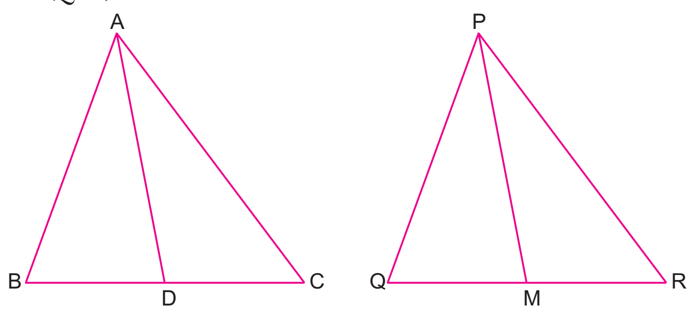


Fig. 4.20

$B = Q$ ($\because$ $\Delta ABC \sim \Delta PQR$) …(*i*)

$\frac{AB}{PQ} = \frac{BC}{QR}$ ($\because$ $\Delta ABC \sim \Delta PQR$)

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Since AD and PM are the medians of ΔABC and ΔPQR respectively] …(*ii*)

From (*i*) and (*ii*), it is proved that

$\Delta ABD \sim \Delta PQM$ (By SAS criterion of similarity)

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \qquad \frac{AB}{PQ} = \frac{AD}{PM}$$

11. In Fig. 4.21, $ABCD$ is a trapezium with $AB \parallel DC$. If ΔAED is similar to ΔBEC, prove that $AD = BC$.

Sol. In ΔEDC and ΔEBA, we have

$1 = \ 2$ [Alternate angles]

$3 = \ 4$ [Alternate angles]

and $CED = \ AEB$ [Vertically opposite angles]

$\Delta EDC \sim \Delta EBA$ [By AA criterion of similarity]

$$\frac{ED}{EB} = \frac{EC}{EA} \qquad \frac{ED}{EC} = \frac{EB}{EA} \qquad \ldots(i)$$

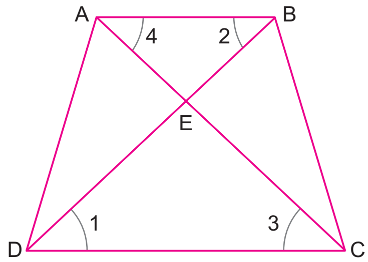


Fig. 4.21

It is given that $\Delta AED \sim \Delta BEC$

$$\frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \qquad (EB)^2 = (EA)^2 \qquad EB = EA$$

Substituting $EB = EA$ in (*ii*), we get

$$\frac{EA}{EA} = \frac{AD}{BC}$$

$$\frac{AD}{BC} = 1 \qquad AD = BC$$

12. ABC is a triangle in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. Prove that $BD = BC$.

Sol. **Given:** ΔABC in which $AB = AC$ and D is a point on the side AC such that

$$BC^2 = AC \times CD$$

To prove: $BD = BC$

Construction: Join BD

Proof: We have, $BC^2 = AC \times CD$

$$\frac{BC}{CD} = \frac{AC}{BC} \qquad \ldots(i)$$

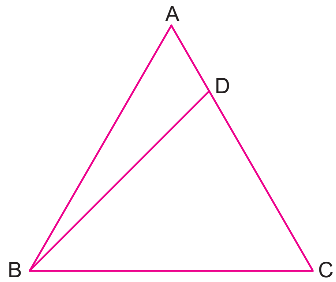


Fig. 4.22

Thus, in ΔABC and ΔBDC, we have

$$\frac{AC}{BC} = \frac{BC}{CD} \qquad \text{[From (i)]}$$

and $C = \ C$ [Common]

$\Delta ABC \sim \Delta BDC$

$$\frac{AB}{BD} = \frac{BC}{CD} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$\frac{AC}{BC} = \frac{AB}{BD}$$

$BD = BC$ $(\because \ AB = AC)$

13. In Fig. 4.23, *ABD* is a triangle right-angled at *A* and *AC* $\quad$ *BD*. Show that
(i) $AB^2 = BC \cdot BD$ $\qquad$ (ii) $AD^2 = BD \cdot CD$ $\qquad$ (iii) $AC^2 = BC \cdot DC$ **[NCERT]**

Sol. **Given:** *ABD* is a triangle right-angled at *A* and *AC* $\quad$ *BD*.

To prove: (i) $AB^2 = BC \cdot BD$

(ii) $AD^2 = BD \cdot CD$

(iii) $AC^2 = BC \cdot DC$

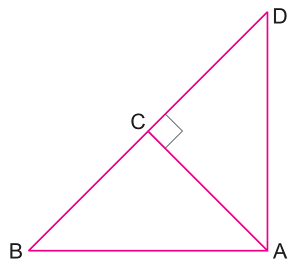


Fig. 4.23

Proof: (i) In ΔACB and ΔDAB, we have

$ACB = \quad DAB = 90°$

$ABC = \quad DBA = \quad B$ $\qquad$ (Common)

$\Delta ACB \sim \Delta DAB$ $\qquad$ (By *AA* criterion of similarity)

$$\frac{BC}{AB} = \frac{AB}{DB}$$

$AB^2 = BC \cdot BD$

(ii) In ΔACD and ΔBAD, we have

$ACD = \quad BAD = 90°$

$CDA = \quad BDA = \quad D$ $\qquad$ (Common)

$\Delta ACD \sim \Delta BAD$ $\qquad$ (By *AA* criterion of similarity)

$$\frac{AD}{BD} = \frac{CD}{AD}$$

$AD^2 = BD \cdot CD$

(iii) We have $\qquad \Delta ACB \sim \Delta DAB$

$\Delta BCA \sim \Delta BAD$ $\qquad$ ...(i)

and $\qquad \Delta ACD \sim \Delta BAD$ $\qquad$ ...(ii)

From (i) and (ii), we have

$\Delta BCA \sim \Delta ACD$

$$\frac{BC}{AC} = \frac{AC}{DC}$$

$AC^2 = BC \cdot DC$

Type C: Problems Based on Areas of Two Similar Triangles

1. Prove that ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Using the above result do the following:

Diagonals of a trapezium *ABCD* with $AB \parallel DC$ intersect each other at the point *O*. If $AB = 2\ CD$, find the ratio of the areas of triangles *AOB* and *COD*.

Sol. **Given:** Two triangles *ABC* and *PQR* such that $\Delta ABC \sim \Delta PQR$

To Prove: $$\frac{\text{ar }(ABC)}{\text{ar }(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Construction: Draw *AM* $\quad$ *BC* and *PN* $\quad$ *QR*.

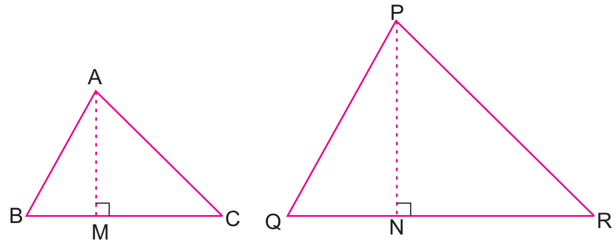


Fig. 4.24

Proof: ar $(ABC) = \frac{1}{2} \times BC \times AM$

and ar $(PQR) = \frac{1}{2} \times QR \times PN$

So, $$\frac{\text{ar }(ABC)}{\text{ar }(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \qquad ...(i)$$

Now, in ΔABM and ΔPQN,

$B = Q$ [As $\Delta ABC \sim \Delta PQR$]

and $M = N$ [Each 90°]

So, $\Delta ABM \sim \Delta PQN$ [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ...(*ii*)

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(*iii*)

Therefore, $\frac{\text{ar }(ABC)}{\text{ar }(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$ [From (*i*) and (*iii*)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (*ii*)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (*iii*), we get $\frac{\text{ar }(ABC)}{\text{ar }(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

In ΔAOB and ΔCOD, we have

$AOB = COD$ (Vertically opposite angles)

and $OAB = OCD$ (Alternate angles)

$\Delta AOB \sim \Delta COD$ (By AA criterion of similarity)

$\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \frac{AB^2}{DC^2}$

$\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \frac{(2DC)^2}{DC^2} = \frac{4}{1}$

Hence, the ratio of areas of ΔAOB and $\Delta COD = 4 : 1$.

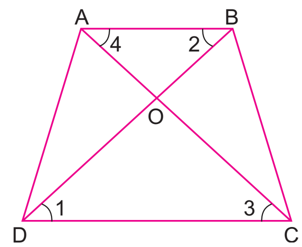


Fig. 4.25

2. Let $\Delta ABC \sim \Delta DEF$ and their areas be respectively 64 $\mathbf{cm^2}$ and 121 $\mathbf{cm^2}$. If $EF = 15\ \ 4$ cm, find BC. **[NCERT]**

Sol. We have, $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \dfrac{BC^2}{EF^2}$ (as $\Delta ABC \sim \Delta DEF$)

$$\frac{64}{121} = \frac{BC^2}{EF^2} \qquad \frac{64}{121} = \frac{BC^2}{(15\ \ 4)^2}$$

$$\frac{BC}{15\ \ 4} = \frac{8}{11} \qquad BC = \frac{8}{11} \times 15\ \ 4 = 11\ \ 2 \text{ cm.}$$

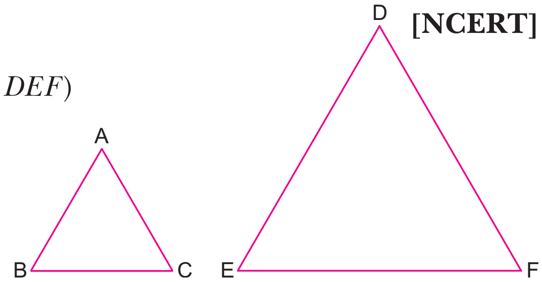


Fig. 4.26

3. Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.

Sol. **Given:** A ΔABC in which $ABC = 90°$ and $AB = BC$. ΔABD and ΔACE are equilateral triangles.

To Prove: ar(ΔABD) $= \dfrac{1}{2} \times$ ar(ΔCAE)

Proof: Let $AB = BC = x$ units.

hyp. $CA = \sqrt{x^2 + x^2} = x\sqrt{2}$ units.

Each of the ΔABD and ΔCAE being equilateral, each angle of each one of them is 60°.

$\Delta ABD \sim \Delta CAE$

But, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{\text{ar}(\Delta ABD)}{\text{ar}(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, ar (ΔABD) $= \dfrac{1}{2} \times$ ar (ΔCAE)

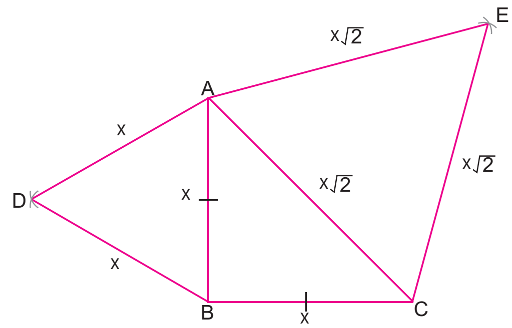


Fig. 4.27

4. If the areas of two similar triangles are equal, prove that they are congruent. **[NCERT]**

Sol. **Given:** Two triangles ABC and DEF, such that

$\Delta ABC \sim \Delta DEF$ and area (ΔABC) = area (ΔDEF)

To prove: $\Delta ABC \quad \Delta DEF$

Proof: $\Delta ABC \sim \Delta DEF$

$A = \ D, \quad B = \ E, \quad C = \ F$

and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

Now, ar (ΔABC) = ar (ΔDEF) (Given)

$\dfrac{\text{ar } (\Delta ABC)}{\text{ar } (\Delta DEF)} = 1$ ...(*i*)

and $\dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2} = \dfrac{\text{ar } (\Delta ABC)}{\text{ar } (\Delta DEF)}$ ($\because \Delta ABC \sim \Delta DEF$) ...(*ii*)

From (*i*) and (*ii*), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \qquad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

$AB = DE,\ BC = EF,\ AC = DF$

Hence, $\Delta ABC \quad \Delta DEF$ (By *SSS* criterion of congruency)

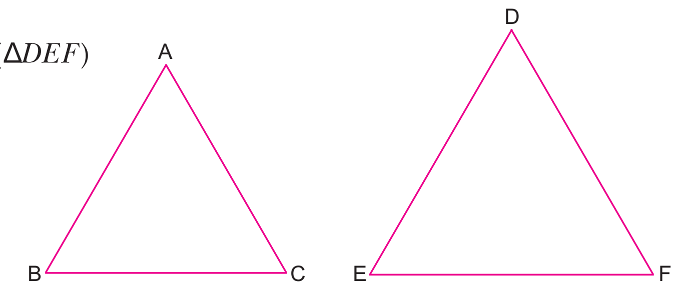


Fig. 4.28

5. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians. **[NCERT]**

Sol. Let ΔABC and ΔPQR be two similar triangles. AD and PM are the medians of ΔABC and ΔPQR respectively.

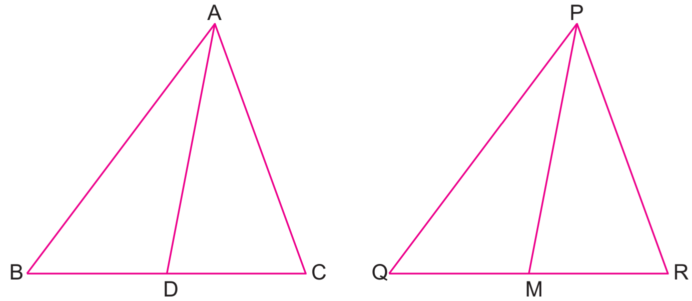


Fig. 4.29

To prove: $\dfrac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = \dfrac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = \frac{AB^2}{PQ^2} \qquad \ldots(i)$$

In ΔABD and ΔPQM

$$\frac{AB}{PQ} = \frac{BD}{QM} \qquad \left(\because \frac{AB}{PQ} = \frac{BC}{QR} = \frac{1/2\,BC}{1/2\,QR}\right)$$

and $B = Q$ $\qquad (\because \Delta ABC \sim \Delta PQR)$

Hence, $\Delta ABD \sim \Delta PQM$ $\qquad$ (By SAS Similarity criterion)

$$\frac{AB}{PQ} = \frac{AD}{PM} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = \frac{AD^2}{PM^2}$$

6. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals. **[NCERT]**

Sol. Let $ABCD$ be a square and ΔBCE and ΔACF have been drawn on side BC and the diagonal AC respectively.

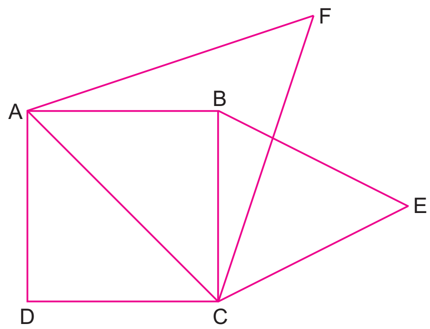


Fig. 4.30

To prove: area $(\Delta BCE) = \dfrac{1}{2}$ area (ΔACF)

Proof: Since ΔBCE and ΔACF are equilateral triangles

$\Delta BCE \sim \Delta ACF$ $\qquad$ (by AAA criterion of similarity)

$$\frac{\text{area }(\Delta BCE)}{\text{area }(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\frac{\text{area }(\Delta BCE)}{\text{area }(\Delta ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} \qquad [\because \text{Diagonal} = \sqrt{2} \text{ side}, AC = \sqrt{2}BC]$$

$$\frac{\text{area }(\Delta BCE)}{\text{area }(\Delta ACF)} = \frac{1}{2}$$

$$\text{area }(\Delta BCE) = \frac{1}{2}\text{ area }(\Delta ACF)$$

Type D: Problems Based on Pythagoras Theorem and its Converse

1. Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Using the above, do the following:

Prove that, in a ΔABC, if AD is perpendicular to BC, then $AB^2 + CD^2 = AC^2 + BD^2$.

Sol. Given: A right triangle ABC right-angled at B.

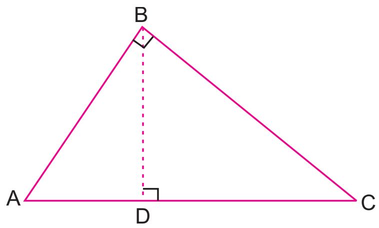


Fig. 4.31

To Prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \quad AC$

Proof: In ΔADB and ΔABC

$$A = \quad A \qquad \text{(Common)}$$

$$ADB = \quad ABC \qquad \text{(Both 90°)}$$

$$\Delta ADB \sim \Delta ABC \qquad (AA \text{ similarity criterion})$$

So, $$\frac{AD}{AB} = \frac{AB}{AC} \qquad \text{(Sides are proportional)}$$

or $$AD \,.\, AC = AB^2 \qquad \ldots(i)$$

In ΔBDC and ΔABC

$$C = \quad C \qquad \text{(Common)}$$

$$BDC = \quad ABC \qquad \text{(Each 90°)}$$

$$\Delta BDC \sim \Delta ABC \qquad (AA \text{ similarity})$$

So, $$\frac{CD}{BC} = \frac{BC}{AC}$$

or, $$CD \,.\, AC = BC^2 \qquad \ldots(ii)$$

Adding (i) and (ii), we get

$$AD \,.\, AC + CD \,.\, AC = AB^2 + BC^2$$

or, $$AC(AD + CD) = AB^2 + BC^2$$

or, $$AC \,.\, AC = AB^2 + BC^2 \text{ or, } AC^2 = AB^2 + BC^2$$

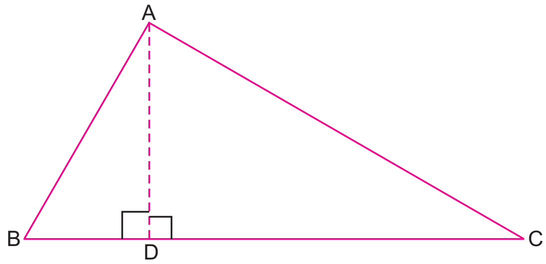


Fig. 4.32

As $AD \quad BC$

Therefore, $ADB = \quad ADC = 90°$

By Pythagoras Theorem, we have

$$AB^2 = AD^2 + BD^2 \qquad \ldots (i)$$

$$AC^2 = AD^2 + DC^2 \qquad \ldots (ii)$$

Subtracting (ii) from (i)

$$AB^2 - AC^2 = AD^2 + BD^2 - (AD^2 + DC^2)$$

$$AB^2 - AC^2 = BD^2 - DC^2$$

$$AB^2 + DC^2 = BD^2 + AC^2$$

2. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides, prove that the angle opposite to the first side is a right angle.
Use the above theorem to find the measure of PKR in Fig. 4.33.

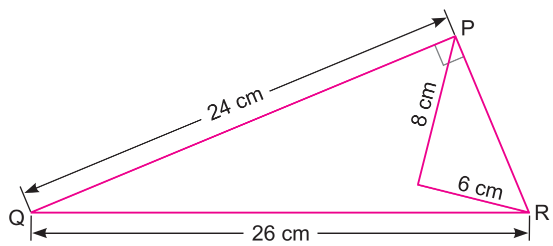


Fig. 4.33

Sol. Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$.

To Prove: $B = 90°$.

Construction: We construct a ΔPQR right-angled at Q such that $PQ = AB$ and $QR = BC$

Proof: Now, from ΔPQR, we have,

$PR^2 = PQ^2 + QR^2$ [Pythagoras Theorem, as $Q = 90°$]

or, $PR^2 = AB^2 + BC^2$ [By construction] ...(*i*)

But $AC^2 = AB^2 + BC^2$ [Given] ...(*ii*)

So, $AC^2 = PR^2$ [From (*i*) and (*ii*)]

$AC = PR$ ...(*iii*)

Now, in ΔABC and ΔPQR,

$AB = PQ$ [By construction]

$BC = QR$ [By construction]

$AC = PR$ [Proved in (*iii*)]

So, $\Delta ABC \quad \Delta PQR$ [SSS congruency]

Therefore,

$B = \quad Q$ (CPCT)

But $Q = 90°$ [By construction]

So, $B = 90°$

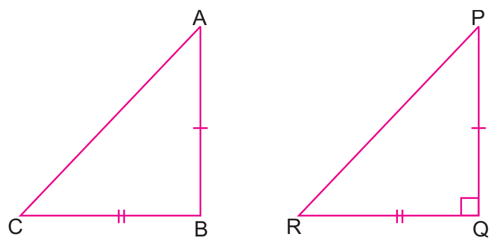


Fig. 4.34

In ΔPQR,

By Pythagoras Theorem, we have

$PR^2 = (26)^2 - (24)^2$

$PR^2 = 676 - 576$

$PR = \sqrt{100} = 10$ cm

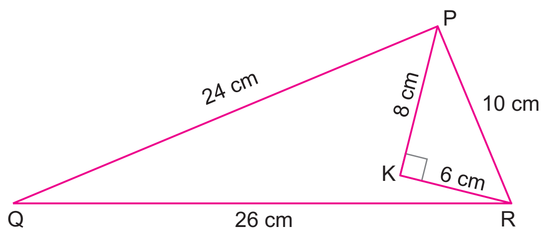


Fig. 4.35

Now, In ΔPKR, we have

$PK^2 + KR^2 = (8)^2 + (6)^2 = 64 + 36 = 100 = PR^2$

Hence, $PKR = 90°$ [By Converse of Pythagoras Theorem]

3. *ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.* **[NCERT]**

Sol. ΔABC is an isosceles triangle right-angled at C.

$AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$AB^2 = AC^2 + AC^2$ [$\because AC = BC$]

$AB^2 = 2AC^2$

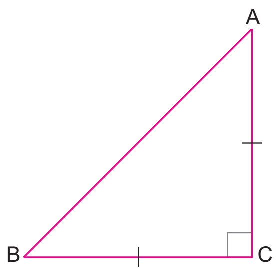


Fig. 4.36

4. Sides of triangle are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(*i*) 7 cm, 24 cm, 25 cm (*ii*) 3 cm, 8 cm, 6 cm **[NCERT]**

Sol. (*i*) Let $a = 7$ cm, $b = 24$ cm and $c = 25$ cm.

Here, largest side, $c = 25$ cm

We have, $a^2 + b^2 = (7)^2 + (24)^2 = 49 + 576$

$= 625 = c^2$ [$\because c = 25$]

So, the triangle is a right triangle.

Hence, c is the hypotenuse of right triangle.

(*ii*) Let $a = 3$ cm, $b = 8$ cm and $c = 6$ cm

Here, largest side, $b = 8$ cm

We have, $a^2 + c^2 = (3)^2 + (6)^2 = 9 + 36 = 45 \neq b^2$

So, the triangle is not a right triangle.

5. *ABC is an equilateral triangle of side 2a. Find each of its altitudes.* **[NCERT]**

Sol. Let ABC be an equilateral triangle of side $2a$ units.

We draw AD BC. Then D is the mid-point of BC.

$$BD = \frac{BC}{2} = \frac{2a}{2} = a$$

Now, ABD is a right triangle right-angled at D.

$$AB^2 = AD^2 + BD^2 \quad \text{[By Pythagoras Theorem]}$$

$$(2a)^2 = AD^2 + a^2$$

$$AD^2 = 4a^2 - a^2 = 3a^2 \qquad AD = \sqrt{3}a$$

Hence, each of altitude $= \sqrt{3}a$ unit.

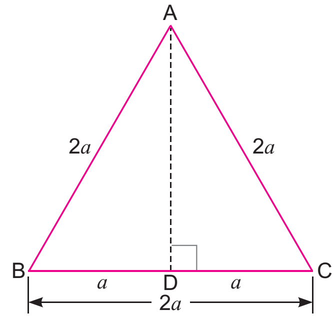


Fig. 4.37

6. *In Fig. 4.38, O is a point in the interior of a triangle ABC, OD   BC, OE   AC and OF   AB. Show that*

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$. **[NCERT]**

Sol. Join OA, OB and OC.

(*i*) In right Δ's OFA, ODB and OEC, we have

$$OA^2 = AF^2 + OF^2 \quad ...(i)$$

$$OB^2 = BD^2 + OD^2 \quad ...(ii)$$

and $$OC^2 = CE^2 + OE^2 \quad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*), we have

$$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(*ii*) We have, $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

$$(OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$$

$$AE^2 + CD^2 + BF^2 = AF^2 + BD^2 + CE^2$$

[Using Pythagoras Theorem in ΔAOE, ΔBOF and ΔCOD]

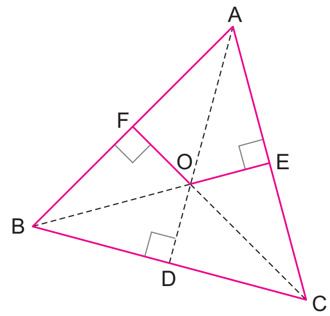


Fig. 4.38

7. *D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that* $AE^2 + BD^2 = AB^2 + DE^2$. **[NCERT]**

Sol. In right angled ΔACE and ΔDCB, we have

$$AE^2 = AC^2 + CE^2 \text{ (Pythagoras Theorem)} \quad ...(i)$$

and $$BD^2 = DC^2 + BC^2 \quad ...(ii)$$

Adding (*i*) and (*ii*), we have

$$AE^2 + BD^2 = AC^2 + CE^2 + DC^2 + BC^2$$

$$AE^2 + BD^2 = (AC^2 + BC^2) + (DC^2 + CE^2)$$

$$AE^2 + BD^2 = AB^2 + DE^2$$

[$\because AC^2 + BC^2 = AB^2$ in right-angled triangle ΔABC and $DC^2 + EC^2 = DE^2$ in right-angled triangle ΔCDE.]

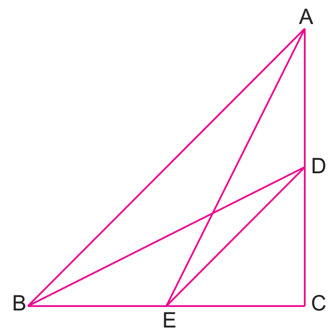


Fig. 4.39

8. *The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see Fig.4.40). Prove that* $2AB^2 = 2AC^2 + BC^2$. **[NCERT]**

Sol. We have, $DB = 3CD$

Now, $BC = BD + CD$

$BC = 3\,CD + CD$ (Given $DB = 3CD$)

$BC = 4\,CD$

$CD = \frac{1}{4}BC$

and $DB = 3\,CD = \frac{3}{4}BC$

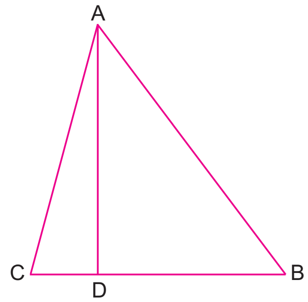


Fig. 4.40

Now, in right-angled triangle ABD, we have

$AB^2 = AD^2 + DB^2$ ...(*i*)

Again, in right-angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ ...(*ii*)

Subtracting (*ii*) from (*i*), we have

$AB^2 - AC^2 = DB^2 - CD^2$

$$AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2$$

$$AB^2 - AC^2 = \frac{1}{2}BC^2$$

$2AB^2 - 2AC^2 = BC^2 \quad\quad 2AB^2 = 2AC^2 + BC^2$

9. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. **[NCERT]**

Sol. Let ABC be an equilateral triangle and let $AD \quad BC$.

$BD = DC$

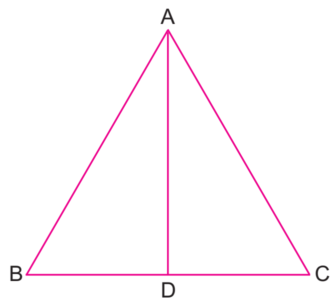


Fig. 4.41

Now, in right-angled triangle ΔADB, we have

$AB^2 = AD^2 + BD^2$ [Using Pythagoras Theorem]

$$AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 \quad\quad AB^2 = AD^2 + \frac{1}{4}BC^2$$

$AB^2 = AD^2 + \frac{AB^2}{4}$ [$\because AB = BC$]

$$AB^2 - \frac{AB^2}{4} = AD^2 \quad\quad \frac{3AB^2}{4} = AD^2 \quad\quad 3AB^2 = 4AD^2$$

10. A point O in the interior of a rectangle $ABCD$ is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$.

Sol. Let $ABCD$ be the given rectangle and O be a point within it. Join OA, OB, OC and OD.

Through O, draw $EOF \parallel AB$. Then, $ABFE$ is a rectangle.

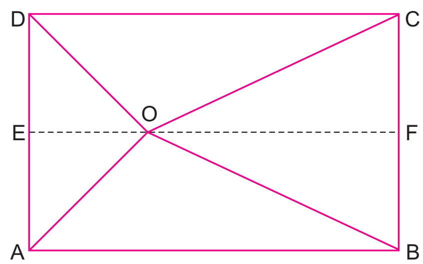


Fig. 4.42

In right triangles OEA and OFC, we have

$OA^2 = OE^2 + AE^2$ and $OC^2 = OF^2 + CF^2$

$OA^2 + OC^2 = (OE^2 + AE^2) + (OF^2 + CF^2)$

$OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ ...(*i*)

Now, in right triangles OFB and ODE, we have

$OB^2 = OF^2 + FB^2$ and $OD^2 = OE^2 + DE^2$

$$OB^2 + OD^2 = (OF^2 + FB^2) + (OE^2 + DE^2)$$
$$OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$$
$$OB^2 + OD^2 = OE^2 + OF^2 + CF^2 + AE^2 \quad [\because DE = CF \text{ and } AE = BF] \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$OA^2 + OC^2 = OB^2 + OD^2$$

11. *ABC* is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ΔABC is right-triangled.

Sol. Given, $AB^2 = 2AC^2$

$$AB^2 = AC^2 + AC^2$$
$$AB^2 = AC^2 + BC^2 \quad [\text{Given, } AC = BC]$$

ΔABC is a right triangle in which $C = 90°$. [Using the converse of Pythagoras Theorem]

HOTS (Higher Order Thinking Skills)

1. In Fig.4.43, *P* is the mid-point of *BC* and *Q* is the mid-point of *AP*. If *BQ* when produced meets *AC* at *R*, prove that $RA = \frac{1}{3} CA$.

Sol. Given: In ΔABC, P is the mid-point of *BC*, *Q* is the mid-point of *AP* such that *BQ* produced meets *AC* at *R*.

To prove: $RA = \frac{1}{3} CA$

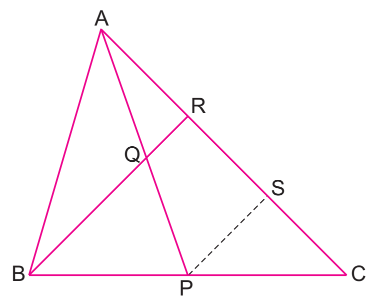


Fig. 4.43

Construction: Draw $PS \parallel BR$, meeting *AC* at *S*.

Proof: In ΔBCR, *P* is the mid-point of *BC* and $PS \parallel BR$.

S is the mid-point of *CR*.

$$CS = SR \quad ...(i)$$

In ΔAPS, *Q* is the mid-point of *AP* and $QR \parallel PS$.

R is the mid-point of *AS*.

$$AR = RS \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$AR = RS = SC$$
$$AC = AR + RS + SC = 3\,AR$$
$$AR = \frac{1}{3} AC = \frac{1}{3} CA$$

2. In Fig. 4.44, $\Delta FEC \quad \Delta GBD$ and $1 = \quad 2$.

Prove that $\Delta ADE \sim \Delta ABC$.

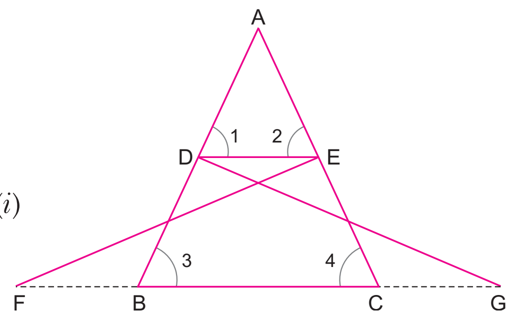


Fig. 4.44

Sol. Since,

$$\Delta FEC \quad \Delta GBD$$
$$EC = BD \quad ...(i)$$

It is given that

$$1 = \quad 2$$
$$AE = AD \quad \left[\begin{array}{l}\text{Sides opposite to equal}\\ \text{angles are equal}\end{array}\right] \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$DE \parallel BC$ [By the converse of basic proportionality theorem]

1 = 3 and 2 = 4 [Corresponding angles]

Thus, in Δ' s *ADE* and *ABC*, we have

$A = A$ [common]

1 = 3

2 = 4 [proved above]

So, by *AAA* criterion of similarity, we have

$$\Delta ADE \sim \Delta ABC$$

3. Sides *AB* and *AC* and median *AD* of a triangle *ABC* are respectively proportional to sides *PQ* and *PR* and median *PM* of another triangle *PQR*. Show that Δ*ABC* ~ Δ*PQR*.

Sol. **Given:** In Δ*ABC* and Δ*PQR*, *AD* and *PM* are their medians respectively.

Such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ ... (*i*)

To prove: Δ*ABC* ~ Δ*PQR*.

Construction: Produce *AD* to *E* such that *AD* = *DE* and produce *PM* to *N* such that *PM* = *MN*. Join *BE*, *CE*, *QN*, *RN*.

Proof: Quadrilateral *ABEC* and *PQNR* are ||gm because their diagonals bisect each other at *D* and *M* respectively.

$BE = AC$ and $QN = PR$

$\frac{BE}{QN} = \frac{AC}{PR}$ $\frac{BE}{QN} = \frac{AB}{PQ}$ [From (*i*)]

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(*ii*)

From (*i*) $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\frac{AB}{PQ} = \frac{AE}{PN}$ ...(*iii*)

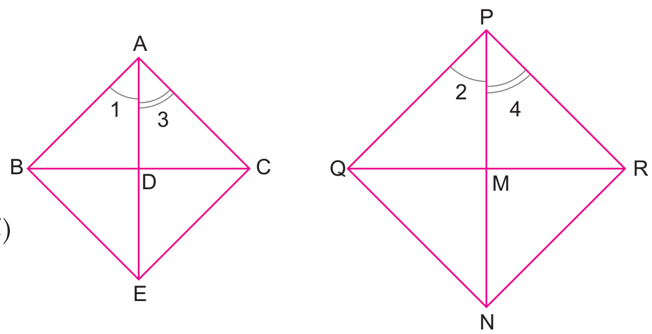


Fig. 4.45

From (*ii*) and (*iii*)

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Delta ABE \sim \Delta PQN$ 1 = 2 ...(*iv*)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN$ 3 = 4 ...(*v*)

Adding (*iv*) and (*v*), we get

1 + 3 = 2 + 4 $A = P$

and $\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

$\Delta ABC \sim \Delta PQR$ (By *SAS* criterion of similarity)

4. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Sol. Let AB and CD be two poles of height a and b metres respectively such that the poles are p metres apart *i.e.,* $AC = p$ metres. Suppose the lines AD and BC meet at O such that $OL = h$ metres.

Let $CL = x$ and $LA = y$. Then, $x + y = p$.

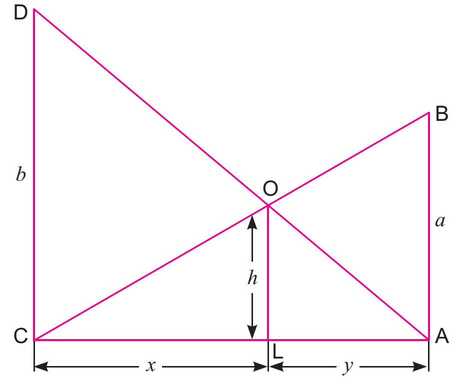


Fig. 4.46

In ΔABC and ΔLOC, we have

$$CAB = CLO \quad \text{[Each equal to 90°]}$$

$$C = C \quad \text{[Common]}$$

$$\Delta ABC \sim \Delta LOC \quad \text{[By } AA \text{ criterion of similarity]}$$

$$\frac{CA}{CL} = \frac{AB}{LO}$$

$$\frac{p}{x} = \frac{a}{h} \qquad x = \frac{ph}{a} \qquad \ldots(i)$$

In ΔALO and ΔACD, we have

$$ALO = ACD \quad \text{[Each equal to 90°]}$$

$$A = A \quad \text{[Common]}$$

$$\Delta ALO \sim \Delta ACD \quad \text{[By } AA \text{ criterion of similarity]}$$

$$\frac{AL}{AC} = \frac{OL}{DC} \qquad \frac{y}{p} = \frac{h}{b}$$

$$y = \frac{ph}{b} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$x + y = \frac{ph}{a} + \frac{ph}{b}$$

$$p = ph\left(\frac{1}{a} + \frac{1}{b}\right) \qquad [\because \; x + y = p]$$

$$1 = h\left(\frac{a+b}{ab}\right) \qquad h = \frac{ab}{a+b} \text{ metres.}$$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

5. In Fig. 4.47, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta DBC)} = \frac{AO}{DO}$

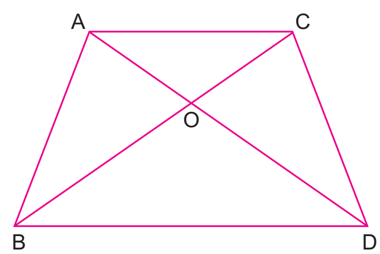


Fig. 4.47

Sol. Given: Two triangles ΔABC and ΔDBC which stand on the same base but on opposite sides of BC.

To prove: $\frac{\text{ar }(ABC)}{\text{ar }(DBC)} = \frac{AO}{DO}$

Construction: We draw AE BC and DF BC.

Proof: In ΔAOE and ΔDOF, we have

$$AEO = DFO = 90°$$

$$AOE = \quad DOF \qquad \text{(Vertically opposite angles)}$$

$$\Delta AOE \sim \Delta DOF \qquad \text{(By AA criterion of similarity)}$$

$$\frac{AE}{DF} = \frac{AO}{DO} \qquad \ldots(i)$$

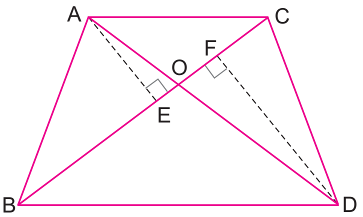


Fig. 4.48

Now, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{AE}{DF} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{AO}{DO}$$

6. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9AD^2 = 7AB^2$.

Sol. Let ABC be an equilateral triangle and D be a point on BC such that $BD = \frac{1}{3} BC$.

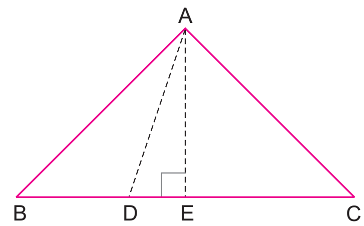


Fig. 4.49

To Prove: $9AD^2 = 7AB^2$

Construction: Draw $AE \quad BC$. Join AD.

Proof : ΔABC is an equilateral triangle and $AE \quad BC$

$$BE = EC$$

Thus, we have

$$BD = \frac{1}{3} BC \quad \text{and} \quad DC = \frac{2}{3} BC \quad \text{and} \quad BE = EC = \frac{1}{2} BC$$

In ΔAEB

$$AE^2 + BE^2 = AB^2 \qquad \text{[Using Pythagoras Theorem]}$$

$$AE^2 = AB^2 - BE^2$$

$$AD^2 - DE^2 = AB^2 - BE^2 \qquad [\because \text{ In } \Delta AED,\ AD^2 = AE^2 + DE^2]$$

$$AD^2 = AB^2 - BE^2 + DE^2$$

$$AD^2 = AB^2 - \left(\frac{1}{2} BC\right)^2 + (BE - BD)^2$$

$$AD^2 = AB^2 - \frac{1}{4} BC^2 + \left(\frac{1}{2} BC - \frac{1}{3} BC\right)^2$$

$$AD^2 = AB^2 - \frac{1}{4} BC^2 + \left(\frac{BC}{6}\right)^2$$

$$AD^2 = AB^2 - BC^2 \left(\frac{1}{4} - \frac{1}{36}\right) \qquad AD^2 = AB^2 - BC^2 \left(\frac{8}{36}\right)$$

$$9AD^2 = 9AB^2 - 2BC^2$$

$$9AD^2 = 9AB^2 - 2AB^2 \qquad [\because AB = BC]$$

$$9AD^2 = 7AB^2$$

Exercise

A. Multiple Choice Questions

Write correct answer for each of the following:

1. In ΔPQR, L and M are points on sides PQ and PR respectively such that $PL : LQ = 1 : 3$. If $MR = 6.6$ cm, then PR is equal to

(*a*) 2.2 cm (*b*) 3.3 cm (*c*) 8.8 cm (*d*) 9.9 cm

2. If ABC and DEF are similar triangles such that $A = 45°$ and $F = 56°$, then C is equal to

(*a*) 45° (*b*) 56° (*c*) 101° (*d*) 79°

3. ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of triangles ABC and BDE is

(*a*) 2 : 1 (*b*) 4 : 1 (*c*) 1 : 4 (*d*) 1 : 2

4. The area of two similar triangles ΔPQR and ΔXYZ are 144 cm^2 and 49 cm^2 respectively. If the shortest side of larger ΔPQR be 24 cm, then the shortest side of the smaller triangle ΔXYZ is

(*a*) 7 cm (*b*) 14 cm (*c*) 16 cm (*d*) 10 cm

5. If ABC and DEF are two triangles such that $\frac{AB}{EF} = \frac{BC}{FD} = \frac{CA}{DE} = \frac{3}{4}$, then ar (ΔDEF) : ar (ΔABC)

(*a*) 3 : 4 (*b*) 4 : 3 (*c*) 9 : 16 (*d*) 16 : 9

6. If $\Delta ABC \sim \Delta RPQ$, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{16}{9}$, $AB = 20$ cm and $AC = 12$ cm, then PR is equal to

(*a*) 15 cm (*b*) 9 cm (*c*) $\frac{45}{4}$ cm (*d*) $\frac{27}{4}$ cm

7. The lengths of the diagonals of a rhombus are 24 cm and 32 cm. Then, the length of the side of the rhombus is

(*a*) 20 cm (*b*) 10 cm (*c*) 40 cm (*d*) 30 cm

8. If in two triangles ABC and PQR, $\frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$, then

(*a*) $\Delta ABC \sim \Delta PRQ$ (*b*) $\Delta CBA \sim \Delta PQR$

(*c*) $\Delta PQR \sim \Delta ACB$ (*d*) $\Delta ACB \sim \Delta RQP$

9. In Fig. 4.50, two line segments AC and BD intersect each other at the point P such that $AP = 8$ cm, $PB = 4$ cm, $PC = 3$ cm and $PD = 6$ cm. If $APB = 50°$ and $CDP = 30°$, then PBA is equal to

(*a*) 50° (*b*) 30°

(*c*) 60° (*d*) 100°

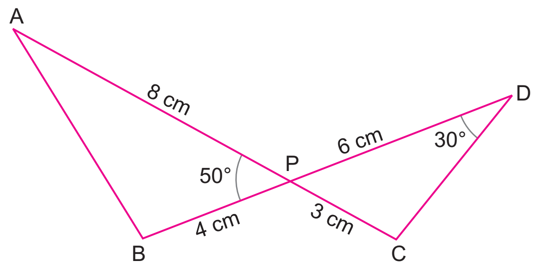


Fig. 4.50

10. Two poles of height 9 m and 15 m stand vertically upright on a plain ground. If the distance between their tops is 10m, the distance between their foot is

(*a*) 9 cm (*b*) 7 cm (*c*) 8 cm (*d*) 6 cm

11. $\Delta ABC \sim \Delta DEF$. If $AB = 4$cm, $BC = 3.5$ cm, $CA = 2.5$ and $DF = 7.5$ cm, then perimeter of ΔDEF is

(*a*) 10 cm (*b*) 14 cm (*c*) 30 cm (*d*) 25 cm

12. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is

(*a*) 100 m (*b*) 120 m (*c*) 25 m (*d*) 200 m

13. In an equilateral triangle ABC, if $AD \quad BC$, then

(*a*) $2AB^2 = 3AD^2$ (*b*) $4AB^2 = 3AD^2$ (*c*) $3AB^2 = 4AD^2$ (*d*) $3AB^2 = 2AD^2$

14. In a ΔABC, points D and E lie on the sides AB and AC respectively, such that $BCED$ is a trapezium. If $DE : BC = 2:5$, then ar (ADE) : ar $(BCED)$

(*a*) 3 : 4 (*b*) 4 : 21 (*c*) 3 : 5 (*d*) 9 : 25

15. If E is a point on side CA of an equilateral triangle ABC such that $BE \quad CA$, then $AB^2 + BC^2 + CA^2$ is equal to

(*a*) $2\,BE^2$ (*b*) $3\,BE^2$ (*c*) $4\,BE^2$ (*d*) $6\,BE^2$

16. If ABC is an isosceles triangle and D is a point on BC such that $AD \quad BC$, then

(*a*) $AB^2 - AD^2 = BD.DC$ (*b*) $AB^2 - AD^2 = BD^2 - DC^2$

(*c*) $AB^2 + AD^2 = BD.DC$ (*d*) $AB^2 + AD^2 = BD^2 - DC^2$

17. In trapezium $ABCD$ with $AB||CD$, the diagonals AC and BD intersect at O. If $AB = 5$ cm and $\frac{AO}{OC} = \frac{OB}{DO} = \frac{1}{2}$, then DC is equal to

(*a*) 12 cm (*b*) 15 cm (*c*) 10 cm (*d*) 20 cm

B. Short Answer Questions Type-I

1. Is the triangle with sides 10 cm, 24 cm and 26 cm a right triangle? Give reason.

2. "Two quadrilaterals are similar, if their corresponding angles are equal". Is it true? Give reason.

3. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar? Why?

4. The ratio of the corresponding altitudes of two similar triangles is $\frac{2}{5}$. Is it correct to say that ratio of their areas is also $\frac{2}{5}$? Why?

5. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, the triangle are similar? Give reason.

6. If $\Delta ABC \sim \Delta ZYX$, then is it true to say that $\quad B = \quad X$ and $\quad A = \quad Z$?

7. L and M are respectively the points on the sides DE and DF of a triangle DEF such that $DL = 4$, $LE = \frac{4}{3}$, $DM = 6$ and $DF = 8$. Is $LM||EF$? Give reason.

8. If the areas of two similar triangles ABC and PQR are in the ratio 9:16 and $BC = 4.5$ cm, what is the length of QR?

9. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

10. In Fig. 4.51, $PQ||BC$ and $AP : PB = 1 : 2$ find $\frac{\text{area}\,(\Delta APQ)}{\text{area}\,(\Delta ABC)}$.

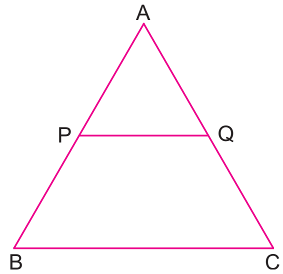


Fig. 4.51

C. Short Answer Questions Type–II

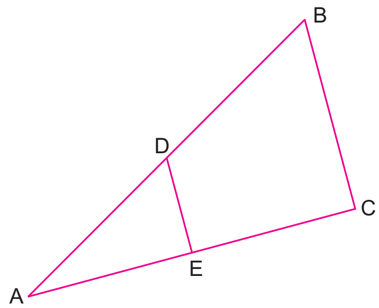


Fig. 4.52

1. If a line intersects sides AB and AC of a ΔABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$.
2. In Fig. 4.52, $DE \parallel BC$. If $\frac{AE}{EC} = \frac{4}{13}$ and $AB = 20.4$ cm, find AD.
3. In ΔABC, $DE \parallel BC$. If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$ and $CE = 5x - 3$, find the value of x.
4. In ΔABC, $DE \parallel BC$. If $AD = 4$ cm, $DB = 4.5$ cm and $AE = 8$ cm, find AC.
5. In ΔABC, $DE \parallel BC$. If $AD = 2.4$ cm, $AE = 3.2$ cm, $DE = 2$ cm and $BC = 5$ cm, find BD and CE.

***L* and *M* are points on the sides *DE* and *DF* respectively of a ΔDEF. For each of the following cases (Q. 6 and 7), state whether $LM \parallel EF$.**

6. $DL = 3.9$ cm, $LE = 3$ cm, $DM = 3.6$ cm and $MF = 2.4$ cm.
7. $DE = 8$ cm, $DF = 15$ cm, $LE = 3.2$ cm and $MF = 6$ cm.
8. The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium.
9. If $\Delta ABC \sim \Delta DEF$, $AB = 4$ cm, $DE = 6$ cm, $EF = 9$ cm and $FD = 12$ cm, find the perimeter of ΔABC.
10. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5m casts a shadow of 3 m, find how far she is away from the base of the pole.
11. CD and GH are respectively the bisectors of ACB and EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG. If $\Delta ABC \sim \Delta FEG$, show that

    (*i*) $\frac{CD}{GH} = \frac{AC}{FG}$ (*ii*) $\Delta DCB \sim \Delta HGE$ (*iii*) $\Delta DCA \sim \Delta HGF$

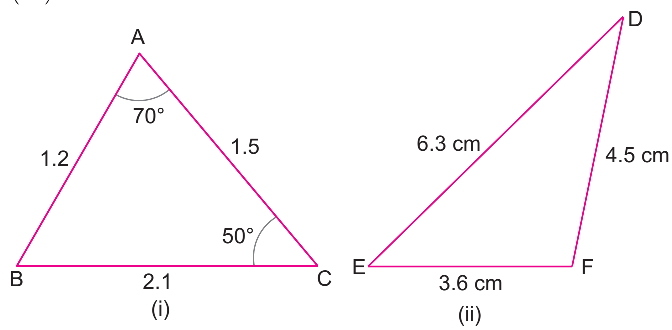


Fig. 4.53

12. D is a point on the side BC of a triangle ABC such that $ADC = BAC$. Show that $CA^2 = CB.CD$.
13. In Fig.4.53, find E.
14. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.
15. A 15 m high tower casts a shadow 24 m long at a certain time and at the same time, a telephone pole casts a shadow 16 m long. Find the height of the telephone pole.
16. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

    (*i*) 13 cm, 12 cm, 5 cm

    (*ii*) 20 cm, 25 cm, 30 cm.

17. O is any point inside a rectangle $ABCD$. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.
18. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
19. In Fig. 4.54 ABC is a right triangle, right-angled at C and D is the mid-point of BC. Prove that $AB^2 = 4AD^2 - 3AC^2$.

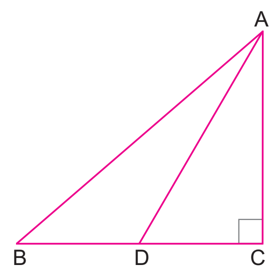


Fig. 4.54

20. In Fig. 4.55, ABC is an isosceles triangle in which $AB = AC$. E is a point on the side CB produced such that FE AC. If AD CB, prove that $AB \times EF = AD \times EC$.

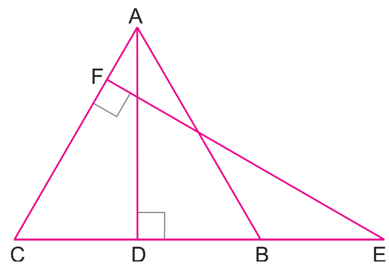


Fig. 4.55

21. In an isosceles triangle PQR, $PQ = QR$ and $PR^2 = PQ^2$. Prove that Q is a right angle.

22. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that:

$$\text{area}\,(\Delta ADE) : \text{area}\,(\Delta ABC) = 3 : 4$$

23. In Fig. 4.56, $D =$ E and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$. Prove that BAC is an isosceles triangle.

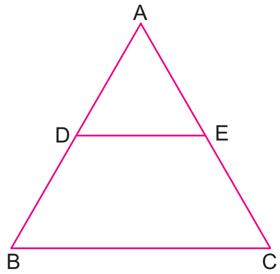


Fig. 4.56

24. In Fig. 4.57, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that $RA = \dfrac{1}{3}CA$.

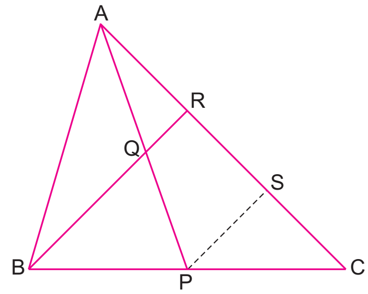


Fig. 4.57

25. In Fig. 4.58, $AB||CD$. If $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$, find x.

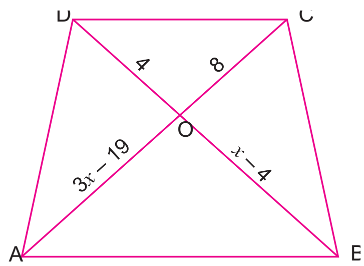


Fig. 4.58

26. In Fig.4.59, AB BC and DE AC. Prove that $\Delta ABC \sim \Delta AED$.

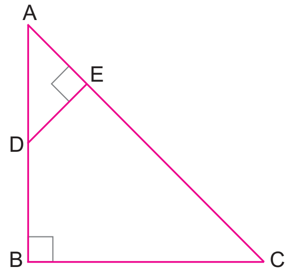


Fig. 4.59

27. Two triangles (Fig. 4.60) BAC and BDC, right-angled at A and D respectively, are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, prove that $AP \times PC = DP \times PB$.

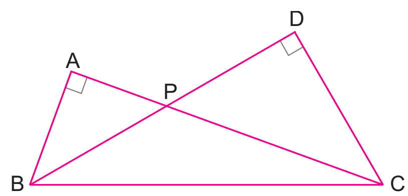


Fig. 4.60

28. In Fig. 4.61, E is a point on side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Prove that $\Delta ABE \sim \Delta CFB$.

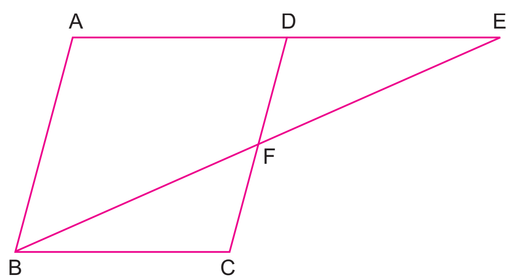


Fig. 4.61

29. In ΔABC (Fig. 4.62), DE is parallel to base BC, with D on AB and E on AC. If $\dfrac{AD}{DB} = \dfrac{2}{3}$, find $\dfrac{BC}{DE}$.

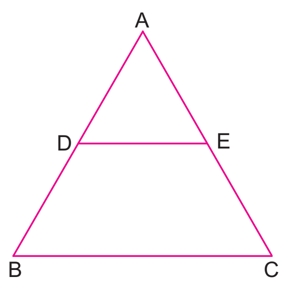


Fig. 4.62

30. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm, $QC = 4.5$ cm, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC.

D. Long Answer Questions

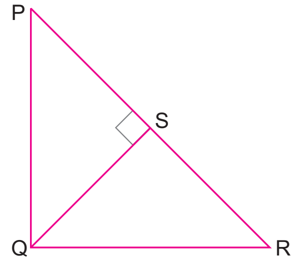


Fig. 4.63

1. In Fig. 4.63, PQR is a right triangle right-angled at Q and QS PR. If $PQ = 6$ cm and $PS = 4$ cm, find the QS, RS and QR.

2. In Fig. 4.64, PA, QB, RC and SD are all perpendicular to a line l, $AB = 6$ cm, $BC = 9$ cm, $CD = 12$ cm and $SP = 36$ cm. Find PQ, QR and RS.

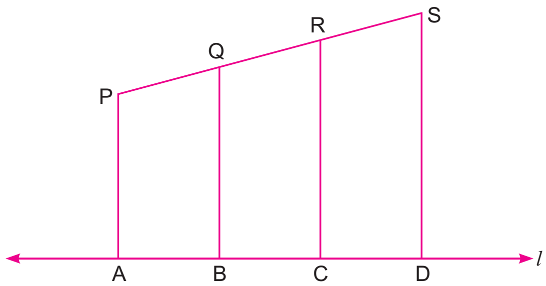


Fig. 4.64

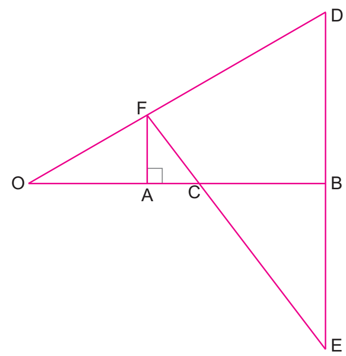


Fig. 4.65

3. In Fig. 4.65, OB is the perpendicular bisector of the line segment DE, FA OB and FE intersects OB at the point C.

Prove that: $\dfrac{1}{OA} + \dfrac{1}{OB} = \dfrac{2}{OC}$

4. In an equilateral triangle ABC, D is a point on side BC such that $BD = \dfrac{1}{3}BC$. Prove that: $9AD^2 = 7AB^2$.

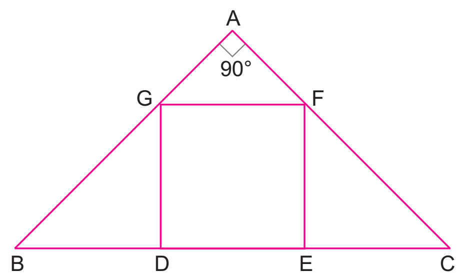


Fig. 4.66

5. In PQR, PD QR such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, prove that: $(a+b)(a-b) = (c+d)(c-d)$.

6. Prove that the area of the semicircle drawn on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

7. In Fig.4.66, $DEFG$ is a square and $BAC = 90°$. Prove that:

(*i*) $\Delta AGF \sim \Delta DBG$ (*ii*) $\Delta AGF \sim \Delta EFC$.

(*iii*) $\Delta DBG \sim \Delta EFC$ (*iv*) $DE^2 = BD \times EC$

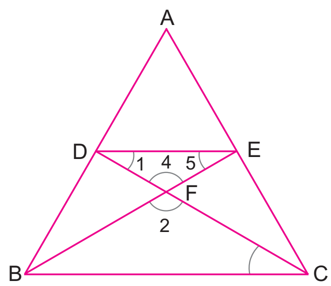


Fig. 4.67

8. If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse, then prove that the triangle on each side of the perpendicular are similar to each other and to the original triangle. Also, prove that the square of the perpendicular is equal to the product of the lengths of the two parts of the hypotenuse.

9. In Fig. 4.67, $DE \| BC$ and $AD : DB = 5 : 4$. Find $\dfrac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)}$.

10. D and E are points on the sides AB and AC respectively of a ΔABC such that $DE \| BC$ and divides ΔABC into two parts, equal in area. Find $\dfrac{BD}{AB}$.

11. P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right-angled at C. Prove that:

(*i*) $4AQ^2 = 4AC^2 + BC^2$ (*ii*) $4BP^2 = 4BC^2 + AC^2$ (*iii*) $(4AQ^2 + BP^2) = 5AB^2$.

12. ABC is a right triangle right-angled at C. Let $BC = a$, $CA = b$, $AB = c$ and let p be the length of perpendicular from C on AB. Prove that:

(*i*) $cp = ab$ (*ii*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

13. In an equilateral triangle with side a, prove that:

(*i*) Altitude $= \frac{a\sqrt{3}}{2}$ (*ii*) Area $= \frac{\sqrt{3}}{4}a^2$.

14. In a triangle ABC, $AC > AB$, D is the mid-point of BC and AE BC. Prove that:

(*i*) $AC^2 = AD^2 + BC.DE + \frac{1}{4}BC^2$

(*ii*) $AB^2 = AD^2 - BC.DE + \frac{1}{4}BC^2$

(*iii*) $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

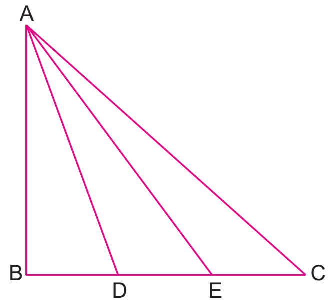


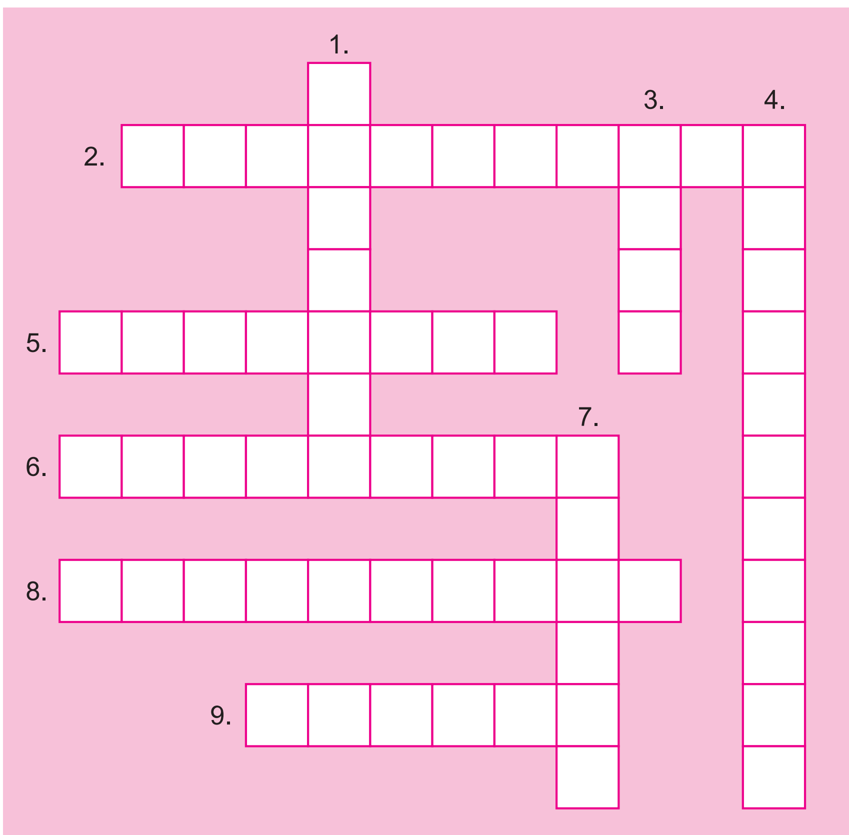
Fig. 4.68

15. In Fig. 4.68, D and E trisects BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.

Formative Assessment

Activity: 1

■ **Solve the following crossword puzzle, hints are given below:**



Across:

2. Triangles whose corresponding angles are equal.
5. If a line divides any two sides of a triangle in the same ratio, then the line is __________ to the third side.
6. Two figures with same shape and size.
8. Mathematician who proved that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
9. The ratio of the areas of two similar triangles is equal to the ratio of the __________ of their corresponding sides.

Down:

1. Two figures with same shape.
4. Triangles in which Pythagoras theorem is applicable.
7. Mathematician with whose name Basic Proportionality Theorem is known.
3. A __________ has no end point.

Activity: 2

Basic Proportionality Theorem

- Draw any XAY (preferably an acute angle).
- On one arm (say AX), mark points at equal distances (say five points B, C, D, E, F)
  $AB = BC = CD = DE = EF$
- Through F, draw any line intersecting the other arm AY at P.
- Through D, draw a line parallel to PF to intersect AP at Q.
- From construction, we have $\dfrac{AD}{DF} = \dfrac{3}{2}$
- Measure AQ and QP

  You will observe $\dfrac{AQ}{QP} = \dfrac{3}{2}$

  So, in ΔAFP, $DQ \| PF$ and $\dfrac{AD}{DF} = \dfrac{AQ}{QP}$

  Thus, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

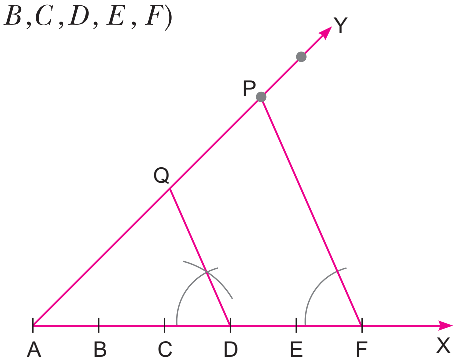


Fig. 4.69

Hands on Activity (Math Lab Activity)

To verify the Pythagoras Theorem by the method of paper folding, cutting and pasting.

Materials Required

Cardboard, coloured pencils, pair of scissors, fevicol, geometry box.

Procedure

1. Take a cardboard piece of size say 15 cm × 15 cm.
2. Cut any right-angled triangle and paste it on the cardboard suppose its sides are a, b and c.
3. Cut a square of side a cm and place it along the side of length a cm of the right-angled triangle.
4. Similarly, cut squares of sides b cm and c cm and place them along the respective sides of the right angled triangle.

5. Label the diagram as shown in Fig. 4.70.
6. Join BH and AI. These are two diagonals of the square $ABIH$. Two diagonals intersect each other at the point O.
7. Through O, draw $RS \,||\, BC$.
8. Draw PQ, the perpendicular bisector of RS, passing through O.
9. Now, the square $ABIH$ is divided in four quadrilaterals. Colour them as shown in Fig. 4.70.
10. From the square $ABIH$, cut the four quadrilaterals. Colour them and name them as shown in Fig. 4.71.

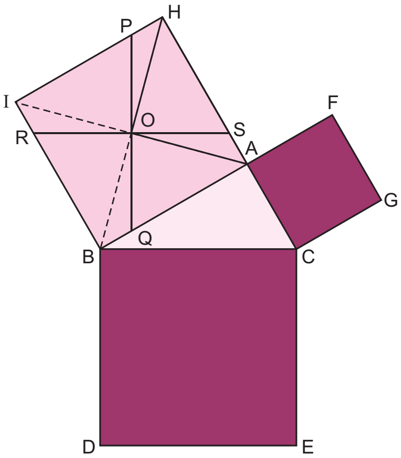


Fig. 4.70

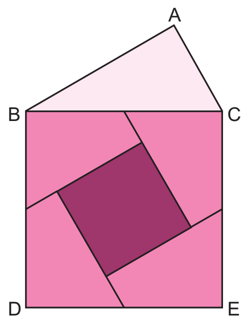


Fig. 4.71

Observations

The square ACGF and the four quadrilaterals cut from the square ABIH completely fill the square BCED. Thus, the theorem is verified.

Conclusion

Pythagoras theorem is verified by paper cutting and pasting.

Suggested Activity

- To verify that the ratio of areas of two similar triangles is equal to the square of ratios of their corresponding sides.

Oral Questions

1. When do we say that two polygons are similar?
2. What is a scale factor?
3. Where do we see the use of the scale factor?
4. Give two examples of pairs of figures which are similar but not congruent.
5. State SAS similarity criterion.
6. State SSS similarity criterion.
7. State AA similarity criterion.
8. $\Delta ABC \quad \Delta PRQ, \quad B = \quad Q$. (True/False)
9. If $\Delta ABC \sim \Delta DEF$, then can we say $AB = DE$?
10. All congruent polygons are also similar. (True/False)
11. All similar polygons are always congruent. (True/False)

Multiple Choice Questions

Tick the correct answer for each of the following:

1. A square and a rhombus are always

(*a*) similar (*b*) congruent

(*c*) similar but not congruent (*d*) neither similar nor congruent

2. Two circles are always

(*a*) congruent (*b*) neither similar nor congruent

(*c*) similar but may not be congruent (*d*) none of these

3. D and E are respectively the points on the sides AB and AC of a triangle ABC such that $AD = 3$ cm, $BD = 5$ cm, $BC = 12.8$ cm and $DE \| BC$. Then length of DE (in cm) is

(*a*) 4.8 cm (*b*) 7.6 cm (*c*) 19.2 cm (*d*) 2.5 cm

4. If $\Delta PRQ \quad \Delta XYZ$, then

(*a*) $\frac{PR}{XZ} = \frac{RQ}{YZ}$ (*b*) $\frac{PQ}{XY} = \frac{PR}{XZ}$ (*c*) $\frac{PQ}{XZ} = \frac{QR}{YZ}$ (*d*) $\frac{QR}{XZ} = \frac{PR}{XY}$

5. The length of each side of a rhombus whose diagonals are of lengths 10 cm and 24 cm is

(*a*) 25 cm (*b*) 13 cm (*c*) 26 cm (*d*) 34 cm

6. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(*a*) $\Delta PQR \quad \Delta CAB$ (*b*) $\Delta PQR \quad \Delta ABC$ (*c*) $\Delta CBA \quad \Delta PQR$ (*d*) $\Delta BCA \quad \Delta PQR$

7. If in triangles ABC and XYZ, $\quad B = \quad X$ and $\quad C = \quad Z$, then which of the following is not true?

(*a*) $\frac{AB}{XY} = \frac{BC}{YZ}$ (*b*) $\frac{AB}{YX} = \frac{BC}{XZ}$ (*c*) $\frac{BC}{XZ} = \frac{CA}{YZ}$ (*d*) $\frac{CA}{ZY} = \frac{AB}{XY}$

8. If ΔABC is not similar to ΔDEF under the correspondence $ABC \leftrightarrow DEF$, then which of the following is surely not true?

(*a*) $BC . EF = AC . FD$ (*b*) $AB . EF = AC . DE$ (*c*) $BC . DE = AB . EF$ (*d*) $BC . DE = AB . FD$

9. In ΔLMN and ΔPQR, $\quad L = \quad P$, $\quad N = \quad R$ and $MN = 2QR$. Then the two triangles are

(*a*) congruent but not similar (*b*) similar but not congruent

(*c*) neither congruent nor similar (*d*) congruent as well as similar

10. In ΔABC and ΔRPQ, $AB = 4.5$ cm, $BC = 5$ cm, $CA = 6\sqrt{2}$ cm, $PR = 12\sqrt{2}$ cm, $PQ = 10$ cm, $QR = 9$ cm. If $\quad A = 75°$ and $\quad B = 55°$, then $\quad P$ is equal to

(*a*) 75° (*b*) 55° (*c*) 50° (*d*) 130°

11. If in triangles ABC and DEF, $\frac{AB}{EF} = \frac{AC}{DE}$, then they will be similar when

(*a*) $\quad A = \quad D$ (*b*) $\quad A = \quad E$ (*c*) $\quad B = \quad E$ (*d*) $\quad C = \quad F$

12. If $\Delta PQR \quad \Delta XYZ$ and $\frac{PQ}{XY} = \frac{5}{2}$, then $\frac{\text{ar}(\Delta XYZ)}{\text{ar}(\Delta PQR)}$ is equal to

(*a*) $\frac{4}{25}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{25}{4}$ (*d*) $\frac{5}{2}$

13. It is given that ar $(\Delta ABC) = 81$ square units and ar $(\Delta DEF) = 64$ square units. If $\Delta ABC \sim \Delta DEF$, then

(*a*) $\frac{AB}{DE} = \frac{81}{64}$

(*b*) $\frac{AB^2}{DE^2} = \frac{9}{8}$

(*c*) $\frac{AB}{DE} = \frac{9}{8}$

(*d*) $AB = 81$ units, $DE = 64$ units

14. If ΔABC ΔDEF, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{9}{25}$, $BC = 21$ cm, then EF is equal to

(*a*) 9 cm (*b*) 6 cm (*c*) 35 cm (*d*) 25 cm

15. ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

(*a*) 2 : 1 (*b*) 1 : 2 (*c*) 1 : 4 (*d*) 4 : 1

16. In ΔABC, if $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then B is

(*a*) 120° (*b*) 60° (*c*) 90° (*d*) 45°

Match the Columns

It is given that $\Delta LNM \sim \Delta YZX$. Match the following columns, which shows the corresponding parts of the two triangles.

Column I	Column II
(*i*) $\frac{XY}{YZ}$	(*a*) $\frac{LM}{MN}$
(*ii*) $\frac{YX}{XZ}$	(*b*) Z
(*iii*) M	(*c*) X
(*iv*) N	(*d*) $\frac{LM}{NL}$

Rapid Fire Quiz

State whether the following statements are true (T) or false (F).

1. All congruent figures need not be similar.
2. A circle of radius 3 cm and a square of side 3 cm are similar figures.
3. Two photographs of the same size of the same person at the age of 20 years and the other at the age of 45 years are not similar.
4. A square and a rectangle are similar figures as each angle of the two quadrilaterals is 90°.
5. If $\Delta ABC \sim \Delta XYZ$, then $\frac{AB}{XY} = \frac{AC}{XZ}$.
6. If $\Delta DEF \sim \Delta QRP$, then $D = Q$ and $E = P$.
7. All similar figures are congruent also.

Fill in the blanks.

8. If a line is drawn parallel to one side of a traingle to intersect the other two sides in distinct points, the other two sides are divided in the ____________ ratio.

9. The ratio of the areas of two similar triangles is equal to the ratio of the ______________ of their corresponding sides.
10. In ________________ triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
11. In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angle opposite to the first is a ___________________ angle.

Word Box

Complete the statements given below by choosing the word from the word box and writing in the spaces provided. Each word may be used once, more than once or not at all.

equiangular	basic proportionality	corresponding sides	parallel
congruent	equal	similar	proportional
Pythagoras	scale factor		

1. Two figures having the same shape and size are said to be _____________ .
2. Two figures are said to be _____________ if they have same shape but not necessarily the same size.
3. All similar figures need not be _____________ .
4. If two polygons are similar, then the same ratio of the corresponding sides is referred to as the _____________ .
5. Two triangles are said to be _____________ if the corresponding angles of two triangles are equal.
6. _____________ theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
7. _____________ theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
8. If a line divides any two sides of a triangle in the same ratio, then the line is _____________ to the third side.
9. The ratio of the areas of two similar triangles is equal to the square of the ratio of their ______________ .
10. All circles are _____________ .
11. All squares with edges of equal length are _____________ .
12. Two polygons of the same number of sides are similar, if their corresponding angles are _____________ and their corresponding sides are _____________ .

Class Worksheet

1. Tick the correct answer for each of the following:

(*i*) P and Q are respectively the points on the sides DE and DF of triangle DEF such that $DE = 6$ cm, $PE = 2.5$ cm, $DQ = 6.3$ cm and $PQ || EF$. Then, length of QF (in cm) is

(*a*) 5 cm (*b*) 12 cm (*c*) 4.5 cm (*d*) 4 cm

(*ii*) If in two triangles DEF and XYZ, $\frac{DF}{YZ} = \frac{ED}{XY} = \frac{EF}{XZ}$, then

(*a*) $\Delta DEF \sim \Delta XYZ$ (*b*) $\Delta DFE \sim \Delta XYZ$ (*c*) $\Delta FED \sim \Delta ZXY$ (*d*) $\Delta EFD \sim \Delta XYZ$

(*iii*) If $\Delta ABC \sim \Delta DEF$ and $\frac{DF}{AC} = \frac{2}{5}$, then $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)}$ is equal to

(*a*) $\frac{5}{2}$ (*b*) $\frac{2}{5}$

(*c*) $\frac{4}{25}$ (*d*) $\frac{25}{4}$

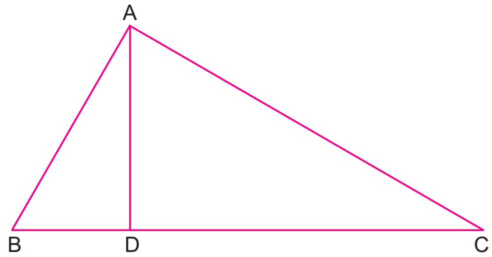


Fig. 4.72

(*iv*) In Fig. 4.72, $BAC = 90°$ and AD BC. Then

(*a*) $BD.CD = BC^2$ (*b*) $AB.AC = BC^2$

(*c*) $BD.CD = AD^2$ (*d*) $AB.AC = AD^2$

2. State whether the following statements are true or false. Justify your answer.

(*i*) A triangle ABC with $AB = 15$ cm, $BC = 20$ cm and $CA = 25$ cm is a right triangle.

(*ii*) Two quadrilaterals are similar, if their corresponding angles are equal.

3. Corresponding sides of two similar triangles are in the ratio 4 : 5. If the area of the smaller triangle is 80 cm^2, find the area of the larger triangle.

4. An aeroplane leaves an Airport and flies due North at 300 km/h. At the same time, another aeroplane leaves the same Airport and flies due West at 400 km/h. How far apart would the two aeroplanes be after $1\frac{1}{2}$ hours?

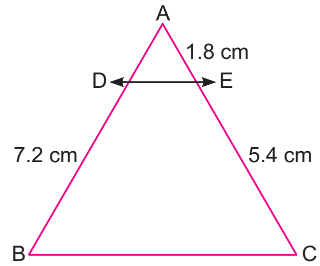


Fig. 4.73

5. (*i*) In Fig. 4.73, if $DE \parallel BC$, find AD.

(*ii*) In Fig. 4.74, is $\Delta ABC \sim \Delta PQR$? If no, why? If yes, name the similarity criterion used.

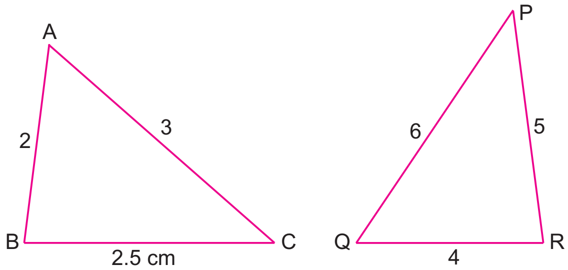


Fig. 4.74

(*iii*) The sides of a triangle are 7 cm, 24 cm, 25 cm. Will it form a right triangle? Why or why not?

6. Fill in the blanks:

(*i*) All equilateral triangles are ________________ . (similar/congruent)

(*ii*) If $\Delta ABC \sim \Delta FED$, then $\frac{AB}{\square} = \frac{\square}{ED} = \frac{AC}{\square}$

(*iii*) Circles with equal radii are ________________. (similar/congruent)

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Tick the correct answer for each of the following:

(*i*) In ΔABC, $AB = 6\sqrt{7}$ cm, $BC = 24$ cm and $CA = 18$ cm. The angle A is

(*a*) an acute angle (*b*) an obtuse angle

(*c*) a right angle (*d*) can't say 1

(*ii*) If in Fig. 4.75, O is the point of intersection of two equal chords AB and CD such that $OB = OD$, then triangles OAC and ODB are

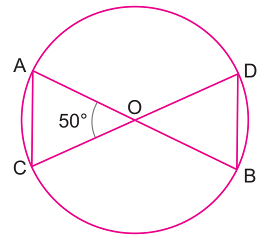


Fig. 4.75

(*a*) equilateral but not similar

(*b*) isosceles but not similar

(*c*) equilateral and similar

(*d*) isosceles and similar 1

(*iii*) It is given that $\Delta PQR \sim \Delta ZXY$, $P = 60°$, $R = 40°$, $PR = 3.6$ cm, $XY = 4$ cm and $YZ = 2.4$ cm. State which of the following is true?

(*a*) $X = 60°$, $PQ = 6$ cm (*b*) $Y = 60°$, $QR = 4$ cm

(*c*) $X = 80°$, $QR = 6$ cm (*d*) $Z = 40°$, $PQ = 4$ cm 1

(*iv*) If $\Delta ABC \sim \Delta DEF$, $\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta ABC)} = \frac{9}{16}$ and $DF = 18$ cm, then AC is equal to

(*a*) 24 cm (*b*) 16 cm (*c*) 8 cm (*d*) 32 cm 2

(*v*) The lengths of the diagonals of a rhombus are 30 cm and 40 cm. The length of the side of the rhombus is

(*a*) 20 cm (*b*) 22 cm (*c*) 25 cm (*d*) 45 cm 2

2. State whether the following statements are true or false. Justify your answer.

(*i*) If $\frac{DE}{PQ} = \frac{EF}{PR}$ and $D = Q$, then $\Delta DEF \sim \Delta PQR$.

(*ii*) P and Q are the points on the sides DE and DF of a triangle DEF such that $DP = 4$ cm, $PE = 14$ cm, $DQ = 6$ cm and $DF = 21$ cm. Then $PQ || EF$. $2 \times 2 = 4$

3. (*i*) In Fig. 4.76, $DE || AC$ and $DF || AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

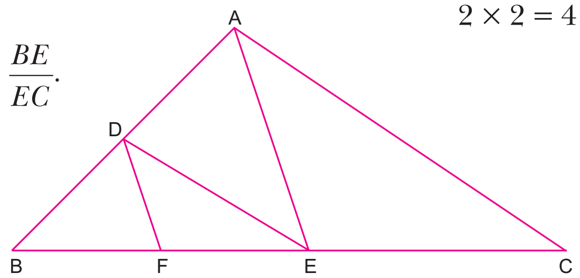


Fig. 4.76

(*ii*) Diagonals of a trapezium $PQRS$ intersect each other at the point O, $PQ || RS$ and $PQ = 3RS$. Find the ratio of the areas of triangles POQ and ROS. $3 \times 2 = 6$

4. (*i*) In Fig. 4.77, if $\Delta ABC \sim \Delta DEF$ and their sides are of lengths (in cm) as marked along them, then find the lengths of the sides of each triangle.

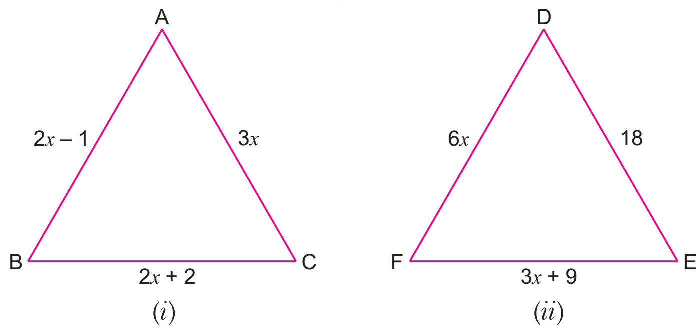


Fig. 4.77

(*ii*) State and prove the converse of Pythagoras Theorem. $4 \times 2 = 8$

Chapter Five

INTRODUCTION TO TRIGONOMETRY

Basic Concepts and Results

- Trigonometry is the branch of Mathematics which deals with the measurement of sides and angles of the triangles.

  Trigonometric Ratios:

  Let ABC be a right triangle, right-angled at B. Let $CAB = \theta$,

  Then,

  $$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}$$

  $$\cot\theta = \frac{AB}{BC} \qquad \sec\theta = \frac{AC}{AB} \qquad \operatorname{cosec}\theta = \frac{AC}{BC}$$

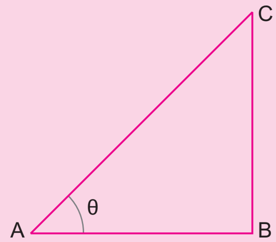


Fig. 5.1

  Relation between trigonometric ratios:

  (*i*) *Reciprocal Relations*

  $$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sin\theta \,.\, \operatorname{cosec}\theta = 1$$

  $$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \sec\theta \,.\, \cos\theta = 1$$

  $$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta \,.\, \cot\theta = 1$$

  (*ii*) *Quotient Relations*

  $$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- An expression having equal to sign (=) is called an **equation.**
- An equation which involves trigonometric ratios of an angle and is true for all values of the angle is called a trigonometric identity.

  Some common trigonometric identities are

  (*i*) $\sin^2\theta + \cos^2\theta = 1$ for $0° \le \theta \le 90°$

  (*ii*) $\sec^2\theta = 1 + \tan^2\theta$ for $0° \le \theta < 90°$

  (*iii*) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ for $0° < \theta \le 90°$

- Trigonometric ratios of complementary angles:

  (*i*) $\sin(90 - \theta) = \cos\theta$ (*ii*) $\cos(90 - \theta) = \sin\theta$

  (*iii*) $\tan(90 - \theta) = \cot\theta$ (*iv*) $\cot(90 - \theta) = \tan\theta$

  (*v*) $\sec(90 - \theta) = \operatorname{cosec}\theta$ (*iv*) $\operatorname{cosec}(90 - \theta) = \sec\theta$

■ Values of Trigonometric Ratios of Standard Angles:

	0°	30°	45°	60°	90°
sin θ	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
sec θ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Note: *There is an easy way to remember the values of* $\sin\theta$ *for* $\theta = 0°, 30°, 45°, 60°$ *and* $90°$.

In brief:

	θ	0°	30°	45°	60°	90°	
sin θ	Write the five numbers in the sequence of 0, 1, 2, 3, 4. Divide by 4 and take their square root.	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	**Increasing order**
cos θ	Write the values of sin θ in reverse order	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	**Decreasing order**
tan θ	Dividing values of sin θ by cos θ *i.e.*, $\tan\theta = \frac{\sin\theta}{\cos\theta}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	**Increasing order**

Note: (*i*) *The values of* sin θ *increases from 0 to 1 as* θ *increases from 0° to 90° and value of* cos θ *decreases from 1 to 0 as* θ *increases from 0 to 90°. The value of* tan θ *also increases from 0 to a bigger number as* θ *increases from 0° to 90°.*

(*ii*) *If* A *and* B *are acute angles such that* $A > B$, *then* $\sin A > \sin B$, $\cos A < \cos B$, $\tan A > \tan B$ *and* $\operatorname{cosec} A < \operatorname{cosec} B$, $\sec A > \sec B$, $\cot A < \cot B$.

Summative Assessment

Multiple Choice Questions

Write correct answer for each of the following:

1. If $\tan A = \frac{3}{2}$, then the value of $\cos A$ is

(*a*) $\frac{3}{\sqrt{13}}$ (*b*) $\frac{2}{\sqrt{13}}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{\sqrt{13}}{2}$

2. If $\sin(\alpha + \beta) = 1$, then $\cos(\alpha - \beta)$ can be reduced to

(*a*) $\cos\beta$ (*b*) $\cos 2\beta$ (*c*) $\sin\beta$ (*d*) $\sin 2\beta$

3. Given that $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cos\beta = \frac{1}{\sqrt{2}}$, then the value of $\tan(\alpha+\beta)$ is

(*a*) 0 (*b*) 1 (*c*) $\sqrt{3}$ (*d*) not defined

4. If ΔABC is right-angled at C, then the value of $\cos(A+B)$ is

(*a*) 0 (*b*) 1 (*c*) 1 (*d*) 0

5. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\sqrt{3}$ (*c*) 1 (*d*) 0

6. The value of the expression $\frac{\sin 60°}{\cos 30°}$ is

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 2

7. The value of the expression cosec $(75°+\theta) - \sec(15°-\theta) - \tan(55°+\theta) + \cot(35°-\theta)$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) $\frac{3}{2}$

8. The value of the expression $\left[\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin^2 27°\right]$ is

(*a*) 3 (*b*) 2 (*c*) 1 (*d*) 0

9. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

10. $\sin 2A = 2\sin A$ is true when A is

(*a*) 0° (*b*) 30° (*c*) 45° (*d*) 60°

11. The value of $\frac{2\tan 30°}{1-\tan^2 30°}$ is equal to

(*a*) $\cos 60°$ (*b*) $\sin 60°$ (*c*) $\tan 60°$ (*d*) $\sin 30°$

12. $9\sec^2 A - 9\tan^2 A$ is equal to

(*a*) 1 (*b*) 9 (*c*) 8 (*d*) 0

13. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\theta)$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) –1

14. If $\sec\theta + \tan\theta = x$, then $\tan\theta$ is equal to

(*a*) $\frac{x^2+1}{x}$ (*b*) $\frac{x^2+1}{2x}$ (*c*) $\frac{x^2-1}{2x}$ (*d*) $\frac{x^2-1}{x}$

15. $\cos^4 A - \sin^4 A$ is equal to

(*a*) $2\cos^2 A + 1$ (*b*) $2\cos^2 A - 1$ (*c*) $2\sin^2 A - 1$ (*d*) $2\sin^2 A + 1$

Short Answer Questions Type–I

Write true or false and justify your answer (1 – 4):

1. The value of the expression $(\cos 80° - \sin 80°)$ is negative.

Sol. True, for $\theta > 45°$, $\sin\theta > \cos\theta$, so $\cos 80° - \sin 80°$ has a negative value.

2. $(\tan\theta+2)(2\tan\theta+1) = 5\tan\theta+\sec^2\theta.$

Sol. False, $(\tan\theta+2)(2\tan\theta+1) = 2\tan^2\theta+5\tan\theta+2 = 5\tan\theta+2(1+\tan^2\theta)$

$$= 5\tan\theta+2\sec^2\theta.$$

3. If $\sin A+\sin^2 A=1$, then $\cos^2 A+\cos^4 A=1$.

Sol. True,

$$\sin A+\sin^2 A=1 \qquad \sin A = 1-\sin^2 A=\cos^2 A$$

$$\cos^2 A+\cos^4 A = \sin A+\sin^2 A=1.$$

4. $\frac{\tan 47°}{\cot 73°} = 1.$

Sol. True,

$$\frac{\tan 47°}{\cot 73} = \frac{\tan(90°-43°)}{\cot 43°} = \frac{\cot 43°}{\cot 43°} = 1.$$

5. If $\sec A = 2x$ and $\tan A = \frac{2}{x}$, find the value of $2\left(x^2-\frac{1}{x^2}\right)$

Sol. $2\left(x^2-\frac{1}{x^2}\right) = 2\left(\frac{\sec^2 A}{4}-\frac{\tan^2 A}{4}\right) = \frac{2}{4}(\sec^2 A-\tan^2 A) = \frac{1}{2}\times 1 = \frac{1}{2}.$

6. Write the value of $\cot^2\theta-\frac{1}{\sin^2\theta}$.

Sol. $\cot^2\theta-\frac{1}{\sin^2\theta} = \cot^2\theta-\operatorname{cosec}^2\theta = 1.$

7. If $\sin\theta=\frac{1}{3}$, then find the value of $2\cot^2\theta+2$.

Sol. $2\cot^2\theta+2 = 2(\cot^2\theta+1) = 2\operatorname{cosec}^2\theta$

$$= \frac{2}{\sin^2\theta} = \frac{2}{\left(\frac{1}{3}\right)^2} = 2\times 9 = 18.$$

8. If $\sec^2\theta(1+\sin\theta)(1-\sin\theta)=k$, then find the value of k.

Sol. $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = \sec^2\theta(1-\sin^2\theta)$ $[(a+b)(a-b)=a^2-b^2]$

$= \sec^2\theta.\cos^2\theta$ $[\because \cos^2\theta+\sin^2\theta=1]$

$= 1$

$k=1.$

9. Write the acute angle θ satisfying $\sqrt{3}\sin\theta=\cos\theta$.

Sol. $\sqrt{3}\sin\theta=\cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \qquad \tan\theta = \frac{1}{\sqrt{3}} \qquad \theta = 30°.$$

10. If $A+B=90°$ and $\tan A=\frac{3}{4}$, what is $\cot B$?

Sol. $\cot B = \cot(90°-A)$ $(\because A+B=90°)$

$= \tan A$ $(\because \cot(90°-\theta)=\tan\theta)$

$= \frac{3}{4}.$

Important Problems

Type A: Problems Based on Trigonometric Ratios

1. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$. **[NCERT]**

Sol. Let us first draw a right ΔABC in which $C = 90°$.

Now, we know that

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{3}{4}$$

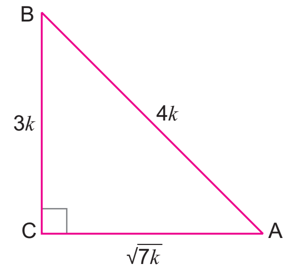


Fig. 5.2

Let $BC = 3k$ and $AB = 4k$, where k is a positive number.

Then, by Pythagoras Theorem, we have

$$AB^2 = BC^2 + AC^2$$

$$(4k)^2 = (3k)^2 + AC^2$$

$$16k^2 - 9k^2 = AC^2 \qquad 7k^2 = AC^2$$

$$AC = \sqrt{7}k$$

$$\cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

and $\quad \tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}.$

2. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$. **[NCERT]**

Sol. Let us first draw a right ΔABC, in which $B = 90°$.

Now, we have, $15 \cot A = 8$

$$\cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{\text{Base}}{\text{Perpendicular}}$$

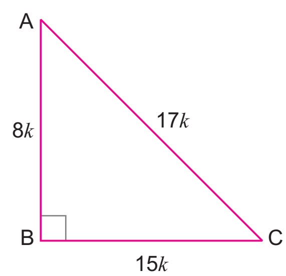


Fig. 5.3

Let $AB = 8k$ and $BC = 15k$

Then, $AC = \sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras theorem)

$$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

and, $\quad \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}.$

3. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$. **[NCERT]**

Sol. We have a right-angled ΔPQR in which $Q = 90°$.

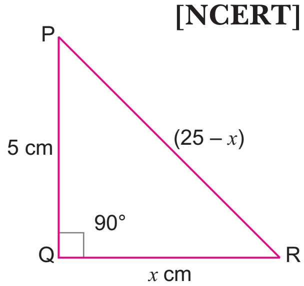


Fig. 5.4

Let $QR = x$ cm

Therefore, $PR = (25 - x)$ cm

By Pythagoras Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$(25 - x)^2 = 5^2 + x^2$$

$$(25-x)^2 - x^2 = 5^2$$

$$(25-x-x)(25-x+x) = 25$$

$$(25-2x)\,25 = 25 \qquad 25-2x = 1$$

$$25-1 = 2x \qquad 24 = 2x$$

$$x = 12 \text{ cm.}$$

Hence, $QR = 12$ cm

$PR = (25-x)$ cm $= 25 - 12 = 13$ cm

$PQ = 5$ cm

$$\sin P = \frac{QR}{PR} = \frac{12}{13}; \quad \cos P = \frac{PQ}{PR} = \frac{5}{13}; \quad \tan P = \frac{QR}{PQ} = \frac{12}{5}$$

4. In Fig. 5.5, find $\tan P - \cot R$. **[NCERT]**

Sol. Using Pythagoras Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$(13)^2 = (12)^2 + QR^2$$

$$169 = 144 + QR^2$$

$$QR^2 = 169 - 144 = 25 \qquad QR = 5 \text{ cm}$$

Now, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$

$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0.$$

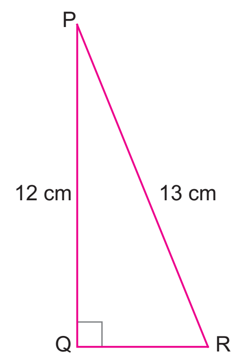


Fig. 5.5

5. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$ (ii) $\cos A \cos C - \sin A \sin C$. **[NCERT]**

Sol. We have a right-angled ΔABC in which $B = 90°$.

and, $\tan A = \frac{1}{\sqrt{3}}$

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $BC = k$ and $AB = \sqrt{3}k$

By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$$

$$AC^2 = 4k^2 \qquad AC = 2k$$

Now, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

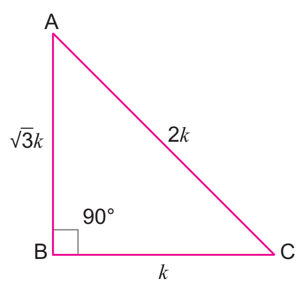


Fig. 5.6

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$$

(*i*) $\sin A . \cos C + \cos A . \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$

(*ii*) $\cos A . \cos C - \sin A . \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$

6. If $\cot \theta = \frac{7}{8}$, evaluate: (*i*) $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$, (*ii*) $\cot^2 \theta$.

Sol. Let us draw a right triangle ABC in which $\angle B = 90°$ and $\angle C = \theta$.

We have

$$\cot \theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} \qquad \text{(given)}$$

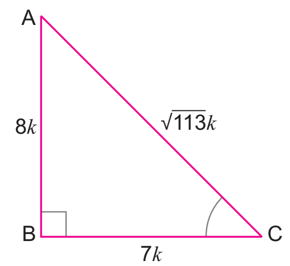


Fig. 5.7

Let $BC = 7k$ and $AB = 8k$

Therefore, by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$AC^2 = 113k^2 \qquad AC = \sqrt{113}k$$

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

and $$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(*i*) $$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64}.$$

Alternate method:

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

(*ii*) $\cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$

7. If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not. **[NCERT]**

Sol. Let us consider a right triangle ABC in which $\angle B = 90°$.

Now, $$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$$

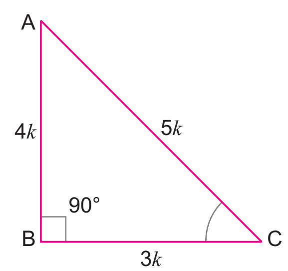


Fig. 5.8

Let $AB = 4k$ and $BC = 3k$

By Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$$

$$AC^2 = 25k^2 \qquad AC = 5k$$

Therefore, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

and, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, L.H.S. $= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{16 - 9}{16 + 9} = \frac{7}{25}$

R.H.S. $= \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Hence, $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$.

8. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$. **[NCERT]**

Sol. Let us consider a right-angled ΔABC in which $B = 90°$.

For A, we have

Base $= AB$

Perpendicular $= BC$

and Hypotenuse $= AC$

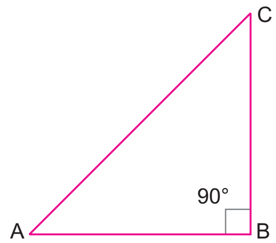


Fig. 5.9

$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$

$\frac{\cot A}{1} = \frac{AB}{BC} \qquad AB = BC \cot A$

Let $BC = k$

$AB = k \cot A$

Then by Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = k^2 \cot^2 A + k^2$$

$$AC = \sqrt{k^2 (1 + \cot^2 A)} = k\sqrt{1 + \cot^2 A}$$

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{k\sqrt{1 + \cot^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}}$

$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{k\sqrt{1 + \cot^2 A}}{k \cot A} = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$

and $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{k}{k \cot A} = \frac{1}{\cot A}$.

9. Write all the other trigonometric ratios of $\angle A$ in terms of sec A. **[NCERT]**

Sol. Let us consider a right-angled ΔABC, in which $\angle B = 90°$.

For $\angle A$, we have

Base = AB, Perpendicular = BC

and Hypotenuse = AC

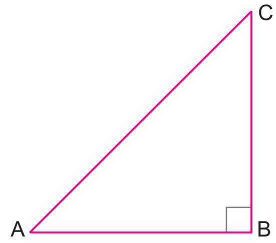


Fig. 5.10

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\frac{\sec A}{1} = \frac{AC}{AB} \qquad \Rightarrow \qquad AC = AB \sec A$$

Let $AB = k$

$AC = k \sec A$

By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$k^2 \sec^2 A = k^2 + BC^2$$

$$BC^2 = k^2 \sec^2 A - k^2 \qquad \Rightarrow \qquad BC = k\sqrt{\sec^2 A - 1}$$

$$\sin A = \frac{BC}{AC} = \frac{k\sqrt{\sec^2 A - 1}}{k \sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k \sec A} = \frac{1}{\sec A}$$

$$\tan A = \frac{BC}{AB} = \frac{k\sqrt{\sec^2 A - 1}}{k} = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\text{cosec } A = \frac{AC}{BC} = \frac{k \sec A}{k\sqrt{\sec^2 A - 1}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}.$$

Type B: Problems Based on Trigonometric Ratios of Standard Angles

1. Evaluate the following:

(*i*) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$ (*ii*) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$ **[NCERT]**

Sol. (*i*) $$\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4} \qquad \text{(on rationalising)}$$

$$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2} = \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}.$$

(*ii*) $$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°} = \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}} = \frac{15 + 64 - 12}{12} = \frac{67}{12}.$$

2. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0° < A + B \le 90°$; $A > B$, find A and B. **[NCERT]**

Sol. We have, $\tan(A+B) = \sqrt{3}$

$\tan(A+B) = \tan 60°$

$A + B = 60°$ ...(*i*)

Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A-B) = \tan 30°$

$A - B = 30°$ ...(*ii*)

Adding (*i*) and (*ii*), we have

$2A = 90°$ $A = 45°$

Putting the value of A in (*i*), we have

$45° + B = 60°$

$B = 60° - 45° = 15°$

Hence, $A = 45°$ and $B = 15°$.

Type C: Problems Based on Trigonometric Ratios of Complementary Angles

1. Express $\sin 67° + \cos 75°$ in terms of trigonometric ratios of angles between 0° and 45°. **[NCERT]**

Sol. $\sin 67° + \cos 75°$

$= \sin(90° - 23°) + \cos(90° - 15°) = \cos 23° + \sin 15°$

2. Evaluate:

(*i*) $\frac{\sin 18°}{\cos 72°}$ (*ii*) $\frac{\tan 26°}{\cot 64°}$

(*iii*) $\cos 48° - \sin 42°$ (*iv*) $\operatorname{cosec} 31° - \sec 59°$ **[NCERT]**

Sol. (*i*) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1$

(*ii*) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

(*iii*) $\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42° = \sin 42° - \sin 42° = 0$

(*iv*) $\operatorname{cosec} 31° - \sec 59° = \operatorname{cosec}(90° - 59°) - \sec 59° = \sec 59° - \sec 59° = 0.$

3. Evaluate:

(*i*) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$ (*ii*) $\sin 25° \cos 65° + \cos 25° \sin 65°$. **[NCERT]**

Sol. (*i*) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \dfrac{\sin^2 (90° - 27°) + \sin^2 27°}{\cos^2 (90° - 73°) + \cos^2 73°}$

$$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°} = \frac{1}{1} = 1$$

(*ii*) $\sin 25° . \cos 65° + \cos 25° . \sin 65°$

$$= \sin (90° - 65°) . \cos 65° + \cos (90° - 65°) . \sin 65°$$
$$= \cos 65° . \cos 65° + \sin 65° . \sin 65°$$
$$= \cos^2 65° + \sin^2 65° = 1.$$

4. If $\tan A = \cot B$, prove that $A + B = 90°$. **[NCERT]**

Sol. We have

$$\tan A = \cot B$$
$$\tan A = \tan (90° - B)$$
$$A = 90° - B \qquad [\because \text{ both } A \text{ and } B \text{ are acute angles}]$$
$$A + B = 90°.$$

5. If $\sec 4A = \text{cosec}\,(A - 20°)$, where $4A$ is an acute angle, find the value of A. **[NCERT]**

Sol. We have

$$\sec 4A = \text{cosec}\,(A - 20°)$$
$$\text{cosec}\,(90° - 4A) = \text{cosec}\,(A - 20°)$$
$$90° - 4A = A - 20°$$
$$90° + 20° = A + 4A$$
$$110 = 5A$$
$$A = \frac{110}{5} = 22°.$$

6. If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}.$$ **[NCERT]**

Sol. Since A, B and C are the interior angles of a ΔABC,

Therefore, $A + B + C = 180°$

$$\frac{A + B + C}{2} = \frac{180°}{2}$$

$$\frac{A}{2} + \frac{(B + C)}{2} = 90°$$

$$\frac{B + C}{2} = 90° - \frac{A}{2}$$

Now, taking sin on both sides, we have

$$\sin\left(\frac{B + C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$\sin\frac{B + C}{2} = \cos\frac{A}{2}.$$

7. Without using tables, evaluate the following:

$$3\cos 68^\circ \,.\, \text{cosec}\, 22^\circ - \frac{1}{2}\tan 43^\circ \,.\, \tan 47^\circ \,.\, \tan 12^\circ \,.\, \tan 60^\circ \,.\, \tan 78^\circ.$$

Sol. We have, $3\cos 68^\circ \,.\, \text{cosec}\, 22^\circ - \frac{1}{2}\tan 43^\circ \,.\, \tan 47^\circ \,.\, \tan 12^\circ \,.\, \tan 60^\circ \,.\, \tan 78^\circ$

$$= 3\cos(90^\circ - 22^\circ) \,.\, \text{cosec}\, 22^\circ - \frac{1}{2}\{\tan 43^\circ \,.\, \tan(90^\circ - 43^\circ)\}$$

$$.\,\{\tan 12^\circ \,.\, \tan(90^\circ - 12^\circ) \,.\, \tan 60^\circ\}$$

$$= 3\sin 22^\circ \,.\text{cosec}\, 22^\circ - \frac{1}{2}(\tan 43^\circ \,.\, \cot 43^\circ)\,.\,(\tan 12^\circ \,.\, \cot 12^\circ)\,.\,\tan 60^\circ$$

$$= 3 \times 1 - \frac{1}{2} \times 1 \times 1 \times \sqrt{3} = 3 - \frac{\sqrt{3}}{2} = \frac{6 - \sqrt{3}}{2}.$$

8. Without using trigonometric tables, evaluate the following:

$$\frac{\cot(90^\circ - \theta)\,.\,\sin(90^\circ - \theta)}{\sin\theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ)$$

Sol. We have $\dfrac{\cot(90^\circ - \theta)\,.\,\sin(90^\circ - \theta)}{\sin\theta} + \dfrac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ)$

$$= \frac{\tan\theta \,.\, \cos\theta}{\sin\theta} + \frac{\cot 40^\circ}{\tan(90^\circ - 40^\circ)} - \{\cos^2 20^\circ + \cos^2(90^\circ - 20^\circ)\}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}\,.\,\cos\theta}{\sin\theta} + \frac{\cot 40^\circ}{\cot 40^\circ} - \{\cos^2 20^\circ + \sin^2 20^\circ\} = 1 + 1 - 1 = 1.$$

9. Without using tables, evaluate the following:

$$\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\text{cosec}^2\, 57^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \,.\, \sec^2 52^\circ - \sin^2 45^\circ.$$

Sol. We have, $\dfrac{\sec^2 54^\circ - \cot^2 36^\circ}{\text{cosec}^2\, 57^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \,.\, \sec^2 52^\circ - \sin^2 45^\circ$

$$= \frac{\sec^2(90^\circ - 36^\circ) - \cot^2 36^\circ}{\text{cosec}^2(90^\circ - 33^\circ) - \tan^2 33^\circ} + 2\sin^2 38^\circ \,.\, \sec^2(90^\circ - 38^\circ) - \sin^2 45^\circ$$

$$= \frac{\text{cosec}^2\, 36^\circ - \cot^2 36^\circ}{\sec^2 33^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \,.\, \text{cosec}^2\, 38 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{1} + 2\,.\,1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}$$

10. Without using trigonometric tables, prove that:

$$\frac{\sec^2\theta - \cot^2(90^\circ - \theta)}{\text{cosec}^2\, 67^\circ - \tan^2 23^\circ} + (\sin^2 40^\circ + \sin^2 50^\circ) = 2.$$

Sol. We have,

$$\text{LHS} = \frac{\sec^2\theta - \cot^2(90^\circ - \theta)}{\text{cosec}^2\, 67^\circ - \tan^2 23^\circ} + (\sin^2 40^\circ + \sin^2 50^\circ)$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\operatorname{cosec}^2(90° - 23°) - \tan^2 23°} + \{\sin^2 40° + \sin^2(90° - 40°)\}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec^2 23° - \tan^2 23°} + (\sin^2 40° + \cos^2 40°)$$

$$= \frac{1}{1} + 1 = 2 = \text{RHS}.$$

11. Without using tables, evaluate the following:

$$\cos(40° + \theta) - \sin(50° - \theta) + \frac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}.$$

Sol. We have, $\cos(40° + \theta) - \sin(50° - \theta) + \dfrac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$

$$= \cos(40° + \theta) - \sin\{90° - (40° + \theta)\} + \frac{\cos^2 40° + \cos^2(90° - 40°)}{\sin^2 40° + \sin^2(90° - 40°)}$$

$$= \cos(40° + \theta) - \cos(40° + \theta) + \frac{\cos^2 40° + \sin^2 40°}{\sin^2 40° + \cos^2 40°} = \frac{1}{1} = 1.$$

12. Without using tables, evaluate:

$$\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0° + \tan 15° \,.\, \tan 25° \,.\, \tan 60° \,.\, \tan 65° \,.\, \tan 75°.$$

Sol. We have,

$$\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0° + \tan 15° \,.\, \tan 25° \,.\, \tan 60° \,.\, \tan 65° \,.\, \tan 75°$$

$$= 2\frac{\cos(90° - 23°)}{\sin 23°} - \frac{\tan 40°}{\cot(90° - 40°)} - \cos 0° + \tan 15°.\tan 25°.\tan 60°.\tan(90 - 25°).\tan(90 - 15°)$$

$$= 2\frac{\sin 23°}{\sin 23°} - \frac{\tan 40°}{\tan 40°} - \cos 0° + \tan 15° \,.\, \tan 25° \,.\tan 60°.\cot 25° \,.\, \cot 15°$$

$$= 2 - 1 - 1 + (\tan 15° \,.\, \cot 15°) \,.\, \tan 60° \,.\, (\tan 25° \,.\, \cot 25°)$$

$$= 1 \,.\, 1 \,.\, \sqrt{3} = \sqrt{3}.$$

13. Evaluate: $\dfrac{\sec\theta \,.\, \operatorname{cosec}(90° - \theta) - \tan\theta.\cot(90° - \theta) + \sin^2 55° + \sin^2 35°}{\tan 10°.\tan 20° \,.\, \tan 60° \,.\, \tan 70° \,.\, \tan 80°}.$

Sol. We have, $\dfrac{\sec\theta \,.\, \operatorname{cosec}(90° - \theta) - \tan\theta.\cot(90° - \theta) + \sin^2 55° + \sin^2 35°}{\tan 10°.\tan 20° \,.\, \tan 60° \,.\, \tan 70° \,.\, \tan 80°}$

$$= \frac{\sec\theta \,.\, \sec\theta - \tan\theta.\tan\theta + \sin^2 55° + \sin^2(90° - 55°)}{\tan 10°.\tan 20° \,.\, \tan 60° \,.\, \tan(90° - 20°) \,.\, \tan(90° - 10°)}$$

$$= \frac{\sec^2\theta - \tan^2\theta + \sin^2 55° + \cos^2 55°}{\tan 10°.\tan 20° \,.\, \tan 60° \,.\, \cot 20°.\cot 10°}$$

$$= \frac{(\sec^2\theta - \tan^2\theta) + (\sin^2 55° + \cos^2 55°)}{(\tan 10°.\cot 10°) \,.\, (\tan 20° \,.\, \cot 20°).\tan 60°}$$

$$= \frac{1 + 1}{(1) \,.\, (1) \,.\, \sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

14. Without using tables, evaluate the following:

$$\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5}.$$

Sol. We have $\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5}$

$$= \frac{2\sin (90^\circ - 22^\circ)}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan (90^\circ - 15^\circ)}$$

$$- \frac{3\tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan (90^\circ - 40^\circ) \cdot \tan (90^\circ - 20^\circ)}{5}$$

$$= \frac{2\cos 22^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\cot 15^\circ} - \frac{3 \cdot \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \cot 40^\circ \cdot \cot 20^\circ}{5}$$

$$= 2 - \frac{2}{5} - \frac{3\tan 45^\circ \cdot (\tan 20^\circ \cdot \cot 20^\circ) \cdot (\tan 40^\circ \cdot \cot 40^\circ)}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} \cdot 1 \cdot 1 \cdot 1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1.$$

Without using tables, evaluate:

15. $\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin (90^\circ - \theta) \cdot \sin \theta}{\tan \theta} + \frac{\cos (90^\circ - \theta) \cdot \cos \theta}{\cot \theta}\right]$

Sol. We have $\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin (90^\circ - \theta) \cdot \sin \theta}{\tan \theta} + \frac{\cos (90^\circ - \theta) \cdot \cos \theta}{\cot \theta}\right]$

$$= \frac{\sin^2 20^\circ + \sin^2 (90^\circ - 20^\circ)}{\cos^2 20^\circ + \cos^2 (90^\circ - 20^\circ)} + \left[\frac{\cos \theta \cdot \sin \theta}{\tan \theta} + \frac{\cos \theta \cdot \sin \theta}{\cot \theta}\right]$$

$$= \frac{\sin^2 20^\circ + \cos^2 20^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} + \left[\frac{\cos \theta \cdot \sin \theta}{\frac{\sin \theta}{\cos \theta}} + \frac{\cos \theta \cdot \sin \theta}{\frac{\cos \theta}{\sin \theta}}\right]$$

$$= \frac{1}{1} + [\cos^2 \theta + \sin^2 \theta]$$

$$= 1 + 1 = 2.$$

16. $\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \cdot \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \cdot \tan 25^\circ \cdot \tan 45^\circ \cdot \tan 65^\circ \cdot \tan 85^\circ)}.$

Sol. $\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \cdot \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \cdot \tan 25^\circ \cdot \tan 45^\circ \cdot \tan 65^\circ \cdot \tan 85^\circ)}$

$$= \frac{3\cos (90^\circ - 35^\circ)}{7\sin 35^\circ} - \frac{4\cos (90^\circ - 20^\circ) \cdot \operatorname{cosec} 20^\circ}{7(\tan (90^\circ - 85^\circ) \cdot \tan (90^\circ - 65^\circ) \cdot 1 \cdot \tan 65^\circ \cdot \tan 85^\circ)}$$

$$= \frac{3\sin 35^\circ}{7\sin 35^\circ} - \frac{4\sin 20^\circ \cdot \operatorname{cosec} 20^\circ}{7\cot 85^\circ \cdot \cot 65^\circ \cdot \tan 65^\circ \cdot \tan 85^\circ}$$

$$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}.$$

Type D: Problems Based on Trigonometric Identities

1. Prove that: $(\text{cosec}\ \theta - \cot\ \theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$. **[NCERT]**

Sol. LHS $= (\text{cosec}\ \theta - \cot\ \theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}.$$

2. Prove that: $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$. **[NCERT]**

Sol. LHS $= \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1+\sin A)\cos A} = \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{RHS}.$$

3. Prove that: $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$. **[NCERT]**

Sol. LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)} = \dfrac{\sin\theta}{\cos\theta}\left[\dfrac{1-2\sin^2\theta}{2(1-\sin^2\theta)-1}\right]$

$$= \tan\theta\left(\frac{1-2\sin^2\theta}{2-2\sin^2\theta-1}\right) = \tan\theta\left(\frac{1-2\sin^2\theta}{1-2\sin^2\theta}\right)$$

$= \tan\theta = \text{RHS}.$

4. Prove that: $(\sin A + \text{cosec}\ A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. **[NCERT]**

Sol. LHS $= (\sin A + \text{cosec}\ A)^2 + (\cos A + \sec A)^2$

$$= \sin^2 A + \text{cosec}^2 A + 2\sin A\,.\,\text{cosec}\ A + \cos^2 A + \sec^2 A + 2\cos A\,.\,\sec A$$

$$= (\sin^2 A + \text{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2) \qquad \begin{bmatrix}\sin A\,.\,\text{cosec}\ A = 1\\ \cos A\,.\,\sec A = 1\end{bmatrix}$$

$$= (\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A) + 4$$

$$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 4$$

$$= 7 + \tan^2 A + \cot^2 A = \text{RHS}.$$

5. Prove that: $(\text{cosec } A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$. **[NCERT]**

Sol. LHS

$$= (\text{cosec } A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right) \times \left(\frac{1 - \cos^2 A}{\cos A}\right) = \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \sin A \,.\, \cos A = \frac{\sin A \,.\, \cos A}{\sin^2 A + \cos^2 A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\dfrac{\sin A \,.\, \cos A}{\sin A \,.\, \cos A}}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \,.\, \cos A}} \qquad \text{[divide numerator and denominator by } \sin A \,.\, \cos A]$$

$$= \frac{1}{\tan A + \cot A} = \text{RHS.}$$

6. Prove that: $\left(\dfrac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$. **[NCERT]**

Sol. LHS

$$= \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \frac{\sec^2 A}{\text{cosec}^2 A}$$

$$= \frac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

RHS

$$= \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \tan A}{1 - \dfrac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1 - \tan A}{\dfrac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1 - \tan A}{\tan A - 1} \times \tan A\right)^2$$

$$= (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

7. Prove that: $\tan^2 A - \tan^2 B = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$.

Sol. LHS

$$= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B}$$

$$= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A\,(1-\cos^2 B)}{\cos^2 A \cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$$

Also $\frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A \cos^2 B} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$ = RHS.

8. Prove that: $\frac{\text{cosec } A}{\text{cosec } A - 1} + \frac{\text{cosec } A}{\text{cosec } A + 1} = 2 + 2\tan^2 A = 2\sec^2 A.$

Sol. LHS $= \frac{\text{cosec } A}{(\text{cosec } A - 1)} + \frac{\text{cosec } A}{(\text{cosec } A + 1)}$

$$= \frac{\text{cosec } A\,(\text{cosec } A + 1) + \text{cosec } A\,(\text{cosec } A - 1)}{(\text{cosec } A - 1)(\text{cosec } A + 1)}$$

$$= \frac{\text{cosec } A\,(\text{cosec } A + 1 + \text{cosec } A - 1)}{(\text{cosec}^2 A - 1)} = \frac{2\,\text{cosec}^2 A}{1 + \cot^2 A - 1} = \frac{2\,\text{cosec}^2 A}{\cot^2 A}$$

$$= 2\,\text{cosec}^2 A \tan^2 A = 2(1+\cot^2 A).\tan^2 A$$

$$= 2\tan^2 A + 2\tan^2 A.\cot^2 A \qquad (\because \tan A \cot A = 1)$$

$$= 2 + 2\tan^2 A = 2(1+\tan^2 A) = 2\sec^2 A = \text{RHS}.$$

9. Prove that: $\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta.$

Sol. LHS $= \frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$$= \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta - \sin\theta} = \frac{\cos\theta \times \cos\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta = \text{RHS}.$$

10. Prove that: $\frac{1+\cos\theta - \sin^2\theta}{\sin\theta\,(1+\cos\theta)} = \cot\theta.$

Sol. LHS $= \frac{1+\cos\theta - \sin^2\theta}{\sin\theta\,(1+\cos\theta)}$

To obtain $\cot\theta$ in RHS, we have to convert the numerator of LHS in cosine function and denominator in sine function.

Therefore converting $\sin^2\theta = 1 - \cos^2\theta$, we get

LHS $= \frac{1+\cos\theta - (1-\cos^2\theta)}{\sin\theta\,(1+\cos\theta)}$

$$= \frac{1+\cos\theta - 1 + \cos^2\theta}{\sin\theta\,(1+\cos\theta)} = \frac{\cos\theta + \cos^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\cos\theta\,(\cos\theta + 1)}{\sin\theta\,(1+\cos\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{RHS}$$

11. Prove that: $\dfrac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}=(\operatorname{cosec}\theta+\cot\theta)^2=1+2\cot^2\theta+2\operatorname{cosec}\theta\cot\theta.$

Sol. LHS $=\dfrac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}$

Rationalising the denominator, we get

$$=\frac{(\operatorname{cosec}\theta+\cot\theta)}{(\operatorname{cosec}\theta-\cot\theta)}\times\frac{(\operatorname{cosec}\theta+\cot\theta)}{(\operatorname{cosec}\theta+\cot\theta)}$$

$$=\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}=\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{1} \qquad [\because \operatorname{cosec}^2\theta-\cot^2\theta=1]$$

$$=\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta.\cot\theta$$

$$=(1+\cot^2\theta)+\cot^2\theta+2\operatorname{cosec}\theta.\cot\theta$$

$$=1+2\cot^2\theta+2\operatorname{cosec}\theta.\cot\theta=\text{RHS}.$$

12. Prove that: $2\sec^2\theta-\sec^4\theta-2\operatorname{cosec}^2\theta+\operatorname{cosec}^4\theta=\cot^4\theta-\tan^4\theta.$

Sol. LHS $=2\sec^2\theta-\sec^4\theta-2\operatorname{cosec}^2\theta+\operatorname{cosec}^4\theta$

$$=2(\sec^2\theta)-(\sec^2\theta)^2-2(\operatorname{cosec}^2\theta)+(\operatorname{cosec}^2\theta)^2$$

$$=2(1+\tan^2\theta)-(1+\tan^2\theta)^2-2(1+\cot^2\theta)+(1+\cot^2\theta)^2$$

$$=2+2\tan^2\theta-(1+2\tan^2\theta+\tan^4\theta)-2-2\cot^2\theta+(1+2\cot^2\theta+\cot^4\theta)$$

$$=2+2\tan^2\theta-1-2\tan^2\theta-\tan^4\theta-2-2\cot^2\theta+1+2\cot^2\theta+\cot^4\theta$$

$$=\cot^4\theta-\tan^4\theta=\text{RHS}.$$

13. Prove that: $(1-\sin A+\cos A)^2=2(1-\sin A)(1+\cos A).$

Sol. LHS $=(1-\sin A+\cos A)^2$

$$=1+\sin^2 A+\cos^2 A-2\sin A+2\cos A-2\sin A\cos A$$

$$=1+1-2\sin A+2\cos A-2\sin A\cos A \qquad [\because \sin^2 A+\cos^2 A=1]$$

$$=2(1-\sin A+\cos A-\sin A\cos A)=2[(1-\sin A)+\cos A(1-\sin A)]$$

$$=2(1-\sin A)(1+\cos A)=\text{RHS}.$$

14. Prove that: $\cot\theta-\tan\theta=\dfrac{2\cos^2\theta-1}{\sin\theta\cos\theta}.$

Sol. LHS $=\cot\theta-\tan\theta=\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}=\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}=\dfrac{\cos^2\theta-(1-\cos^2\theta)}{\sin\theta\cos\theta}$

$$=\frac{\cos^2\theta-1+\cos^2\theta}{\sin\theta\cos\theta}=\frac{2\cos^2\theta-1}{\sin\theta\cos\theta}=\text{RHS}.$$

15. Prove that: $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2=(1+\sec\theta\operatorname{cosec}\theta)^2.$

Sol. LHS $=(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2$

$$=\left(\sin\theta+\frac{1}{\cos\theta}\right)^2+\left(\cos\theta+\frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta}\right)^2 + \left(\frac{\cos\theta\sin\theta + 1}{\sin\theta}\right)^2$$

$$= \frac{(\sin\theta\cos\theta + 1)^2}{\cos^2\theta} + \frac{(\cos\theta\sin\theta + 1)^2}{\sin^2\theta} = (\sin\theta\cos\theta + 1)^2\left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right)$$

$$= (\sin\theta\cos\theta + 1)^2\left(\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}\right) = (\sin\theta\cos\theta + 1)^2\left(\frac{1}{\cos^2\theta\sin^2\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta\sin\theta}\right)^2 = \left(1 + \frac{1}{\cos\theta\sin\theta}\right)^2 = (1 + \sec\theta\,\mathrm{cosec}\,\theta)^2 = \text{RHS.}$$

16. Prove that: $\frac{1}{(\mathrm{cosec}\,x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\mathrm{cosec}\,x - \cot x)}$.

Sol. In order to show that,

$$\frac{1}{(\mathrm{cosec}\,x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\mathrm{cosec}\,x - \cot x)}$$

It is sufficient to show

$$\frac{1}{\mathrm{cosec}\,x + \cot x} + \frac{1}{(\mathrm{cosec}\,x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\frac{1}{(\mathrm{cosec}\,x + \cot x)} + \frac{1}{(\mathrm{cosec}\,x - \cot x)} = \frac{2}{\sin x} \qquad ...(i)$$

Now, LHS of above is

$$\frac{1}{(\mathrm{cosec}\,x + \cot x)} + \frac{1}{(\mathrm{cosec}\,x - \cot x)} = \frac{(\mathrm{cosec}\,x - \cot x) + (\mathrm{cosec}\,x + \cot x)}{(\mathrm{cosec}\,x - \cot x)(\mathrm{cosec}\,x + \cot x)}$$

$$= \frac{2\,\mathrm{cosec}\,x}{\mathrm{cosec}^2 x - \cot^2 x} \qquad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{2\,\mathrm{cosec}\,x}{1} = \frac{2}{\sin x} = \text{RHS of } (i)$$

Hence, $\frac{1}{(\mathrm{cosec}\,x + \cot x)} + \frac{1}{(\mathrm{cosec}\,x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x}$

or $\frac{1}{(\mathrm{cosec}\,x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\mathrm{cosec}\,x - \cot x)}$.

17. Prove that: $\frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1} = \mathrm{cosec}\,A + \sec A$

Sol. LHS $= \frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1}$

$$= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A - 1)(\cos A + \sin A + 1)}$$

$$= \frac{2(\cos A + \sin A)}{(\cos A + \sin A)^2 - 1} = \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2\cos A\sin A - 1}$$

$$= \frac{\cos A + \sin A}{\cos A\sin A} = \frac{\cos A}{\cos A\sin A} + \frac{\sin A}{\cos A\sin A} = \frac{1}{\sin A} + \frac{1}{\cos A}$$

$$= \mathrm{cosec}\,A + \sec A = \text{RHS.}$$

HOTS (Higher Order Thinking Skills)

1. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2) = 4\sqrt{mn}$.

Sol. We have given $\tan\theta + \sin\theta = m$, and $\tan\theta - \sin\theta = n$, then

LHS $= (m^2 - n^2) = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta = 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta - \sin\theta)(\tan\theta + \sin\theta)} = 4\sqrt{mn} =$ RHS

2. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, prove that $l^2m^2(l^2+m^2+3)=1$.

Sol. LHS, $= l^2m^2(l^2 + m^2 + 3)$

$= (\operatorname{cosec}\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2\{(\operatorname{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$

$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$

$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$

$= \frac{\cos^4\theta}{\sin^2\theta}\,\frac{\sin^4\theta}{\cos^2\theta}\left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$

$= \cos^2\theta\sin^2\theta\left\{\frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}\right\}$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$

$= [(\cos^2\theta)^3 + (\sin^2\theta)^3] + 3\cos^2\theta\sin^2\theta$

$= [(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)] + 3\cos^2\theta\sin^2\theta$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= 1 - 3\cos^2\theta\sin^2\theta + 3\cos^2\theta\sin^2\theta$ $\quad [\because \cos^2\theta + \sin^2\theta = 1]$

$= 1 =$ RHS.

3. Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta = 1 + \tan\theta + \cot\theta$.

Sol. LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$$= \frac{\sin\theta \times \sin\theta}{\cos\theta\,(\sin\theta - \cos\theta)} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta\,\{-(\sin\theta - \cos\theta)\}}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\,(\sin\theta - \cos\theta)\sin\theta} = \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\cos\theta\sin\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} + 1 \qquad ...(i)$$

$$= \sec\theta\,\text{cosec}\,\theta + 1 \qquad ...(ii)$$

For second part

Now from (*i*), we have

$$= \frac{1}{\sin\theta\cos\theta} + 1 \qquad [\text{Putting } 1 = \sin^2\theta + \cos^2\theta]$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} + 1 = \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta} + 1$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} + 1 = \tan\theta + \cot\theta + 1.$$

4. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$

Sol. We have to find $\cos^2 A$ in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now, $\tan A = n \tan B$ $\qquad \tan B = \frac{1}{n}\tan A$ $\qquad \cot B = \frac{n}{\tan A}$

and $\sin A = m \sin B$ $\qquad \sin B = \frac{1}{m}\sin A$ $\qquad \text{cosec}\, B = \frac{m}{\sin A}$

Substituting the values of $\cot B$ and cosec B in $\text{cosec}^2 B - \cot^2 B = 1$, we get

$$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1 \qquad \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$$

$$\frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1 \qquad m^2 - n^2\cos^2 A = \sin^2 A$$

$$m^2 - n^2\cos^2 A = 1 - \cos^2 A \qquad m^2 - 1 = n^2\cos^2 A - \cos^2 A$$

$$m^2 - 1 = (n^2 - 1)\cos^2 A \qquad \frac{m^2 - 1}{n^2 - 1} = \cos^2 A.$$

5. If $x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove $x^2 + y^2 = 1$.

Sol. We have,

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$(x \sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta \qquad [\because x\sin\theta = y\cos\theta]$$

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$x\sin\theta = \sin\theta\cos\theta \qquad x = \cos\theta$$

Now, we have $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta \qquad [\because x = \cos\theta]$$

$$y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$.

6. Prove the following identity, where the angle involved is acute angle for which the expressions are defined.

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Sol. LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{\dfrac{\cos A - \sin A + 1}{\sin A}}{\dfrac{\cos A + \sin A - 1}{\sin A}}$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \qquad [\because \operatorname{cosec}^2 A - \cot^2 A = 1]$$

$$= \frac{(\cot A + \operatorname{cosec} A) - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A + 1)} = \operatorname{cosec} A + \cot A = \text{RHS}.$$

Exercise

A. Multiple Choice Questions

Write the correct answer for each of the following:

1. If $\sin B = \frac{12}{13}$, then $\cot B$ is

(a) $\frac{5}{12}$ (b) $\frac{5}{13}$ (c) $\frac{12}{5}$ (d) $\frac{13}{5}$

2. Given that $\cos\theta = \frac{a}{b}$, then $\operatorname{cosec}\theta$ is

(a) $\frac{b}{a}$ (b) $\frac{b}{\sqrt{b^2 - a^2}}$ (c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

3. If $\cos A + \cos^2 A = 1$, then the value of the expression $\sin^2 A + \sin^4 A$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

4. Given $\tan\alpha = \sqrt{3}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, then the value of $\cot(\alpha + \beta)$ is

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) 0 (*d*) 1

5. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

6. The value of $(\sin 45° - \cos 45°)$ is

(*a*) 0 (*b*) $\frac{1}{\sqrt{2}}$ (*c*) $\sqrt{2}$ (*d*) 1

7. $\frac{2\tan 30°}{1+\tan^2 30°}$ is equal to

(*a*) $\sin 60°$ (*b*) $\cos 60°$ (*c*) $\tan 60°$ (*d*) $\sin 30°$

8. The value of $\sin^2 39° + \sin^2 51°$ is

(*a*) 1 (*b*) 0 (*c*) $2\sin^2 39°$ (*d*) $2\cos^2 51°$

9. If ΔABC is right-angled at A, then $\sec(B+C)$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) not defined

10. The value of the expression $\frac{\tan^2 45° - \sin^2 40° - \sin^2 50°}{\tan 10° \tan 80°}$ is

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) $\frac{1}{2}$

11. The value of the expression $\left[\frac{\cot(20° - \theta) + \tan(70° + \theta)}{\sin(70° + \theta)}\right]\sin(20° - \theta)$ is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) 2

12. $(\sec A + \tan A)(1 - \sin A)$ is equal to

(*a*) $\sec A$ (*b*) $\sin A$ (*c*) $\operatorname{cosec} A$ (*d*) $\cos A$

13. $\frac{\sin\theta}{1+\cos\theta}$ is equal to

(*a*) $\frac{1+\cos\theta}{\sin\theta}$ (*b*) $\frac{1-\cos\theta}{\sin\theta}$ (*c*) $\frac{1-\cos\theta}{\cos\theta}$ (*d*) $\frac{1-\sin\theta}{\cos\theta}$

14. The value of the expression $\frac{\sin^2 60° - \cos^2 30° + \tan^2 60° + \cot^2 30°}{\tan^2 45° + \sec^2 45°}$ is

(*a*) 1 (*b*) 0 (*c*) 2 (*d*) $\frac{1}{2}$

15. If $\sec\theta + \tan\theta = x$, then $\sec\theta$ is equal to

(*a*) $\frac{x^2+1}{x}$ (*b*) $\frac{x^2-1}{x}$ (*c*) $\frac{x^2+1}{2x}$ (*d*) $\frac{x^2-1}{2x}$

16. If $\cos\theta = \frac{3}{5}$, where θ is an acute angle, then $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$ is equal to

(a) $\frac{16}{625}$ (b) $\frac{1}{36}$ (c) $\frac{3}{160}$ (d) $\frac{160}{3}$

17. If $b\tan\theta = a$, then $\frac{b\sin\theta - a\cos\theta}{b\sin\theta + a\cos\theta}$ is equal to

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) 1 (d) 0

18. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is equal to

(a) $\frac{5}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{12}$ (d) $\frac{3}{4}$

19. If $x\sin(90° - \theta)\cot(90° - \theta) = \cos(90° - \theta)$, then x is equal to

(a) 0 (b) 1 (c) –1 (d) 2

20. If 5 cos θ = 7 sin θ, then $\frac{2\sin\theta + 5\cos\theta}{3\sin\theta - 5\cos\theta}$ is

(a) $\frac{-9}{4}$ (b) $\frac{9}{4}$ (c) $\frac{7}{5}$ (d) $-\frac{5}{7}$

21. The value of $\cos 1°\cos 2°\cos 3°............\cos 90°$ is

(a) 1 (b) –1 (c) 0 (d) None of these

22. If $\sin\theta = \frac{1}{3}$, then the value of $(9\cot^2\theta + 9)$ is

(a) 1 (b) 81 (c) 9 (d) $\frac{1}{81}$

23. If for some angle θ, $\cot 2\theta = \frac{1}{\sqrt{3}}$, then the value of $\sin 3\theta$, where $2\theta \le 90°$ is

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) 0 (d) $\frac{\sqrt{3}}{2}$

B. Short Answer Questions Type-I

Write true or false and justify your answer in each of the following: (1–5)

1. tan θ increases faster than sin θ as θ increases.

2. The value of sin θ is $\left(a + \frac{1}{a}\right)$ where 'a' is a positive number.

3. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.

4. The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ is 1.

5. $\cos\theta = \frac{a^2 + b^2}{2ab}$, where a and b are two distinct numbers such that $ab > 0$.

6. If $\text{cosec}\,\theta = 3x$ and $\cot\theta = \frac{3}{x}$, then find the value of $\left(x^2 - \frac{1}{x^2}\right)$

7. What is the value of $(1+\cot^2\theta)\sin^2\theta$?

8. What is the value of $\sin^2\theta + \dfrac{1}{1+\tan^2\theta}$?

9. Write the value of $\sin A \cos(90° - A) + \cos A \sin(90° - A)$.

10. If $\text{cosec}^2\theta(1+\cos\theta)(1-\cos\theta) = \alpha$, then find the value of α.

11. What is the maximum value of $\dfrac{2}{\text{cosec}\theta}$? Justify your answer.

C. Short Answer Questions Type-II

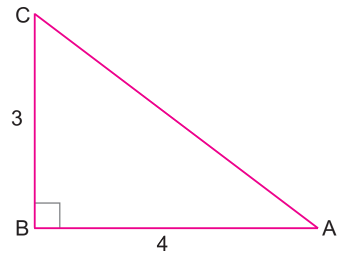


Fig. 5.11

1. In Fig. 5.11, find $\sin A, \tan A$ and $\cot A$.

2. In ΔABC, right-angled at C, find $\cos A, \tan A$ and $\text{cosec } B$ if $\sin A = \dfrac{24}{25}$.

3. If $12\sec A = 13$, find $\sin A$ and $\cot A$.

4. Given $\text{cosec}\theta = \dfrac{4}{3}$, calculate all other trigonometric ratios.

5. In ΔABC, right-angled at A, if $\cot B = 1$, find the value of
   (*i*) $\cos B \cos C + \sin B \sin C$ (*ii*) $\sin B \cos C - \cos B \sin C$.

6. If $\cot\theta = \dfrac{1}{\sqrt{3}}$, show that $\dfrac{1-\cos^2\theta}{2-\sin^2\theta} = \dfrac{3}{5}$.

7. In ΔOPQ, right-angled at P, $OP = 7$ cm, and $OQ - PQ = 1$ cm. Determine the values of $\sin Q$ and $\cos Q$.

8. Write all the other trigonometric ratios of B in terms of $\tan B$.

9. If $\tan\theta = \dfrac{1}{3}$, find other five trigonometric ratios.

Evaluate the following: (10 – 15)

10. $\cos 90° \sin 0° - \sin 0° \cos 90°$.

11. $\dfrac{\cos 60° - \cot 45° + \text{cosec } 30°}{\sec 60° + \tan 45° - \sin 30°}$.

12. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$.

13. $\cot^2 30° - 2\cos^2 60° - \dfrac{3}{4}\sec^2 45° - 4\sec^2 30°$

14. $\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec } 30° + \sec 60° - \cot^2 30°}$.

15. $\dfrac{\tan 45°}{\text{cosec } 30°} + \dfrac{\sec 60°}{\cot 45°} - \dfrac{3\sin 90°}{2\cos 0°}$.

16. In ΔABC, right-angled at B, $AB = 3$ cm and $BAC = 60°$. Determine the lengths of the sides BC and AC.

17. If $\sin(A-B) = 0$, $\cos(A+B) = 0$, $0° < A+B \le 90°$, find A and B.

18. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 0$, $0° < A+B \le 90°$, find $\sin(A+B)$ and $\cos(A-B)$.

Prove the following: (19–24)

19. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

20. $(\sin^4\theta - \cos^4\theta + 1)\,\text{cosec}^2\theta = 2$

21. $\tan\theta + \tan(90° - \theta) = \sec\theta\sec(90° - \theta)$

22. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

23. $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A} = 1 + \tan A + \cot A$

24. $(1+\cot A - \text{cosec} A)(1+\tan A + \sec A) = 2$

25. Simplify: $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

26. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$

27. Given that $\alpha + \beta = 90°$, show that $\sqrt{\cos\alpha\,\text{cosec}\beta - \cos\alpha\sin\beta} = \sin\alpha$.

28. If $\tan\theta = \dfrac{a}{b}$, prove that $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{a^2 - b^2}{a^2 + b^2}$.

29. If $\sec\theta = \dfrac{5}{4}$, find the value of $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

Find the value of *x* if (30-31)

30. $\sqrt{3}\sin x = \cos x$

31. $\tan x = \sin 45°\cos 45° + \sin 30°$

32. If $\theta = 30°$, verify that $\tan^2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

33. If $\theta = 30°$, verify that $\cos^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

34. If $A = 30°$ and $B = 60°$, verify that $\cos(A+B) = \cos A\cos B - \sin A\sin B$.

35. If $A = 30°$ and $B = 60°$, verify that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

Evaluate the following: (36 – 41)

36. $\dfrac{\sec 70°}{\text{cosec} 20°} + \dfrac{\sin 59°}{\cos 31°}$

37. $\tan 48°\tan 23°\tan 42°\tan 67°$

38. $\dfrac{\cos 80°}{\sin 10°} + \cos 59°\,\text{cosec} 31°$

39. $\text{cosec}(65°+\theta) - \sec(25°-\theta) - \tan(55°-\theta) + \cot(35°+\theta)$

40. $\dfrac{3\cos 55°}{7\sin 35°} - \dfrac{4\cos 70°\,\text{cosec}\,20°}{7(\tan 5°\tan 25°\tan 45°\tan 65°\tan 85°)}$

41. If $\sec 2A = \text{cosec}(A - 42°)$ where 2A is an acute angle, find the value of A.

Prove the following trigonometric identities: (42–43)

42. $\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

43. $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

44. If $\cot\theta = \dfrac{15}{8}$, then evaluate $\dfrac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

45. If $\sec\theta = x + \dfrac{1}{x}$, prove that $\sec\theta\tan\theta = 2x$ or $\dfrac{1}{2x}$.

46. If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$.

47. If $\text{cosec}\,\theta = \dfrac{13}{12}$, find the value of $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$.

48. If $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$, find $1 + \tan\theta\cos\theta$.

49. Prove the identity: $(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta) = \dfrac{\sec\theta}{\operatorname{cosec}^2\theta} - \dfrac{\operatorname{cosec}\theta}{\sec^2\theta}$

50. Evaluate: $\left(\dfrac{3\cos 43^\circ}{\sin 47^\circ}\right)^2 - \dfrac{\cos 37^\circ \operatorname{cosec} 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

D. Long Answer Questions

1. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

2. If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

3. If $\sec\theta + \tan\theta = p$, show that $\dfrac{p^2 - 1}{p^2 + 1} = \sin\theta$.

4. Given that $\sin\theta + 2\cos\theta = 1$, then prove that $2\sin\theta - \cos\theta = 2$.

5. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\dfrac{1}{2}$.

6. If $a\sin\theta + \cos\theta = c$, then prove that $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$.

Prove that the following identities (Q. 7 to 22)

7. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

8. $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

9. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

10. $\sin A(1 + \tan\theta)^2 + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

11. $(\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2 = (1 - \sec\theta\operatorname{cosec}\theta)^2$

12. $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A} = \dfrac{2}{\sin^2 A - \cos^2 A} = \dfrac{2}{2\sin^2 A - 1}$

13. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

14. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

15. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \dfrac{1}{\tan\theta + \cot\theta}$

16. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

17. $\dfrac{1}{\sec A - 1} + \dfrac{1}{\sec A + 1} = 2\operatorname{cosec} A\cot A$

18. $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

19. $\dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \cot\theta$

20. $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

21. $(1 + \cot A + \tan A)(\sin A - \cos A) = \dfrac{\sec A}{\operatorname{cosec}^2 A} - \dfrac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$

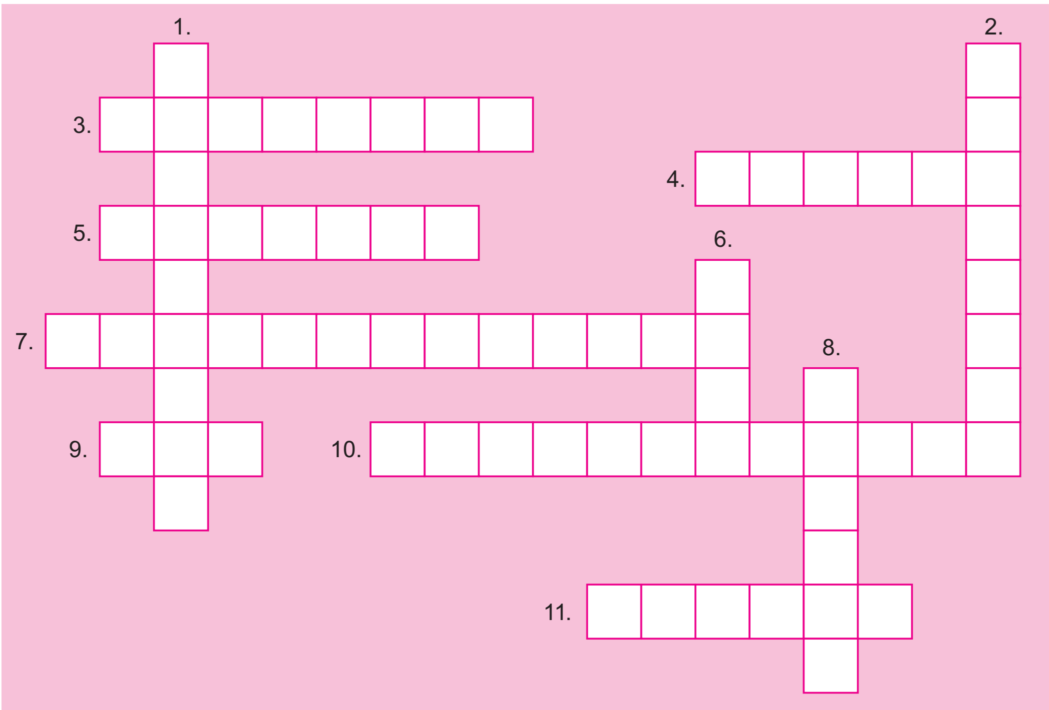
22. $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

23. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Formative Assessment

Activity

■ **Solve the following crossword puzzle, hints are given below:**



Across

3. Reciprocal of sine of an angle.
4. Sum of ________ of sine and cosine of an angle is one.
5. Sine of an angle divided by cosine of that angle.
7. Triangles in which we study trigonometric ratios.
9. Maximum value for sine of any angle.
10. Branch of Mathematics in which we study the relationship between the sides and angles of a triangle.
11. Sine of $(90° - \theta)$.

Down

1. Reciprocal of tangent of an angle.
2. An equation which is true for all values of the variables involved.
6. Cosine of 90°.
8. Reciprocal of cosine of an angle.

Hands on Activity (Math Lab Activity)

■ **To find trigonometric ratios of some specific angles.**

Trigonometric Ratios of 0° and 90°

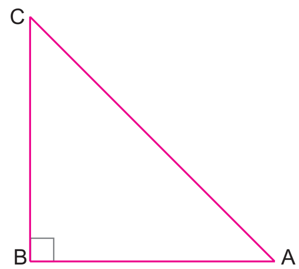


Fig. 5.12

- Consider a ΔABC right-angled at B.
- Let us see what happens to the trigonometric ratios of angle A, if we make A smaller and smaller, till it becomes zero.
- On observing Fig. 5.13, we find that as A gets smaller and smaller, the length of the side BC decreases and when A becomes very close to 0°, AC becomes almost the same as AB.

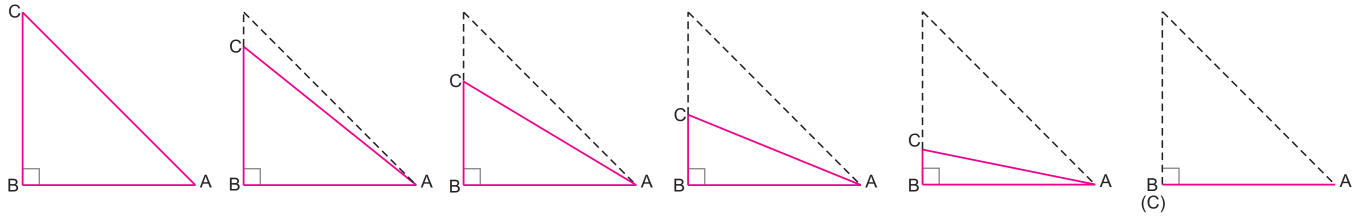


Fig. 5.13

- Since $\sin A = \dfrac{BC}{AC}$, and the value of BC is very close to O when A is very close to 0°, therefore,

  $$\sin 0° = 0$$

  Similarly, the value of AC is nearly the same as AB, when A is very close to 0°

  $$\cos 0° = \frac{AB}{AC} = 1$$

- Hence, $\sin 0° = 0$, $\cos 0° = 1$

  $$\tan 0° = \frac{\sin 0°}{\cos 0°} = \frac{0}{1} = 0, \cot 0° = \frac{1}{\tan 0°} = \frac{1}{0}, \text{ which is not defined, } \sec 0° = \frac{1}{\cos 0°} = 1,$$

  $$\text{cosec } 0° = \frac{1}{\sin 0°} = \frac{1}{0} \text{ which is not defined}$$

Trigonometric Ratios of 45°

- Consider ΔABC right-angled at B.
- If one of the acute angles, say A is 45°, then $C = 45°$
  So, $AB = BC$ (Sides opposite to equal angles are equal)
- Let $AB = BC = a$
  Then, by Pythagoras theorem, $AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

  $$AC = \sqrt{2}a$$

- Thus, we have

  $$\sin A = \sin 45° = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}, \text{ cosec } 45° = \frac{1}{\sin 45°} = \sqrt{2}$$

  $$\cos 45° = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}, \sec 45° = \frac{1}{\cos 45°} = \sqrt{2}$$

  $$\tan 45° = \frac{BC}{AB} = \frac{a}{a} = 1, \cot 45° = \frac{1}{\tan 45°} = 1$$

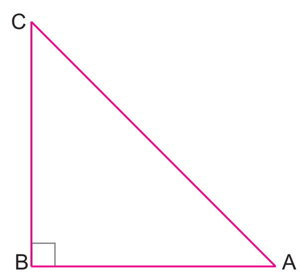


Fig. 5.14

■ **Trigonometric ratios of 30° and 60°**

- Consider an equilateral triangle ABC
  Then $A = B = C = 60°$ (Each angle of an equilateral triangle is 60°)

- Draw $AD \quad BC$

  In ΔABD and ΔACD

$$AB = AC \qquad \text{(sides of an equilateral triangle)}$$
$$ADB = \quad ADC \qquad \text{(each 90°)}$$
$$AD = DA \qquad \text{(common)}$$
$$\Delta ABD \quad \Delta ACD \qquad \text{(By RHS congruence condition)}$$
$$BD = DC \qquad \text{(CPCT)}$$
$$BAD = \quad CAD$$
$$BAD = \quad CAD = \frac{1}{2} \quad BAC = 30° \quad \text{(CPCT)}$$

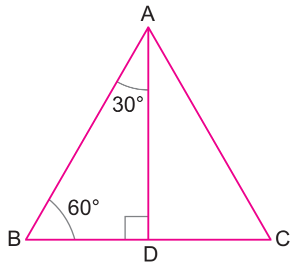


Fig. 5.15

- Let $AB = 2a$

  Then $BD = \frac{1}{2}BC = a$

$$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2 = 3a^2 \qquad i.e. \quad AD = \sqrt{3}\,a$$

- In right ΔABD

$$\sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}, \qquad \text{cosec } 30° = \frac{1}{\sin 30°} = 2$$
$$\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}, \qquad \sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}}$$
$$\tan 30° = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}, \qquad \cot 30° = \frac{1}{\tan 30°} = \sqrt{3}$$

  Similarly, find all the trigonometric ratios of 60°

Seminar

Make PPTs/Charts on following topics and present in the class in the presence of teachers.

(*i*) What is the relationship between any t-ratio and another t-ratio with suffix co-added or removed, like sine and cosine, cotangent and tangent, etc.

(*ii*) Interpretation of t-ratios of 0° and 90° in a right angled triangle.

Group Discussion

Divide the class into small groups and ask them to discuss practical uses of trigonometry.

Multiple Choice Questions

Tick the correct answer for each of the following:

1. The reciprocal of $\cos\theta$ is

(*a*) $\sin\theta$ (*b*) cosec θ (*c*) $\tan\theta$ (*d*) $\sec\theta$

2. Which of the following is not a trigonometric identity?

(*a*) $\cos^2\theta + \sin^2\theta = 1$ (*b*) $\cot^2\theta + 1 = \tan^2\theta$ (*c*) $\cot^2\theta + 1 = \text{cosec}^2\theta$ (*d*) $\tan^2\theta + 1 = \sec^2\theta$

3. The value of tangent of 90° is

(*a*) 0 (*b*) 1 (*c*) $\sqrt{3}$ (*d*) not defined

4. If $\sin A = \frac{5}{13}$, the value of $\tan A$ is

(*a*) $\frac{5}{12}$ (*b*) $\frac{12}{13}$ (*c*) $\frac{13}{12}$ (*d*) $\frac{12}{5}$

5. If $\cos A = \frac{1}{\sqrt{2}}$, the value of $\cot A$ is

(*a*) $\sqrt{2}$ (*b*) 1 (*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{\sqrt{3}}$

6. Maximum value of $\frac{1}{\operatorname{cosec}\theta}$, $0° < \theta < 90°$ is

(*a*) –1 (*b*) 2 (*c*) 1 (*d*) $\frac{1}{2}$

7. The value of $\sin^2 37° + \cos^2 37°$ is

(*a*) 1 (*b*) 0 (*c*) $\frac{\sqrt{3}}{4}$ (*d*) $\frac{1}{2}$

8. The value of $\frac{\tan 60°}{\tan 30°}$ is

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\sqrt{3}$ (*c*) 3 (*d*) $\frac{1}{3}$

9. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(*a*) $\frac{b}{\sqrt{b^2-a^2}}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{\sqrt{b^2-a^2}}{b}$ (*d*) $\frac{a}{\sqrt{b^2-a^2}}$

10. If $\cos(\alpha+\beta) = 0$, then $\sin(\alpha-\beta)$ can be reduced to

(*a*) $\cos\beta$ (*b*) $\cos 2\beta$ (*c*) $\sin\alpha$ (*d*) $\sin 2\alpha$

11. If ΔABC is right-angled at A, then $\cos(B+C)$ is

(*a*) 1 (*b*) $\frac{1}{\sqrt{2}}$ (*c*) $\frac{1}{2}$ (*d*) 0

12. The value of the expression $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 63° + \cos^2 27°} - \sin^2 90°$ is

(*a*) 1 (*b*) 0 (*c*) $\frac{1}{2}$ (*d*) 2

13. The value of the expression $\frac{\tan^2 25° - \cot 65° \tan 25°}{\sin^2 25° + \sin^2 65°}$ is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

14. $\sin(45°+\theta) - \cos(45°-\theta)$ is equal to

(*a*) $2\cos\theta$ (*b*) 0 (*c*) $2\sin\theta$ (*d*) 1

15. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

Match the Columns

Simplify and match the expressions in column I with their values in column II.

Column I	Column II
(*i*) $\sin^2 37°+\sin^2 53°+\sin^2 90°$	(*a*) 0
(*ii*) $\tan 35° \tan 45° \tan 55°$	(*b*) 3
(*iii*) $\dfrac{\sec 72° \sin 18° + \tan 72° \cot 18°}{\cos 60°}$	(*c*) 1
(*iv*) $\dfrac{\tan 60°}{\tan 30°}$	(*d*) 2
(*v*) $\sin^2 30 + \cos^2 30 - \sin^2 60 - \cos^2 60$	(*e*) 4

Project Work

History of Trigonometry

- Each student must make presentation based on the following topics:
  - Mathematicians who worked for the development of trigonometry.
  - List of formulae.
  - Uses of Trigonometry in various fields.

  Students should mention all the sources they used to collect the information.

Rapid Fire Quiz

State whether the following statements are true (T) or false (F).

1. The reciprocal of $\sin A$ is $\cos A$, $A \neq 0$.
2. $\cot A$ is the reciprocal of $\tan A$, $A \neq 90°$.
3. Sum of the squares of $\sin A$ and $\cos A$ is 1.
4. The value of $\cos 90°$ is 1.
5. The trigonometric ratios can be applied in any triangle.
6. The values of $\sin A$ and $\sin B$ will always be same for a right ΔABC right-angled at C.
7. The values of $\sin A$ and $\cos A$ can never exceed 1.
8. $\sec A$ and $\operatorname{cosec} A$ can take any value on the real number line.
9. $\sin(90° - A) = \cos A$
10. $\cos(90° - A) = \sec A$
11. The value of $\sin\theta + \cos\theta$ is always greater than 1
12. $\tan 70° \tan 20° = 1$
13. The value of the expression $(\cos^2 20° - \sin^2 67°)$ is positive.
14. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$
15. $\tan\theta$ increases faster than $\sin\theta$ as θ increases.

16. cos A is the abbreviation used for the cosecant of angle A.
17. $\sin^2 A = (\sin A)^2$
18. $\sin\theta = \frac{5}{3}$ for some angle θ.
19. cot A is not defined for $A = 0°$
20. Trigonometry deals with measurement of components of triangles.

Oral Questions

1. What is the reciprocal of sec A?
2. Is tan A the reciprocal of cot A?
3. What is the value of sine of 0°?
4. What is $1+\tan^2\theta$?
5. What is the value of $\text{cosec}^2\theta - \cot^2\theta$?
6. Name the side adjacent to angle A if ΔABC is a triangle right-angled at B.
7. Define an identity.
8. What is the maximum possible value for sine of any angle?
9. Can the value of secant of an angle be greater than 1?
10. What is $\tan(90°-A)$ equal to?
11. What do we call the side opposite to the right angle in a right triangle?
12. If we increase the lengths of the sides of a right triangle keeping the angle between them same, then the values of the trigonometric ratios will also increase. State True or False.
13. Does the value of $\tan\theta$ increase or decrease as we increase the value of θ? Give reason.
14. What will be the change in the value of $\cos\theta$ if we decrease the value of θ?
15. What is the relation between $\sin\theta, \cos\theta$ and $\cot\theta$?
16. What is the relation between $\tan\theta$ and $\sec\theta$?
17. The value of tan A is always less than 1. State True or False.
18. Can the value of $\cos\theta$ be $\frac{5}{4}$ for some angle θ?

Class Worksheet

1. Tick the correct answer for each of the following:

   (*i*) Which of the following is not a trigonometric identity?

   (*a*) $\sec^2\theta - \tan^2\theta = 1$ (*b*) $\text{cosec}^2\theta - \sin^2\theta = 1$ (*c*) $\cot^2\theta - \text{cosec}^2\theta = -1$ (*d*) $1-\cos^2\theta = \sin^2\theta$

   (*ii*) The value of the expression $\frac{1}{2}\tan 60° - \sin 60° + 2\cos 60°$ is

   (*a*) $\frac{1}{2}$ (*b*) 2 (*c*) 1 (*d*) 0

   (*iii*) The value of (tan 1° tan 2° tan 3° . . . tan 89°) is

   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) $\frac{1}{2}$

(*iv*) If ΔABC is right-angled at C, then $\cot(A+B)$ is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{\sqrt{2}}$ (*d*) not defined

(*v*) If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) 1

(*vi*) $\sqrt{1+\tan^2\theta}$ is equal to:

(*a*) $\cot\theta$ (*b*) $\cos\theta$ (*c*) $\operatorname{cosec}\theta$ (*d*) $\sec\theta$

(*vii*) If $5\tan\theta = 12$, then $\left(\frac{5\sin\theta - \cos\theta}{5\sin\theta + \cos\theta}\right)$ is equal to

(*a*) $\frac{12}{13}$ (*b*) $\frac{5}{13}$ (*c*) $\frac{11}{13}$ (*d*) $\frac{13}{11}$

(*viii*) If $\cos\theta = \frac{1}{2}$, $\sin\varphi = \frac{1}{2}$, then value of $\theta + \varphi$

(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 120°

2. State true or false for the following statements and justify your answer.

(*i*) The value of the expression $(\sin 70° - \cos 70°)$ is negative.

(*ii*) If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.

3. Write True or False.

(*i*) $\cos A = \cos \times A$

(*ii*) $\cos\theta = \frac{7}{6}$ for some angle θ

(*iii*) $\sec A = \frac{1}{\cos A}$, for an acute θ

(*iv*) $\sin 60° = 2\sin 30°$

(*v*) $\cos 75° = \cos 60° + \cos 15°$

(*vi*) If $\tan A = \frac{3}{4}$, then $\cos A = \frac{4}{5}$

4. Fill in the blanks.

(*i*) 5 cos 0° + sin 90° = ____________.

(*ii*) tan 0° = ____________.

(*iii*) cot 90° is ____________.

(*iv*) If $\cos\theta = 1$, then θ = ____________.

(*v*) $3\tan^2 45°$ = ____________.

(*vi*) $2\sin^2 45°$ = ____________.

5. Fill in the blanks.

(*i*) sin *A* ____________ when A increases from 0° to 90°.

(*ii*) cos *A* ____________ when A increases from 0° to 90°.

(*iii*) $\frac{\sin 58°}{\cos 32°}$ = ____________.

(*iv*) cos 0° × cos 10° × cos 30° × cos 80° × cos 90° = ____________ .

(*v*) The word 'Trigonometry' is derived from the Greek words ____________, ____________ and ____________.

6. Write True or False.

(*i*) In ΔABC, if $A + C = 90°$, then $\sin A = \cos C$

(*ii*) cot 60° = tan (90° – 30°)

(*iii*) $\sin\theta + \cos\theta = 1$

(*iv*) $\tan^2 A = \sec^2 A - 1$

(*v*) $\sin^2 56° + \cos^2 34° = 1$

(*vi*) $\operatorname{cosec} 50° = \sec 40°$

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Tick the correct answer for each of the following:

(*i*) If $\tan A = \frac{3}{4}$, then the value of $\sec A$ is 1

(*a*) $\frac{5}{3}$ (*b*) $\frac{5}{4}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{4}{5}$

(*ii*) The value of the expression $\dfrac{\text{cosec}\,(58^\circ+\theta)-\sec(32^\circ-\theta)}{\tan 45^\circ+\tan(45^\circ+\theta)-\cot(45^\circ-\theta)}$ is 1

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 0 (*d*) 2

(*iii*) The value of $\dfrac{\tan^2 60^\circ-\sin^2 30^\circ}{\tan^2 45^\circ+\cos^2 30}$ is 1

(*a*) $\frac{7}{11}$ (*b*) $\frac{11}{13}$ (*c*) $\frac{13}{11}$ (*d*) $\frac{11}{7}$

(*iv*) If $\sin A+\sin^2 A=1$, then the value of the expression $\cos^2 A+\cos^4 A$ is 1

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) 3

(*v*) Given that $\tan\alpha=\sqrt{3}$ and $\tan\beta=\frac{1}{\sqrt{3}}$, then the value of $(\alpha+\beta)$ is 1

(*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

(*vi*) Given that $3\cot\theta=4$, then $\left(\dfrac{5\sin\theta-3\cos\theta}{5\sin\theta+3\cos\theta}\right)$ is equal to 2

(*a*) $\frac{1}{9}$ (*b*) 9 (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{2}$

2. State whether the following statements are true or false. Justify your answer.

(*i*) The value of $2\sin\theta$ can be $\left(a+\frac{1}{a}\right)$, where a is a positive number and $a\neq 1$.

(*ii*) $\tan\theta$ increases faster than $\sin\theta$ as θ increases. $2\times 2=4$

3. (*i*) Show that: $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}=1.$

(*ii*) If $2\sin^2\theta-\cos^2\theta=2$, then find the value of θ. $3\times 2=6$

4. (*i*) Prove that: $\dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\dfrac{1-\sin\theta}{\cos\theta}$

(*ii*) If $\cot\theta=\frac{7}{8}$, check whether $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta-\sin^2\theta$ or not. $4\times 2=8$

Chapter Six

STATISTICS

Basic Concepts and Results

- ***Arithmetic mean***

  The ***arithmetic mean*** (or, simply ***mean***) *of a set of numbers is obtained by dividing the sum of numbers of the set by the number of numbers.*

  The mean of n numbers $x_1, x_2, x_3, \ldots, x_n$ denoted by $\overline{X}$ (read as X bar) is defined as:

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\Sigma x}{n}$$

  where Σ is a Greek alphabet called sigma. It stands for the words *"the sum of"*. Thus, Σx means sum of all x.

- ***Mean of grouped data***

  (*i*) *Direct method*: If the variates observations $x_1, x_2, x_3, \ldots, x_n$ have frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then the mean is given by :

$$\text{Mean } (\overline{X}) = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

  This method of finding the mean is called the *direct method*.

  (*ii*) *Short cut method*: In some problems, where the number of variates is large or the values of x_i or f_i are larger, then the calculations become tedious. To overcome this difficulty, we use *short cut* or *deviation method*. In this method, an approximate mean, called *assumed mean* or *provisional mean* is taken. This assumed mean is taken preferably near the middle, say A, and the deviation $d_i = x_i - A$ for each variate x_i. The mean is given by the formula :

$$\text{Mean } (\overline{X}) = A + \frac{\Sigma f_id_i}{\Sigma f_i}$$

- ***Mean for a grouped frequency distribution***

  Find the class mark or mid-value x_i of each class, as

$$x_i = \text{class mark} = \left(\frac{\text{lower limit} + \text{upper limit}}{2}\right)$$

  Then $\overline{X} = \dfrac{\Sigma f_ix_i}{\Sigma f_i}$ or $\overline{X} = A + \dfrac{\Sigma f_id_i}{\Sigma f_i}$, $d_i = x_i - A$

- ***Step Deviation method for computing mean***

  In this method an arbitrary constant A is chosen which is called as origin or assumed mean somewhere in the middle of all values of x_i. If h is the difference of any two consecutive values of x_i, then $u_i = \dfrac{x_i - A}{h}$

$$\text{Mean} = A + \frac{\Sigma f_iu_i}{\Sigma f} \times h$$

- ***Median*:** The median is the middle value of a distribution *i.e,* median of a distribution is the value of the variable which divides it into two equal parts. It is the value of the variable such that the number of observations above it is equal to the number of observations below it.

Median of a grouped or continuous frequency distribution $= l + \left(\dfrac{\dfrac{n}{2} - cf}{f}\right) \times h$

where, l = lower limit of the median class
$\Sigma f_i = n$ = number of observations
f = frequency of the median class
h = size of the median class (assuming class size to be equal)
cf = cumulative frequency of the class preceding the median class

- ***Mode*:** The mode or modal value of a distribution is that value of the variable for which the frequency is maximum.

Mode for a continuous frequency distribution with equal class interval

$$= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

where, l = lower limit of the modal class
f_1 = frequency of the modal class
f_0 = frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class
h = size of the modal class

- ***Graphical representation of cumulative frequency distribution***

(*i*) *Cumulative frequency curve or an ogive of the less than type*:

(*a*) Mark the upper limit of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

(*b*) Plot the points corresponding to the ordered pairs given by upper limit and corresponding cumulative frequency. Join them by a freehand smooth curve.

(*ii*) *Cumulative frequency curve or an ogive of the more than type*:

(*a*) Mark the lower limit of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on vertical axis (y-axis).

(*b*) Plot the points corresponding to the ordered pairs given by lower limit and corresponding cumulative frequency. Join them by a freehand smooth curve.

- Median of a ground data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives more than type and less than type.

3 Median = Mode + 2 Mean

Summative Assessment

Multiple Choice Questions

Write correct answer for each of the following:

1. The arithmetic mean of 1, 2, 3, ... n is

(*a*) $\frac{n}{2}$ (*b*) $\frac{n}{2} + 1$ (*c*) $\frac{n(n+1)}{2}$ (*d*) $\frac{n-1}{2}$

2. If the mean of the following distribution is 6.4, then the value of p is

x	2	4	6	8	10	12
f	3	*p*	5	3	2	1

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

3. Consider the following distribution

Monthly Income Range (in ₹)	Number of Families
More than or equal to 5,000	150
More than or equal to 10,000	132
More than or equal to 15,000	115
More than or equal to 20,000	85
More than or equal to 25,000	68
More than or equal to 30,000	42

The number of families having income range (in ₹) 15,000 – 20,000 is

(*a*) 14 (*b*) 33 (*c*) 118 (*d*) 85

4. For the following distribution:

Marks	Number of Students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The modal class is

(*a*) 10 – 20 (*b*) 20 – 30 (*c*) 30 – 40 (*d*) 50 – 60

5. Consider the data:

Class	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is:

(*a*) 0 (*b*) 19 (*c*) 20 (*d*) 38

Short Answer Questions Type–I

1. Find the class marks of classes 15.5 – 18.5 and 50 – 75.

Sol. Class marks $= \dfrac{\text{upper limit} + \text{lower limit}}{2}$

Class marks of 15.5 – 18.5 $= \dfrac{18.5 + 15.5}{2} = \dfrac{34}{2} = 17$

Class marks of 50 – 75 $= \dfrac{75 + 50}{2} = \dfrac{125}{2} = 62.5.$

2. Find the median class of the following distribution:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	4	8	10	12	8	4

Sol. First we find the cumulative frequency

Classes	Frequency	Cumulative Frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50
Total	**50**	

Here, $\frac{n}{2} = \frac{50}{2} = 25$

Median class = 30 – 40.

3. Write the modal class for the following frequency distribution:

Class Interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	33	38	65	52	19	48

Sol. Maximum frequency, *i.e.*, 65 corresponds to the class 30 – 40

Modal class is 30 – 40.

4. A student draws a cumulative frequency curve for the marks obtained by 50 students of a class as shown below. Find the median marks obtained by the students of the class.

Fig. 6.1

Sol. Here $n = 60$ $\qquad \frac{n}{2} = 30$

Corresponding to 30 on y-axis, the marks on x-axis is 40.

Median marks = 40.

Important Problems

Type A: Problems Based on Mean of Grouped Data

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean and why? **[NCERT]**

Sol. Calculation of mean number of plants per house.

Number of plants	Number of houses (f_i)	Class mark (x_i)	$f_i x_i$
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
Total	$\Sigma f_i = 20$		$\Sigma f_i x_i = 162$

Hence, Mean $(\overline{X}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{162}{20} = 8.1$

Here, we used direct method to find mean because numerical values of x_i and f_i are small.

2. Find the mean of the following distribution:

x	4	6	9	10	15
f	5	10	10	7	8

Sol. Calculation of arithmetic mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
Total	$\Sigma f_i = 40$	$\Sigma f_i x_i = 360$

$$\text{Mean } (\overline{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{40} = 9$$

3. If the mean of the following distribution is 6, find the value of p.

x	2	4	6	10	$p + 5$
f	3	2	3	1	2

Sol. Calculation of mean

x_i	f_i	$f_i x_i$
2	3	6
4	2	8
6	3	18
10	1	10
$p + 5$	2	$2p + 10$
Total	$\Sigma f_i = 11$	$\Sigma f_i x_i = 2p + 52$

We have, $\Sigma f_i = 11, \Sigma f_i x_i = 2p + 52, \overline{X} = 6$

$$\text{Mean } (\overline{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$6 = \frac{2p + 52}{11} \qquad 66 = 2p + 52$$

$$2p = 14 \qquad p = 7$$

4. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45–55	55–65	65–75	75–85	85–95
Number of cities	3	10	11	8	3

Sol. Here, we use step deviation method to find mean.

Let assumed mean $A = 70$ and class size $h = 10$

So, $$u_i = \frac{x_i - 70}{10}$$

Now, we have

Literacy rate (in %)	**Frequency (f_i)**	**Class mark (x_i)**	$u_i = \frac{x_i - 70}{10}$	$f_i u_i$
45 – 55	3	50	– 2	– 6
55 – 65	10	60	– 1	– 10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	$\Sigma f_i = 35$			$\Sigma f_i u_i = -2$

$$\text{Mean } (\overline{X}) = A + h \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 70 + 10 \times \frac{-2}{35} = 70 - 0 \;\; 57 = 69 \;\; 43\%$$

5. Find the mean of the following frequency distribution:

Class interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of workers	15	18	21	29	17

Sol. Calculation of mean

Class interval	Mid-values (x_i)	Frequency (f_i)	$u_i = \frac{x_i - A}{20} = \frac{x_i - 50}{20}$	$f_i u_i$
0 – 20	10	15	– 2	– 30
20 – 40	30	18	– 1	– 18
40 – 60	50	21	0	0
60 – 80	70	29	1	29
80 – 100	90	17	2	34
Total		$\Sigma f_i = 100$		$\Sigma f_i u_i = 15$

We have, $A = 50, h = 20, \Sigma f_i = 100$ and $\Sigma f_i u_i = 15$.

$$\text{Mean } (\bar{X}) = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) = 50 + 20 \times \frac{15}{100} = 50 + 3 = 53.$$

6. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency f. **[NCERT]**

Daily pocket allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of children	7	6	9	13	f	5	4

Sol. Let the assumed mean $A = 16$ and class size $h = 2$, here we apply step deviation method.

So, $u_i = \frac{x_i - A}{h} = \frac{x_i - 16}{2}$

Now, we have,

Class interval	Frequency (f_i)	Class mark (x_i)	$u_i = \frac{x_i - 16}{2}$	$f_i u_i$
11 – 13	7	12	– 2	– 14
13 – 15	6	14	– 1	– 6
15 – 17	9	16	0	0
17 – 19	13	18	1	13
19 – 21	f	20	2	$2f$
21 – 23	5	22	3	15
23 – 25	4	24	4	16
Total	$\Sigma f_i = f + 44$			$\Sigma f_i u_i = 2f + 24$

We have, Mean $(\bar{X}) = 18$, $A = 16$ and $h = 2$

$$\bar{X} = A + h \times \frac{\Sigma f_i u_i}{\Sigma f_i}$$

$$18 = 16 + 2 \times \left(\frac{2f + 24}{f + 44}\right) \qquad 2 = 2 \times \left(\frac{2f + 24}{f + 44}\right)$$

$$1 = \frac{2f + 24}{f + 44} \qquad f + 44 = 2f + 24$$

$$f = 44 - 24 \qquad f = 20$$

Hence, the missing frequency is 20.

7. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	x	12	7	8

Sol. We have

Class interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 20	5	10	50
20 – 40	8	30	240
40 – 60	x	50	$50x$
60 – 80	12	70	840
80 – 100	7	90	630
100 – 120	8	110	880
Total	$\Sigma f_i = 40 + x$		$\Sigma f_i x_i = 2640 + 50x$

Here, $\Sigma f_i x_i = 2640 + 50x, \Sigma f_i = 40 + x, \overline{X} = 62.8$

$$\text{Mean } (\overline{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$62.8 = \frac{2640 + 50x}{40 + x} \qquad 2512 + 62.8x = 2640 + 50x$$

$$62.8x - 50x = 2640 - 2512 \qquad 12.8x = 128$$

$$x = \frac{128}{12.8} = 10$$

Hence, the missing frequency is 10.

Type B: Problems Based on Mode of Grouped Data

1. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components: **[NCERT]**

Lifetimes (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol. Here, the maximum class frequency is 61 and the class corresponding to this frequency is 60–80. So, the modal class is 60–80.

Here, $l = 60, h = 20, f_1 = 61, f_0 = 52, f_2 = 38$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 60 + \frac{61 - 52}{2 \times 61 - 52 - 38} \times 20 = 60 + \frac{9}{122 - 90} \times 20 = 60 + \frac{9}{32} \times 20$$

$$= 60 + \frac{45}{8} = 60 + 5.625 = 65.625$$

Hence, modal lifetime of the components is 65.625 hours.

2. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

Number of cars	0 – 10	10 –20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

[NCERT]

Sol. Here, the maximum frequency is 20 and the corresponding class is 40–50. So 40–50 is the modal class.

We have, $l = 40, h = 10, f_1 = 20, f_0 = 12, f_2 = 11$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 40 + \frac{20 - 12}{2\times 20 - 12 - 11} \times 10$$

$$= 40 + \frac{8}{40 - 23} \times 10 = 40 + \frac{80}{17} = 40 + 4.7 = 44.7$$

Hence, the mode of the given data is 44.7 cars.

3. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	**Number of batsmen**	**Runs scored**	**Number of batsmen**
3000 – 4000	4	7000 – 8000	6
4000 – 5000	18	8000 – 9000	3
5000 – 6000	9	9000 – 10000	1
6000 – 7000	7	10000 – 11000	1

Find the mode of the data. **[NCERT]**

Sol. Here, the maximum frequency is 18 and the class corresponding to this frequency is 4000–5000. So the modal class is 4000–5000.

Now we have,

$$l = 4000, h = 1000, f_1 = 18, f_0 = 4, f_2 = 9$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 4000 + \frac{18 - 4}{2 \times 18 - 4 - 9} \times 1000$$

$$= 4000 + \frac{14}{36 - 13} \times 1000 = 4000 + \frac{14}{23} \times 1000$$

$$= 4000 + 608.696 = 4608.696 = 4608.7 \text{ (approx.)}$$

Hence, the mode of the given data is 4608.7 runs.

Type C: Problems Based on Median of Grouped Data

1. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

[NCERT]

Sol. Calculation of median

Weight (in kg)	Number of students (f_i)	Cumulative frequency (cf)
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
Total	$\Sigma f_i = 30$	

We have, $\sum fi = n = 30$ $\quad \frac{n}{2} = 15$

The cumulative frequency just greater than $\frac{n}{2} = 15$ is 19, and the corresponding class is $55 - 60$.

$55 - 60$ is the median class.

Now, we have $\quad \frac{n}{2} = 15, l = 55, cf = 13, f = 6, h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 55 + \left(\frac{15 - 13}{6}\right) \times 5 = 55 + \frac{2}{6} \times 5 = 55 + 1.67 = 56.67$$

Hence, median weight is 56.67 kg.

2. The lengths of 40 leaves of a plant are measured correctly to the nearest millimetre, and the data obtained is represented in the following table:

Length (in mm)	118–126	127–135	136–144	145–153	154–162	163–171	172–180
Number of Leaves	3	5	9	12	5	4	2

Find the median length of the leaves. **[NCERT]**

Sol. Here, the classes are not in inclusive form. So, we first convert them in inclusive form by subtracting $\frac{h}{2}$ from the lower limit and adding $\frac{h}{2}$ to the upper limit of each class, where h is the difference between the lower limit of a class and the upper limit of preceding class.

Now, we have

Class interval	Number of leaves (f_i)	Cumulative frequency (cf)
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40
Total	$\Sigma f_i = 40$	

We have, $n = 40$ $\frac{n}{2} = 20$

And, the cumulative frequency just greater than $\frac{n}{2}$ is 29 and corresponding class is 144.5 – 153.5. So median class is 144.5 – 153.5.

Here, we have $\frac{n}{2} = 20,\ l = 144.5,\ h = 9,\ f = 12,\ cf = 17$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$= 144.5 + \frac{3}{12} \times 9 = 144.5 + \frac{9}{4} = 144.5 + 2.25 = 146.75 \text{ mm.}$$

Hence, the median length of the leaves is 146.75 mm.

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders	Age (in years)	Number of policy holders
Below 20	2	Below 45	89
Below 25	6	Below 50	92
Below 30	24	Below 55	98
Below 35	45	Below 60	100
Below 40	78		

Sol. We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

Class interval	Frequency (f_i)	Cumulative frequency (cf)
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100
Total	$\Sigma f_i = 100$	

Here, $n = 100$ $\frac{n}{2} = 50$

And, cumulative frequency just greater than $\frac{n}{2} = 50$ is 78 and the corresponding class is 35 – 40. So 35 – 40 is the median class.

$\frac{n}{2} = 50, l = 35, cf = 45, f = 33, h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 35 + \left(\frac{50 - 45}{33}\right) \times 5 = 35 + \frac{5}{33} \times 5 = 35 + \frac{25}{33} = 35 + 0.76 = 35.76$$

Hence, the median age is 35.76 years.

Type D: Problems Based on Graphical Representation of Cumulative Frequency Distribution

1. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive. **[NCERT]**

Sol. Now, converting given distribution to a less than type cumulative frequency distribution, we have,

Daily income (in ₹)	Cumulative frequency
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now, let us plot the points corresponding to the ordered pairs (120, 12), (140, 26), (160, 34), (180, 40), (200, 50) on a graph paper and join them by a freehand smooth curve.

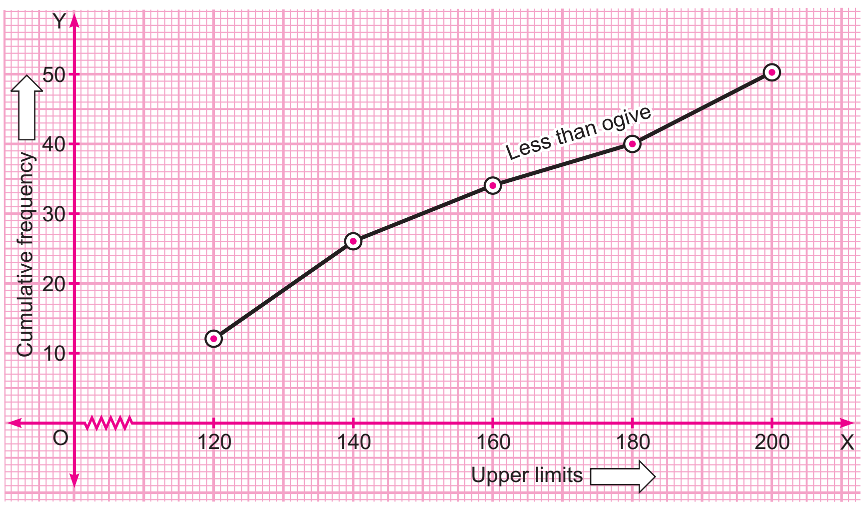


Fig. 6.2

Thus, obtained curve is called the less than type ogive.

2. The distribution below gives the marks of 100 students of a class.

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	4	6	10	10	25	22	18	5

Draw a less than type and a more than type ogive from the given data. Hence, obtain the median marks from the graph.

Sol.

Marks	Cumulative Frequency	Marks	Cumulative Frequency
Less than 5	4	More than 0	100
Less than 10	10	More than 5	96
Less than 15	20	More than 10	90
Less than 20	30	More than 15	80
Less than 25	55	More than 20	70
Less than 30	77	More than 25	45
Less than 35	95	More than 30	23
Less than 40	100	More than 35	5

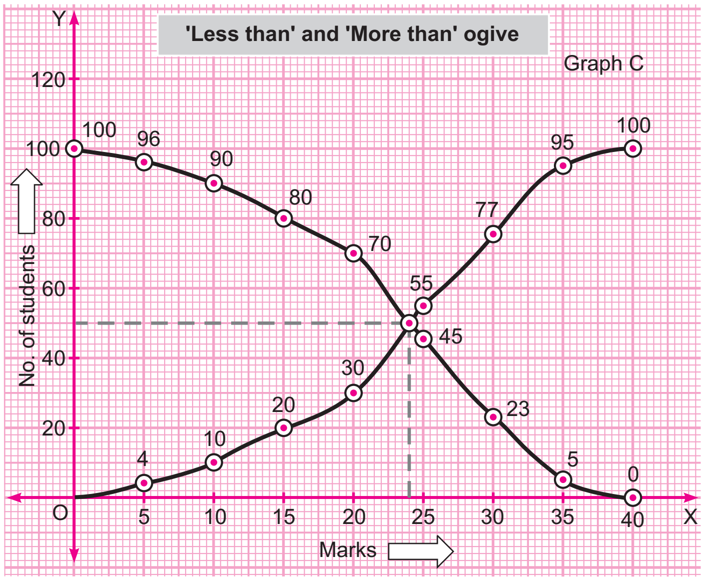


Fig. 6.3

Hence, Median Marks = 24

3. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students	Weight (in kg)	Number of students
Less than 38	0	Less than 46	14
Less than 40	3	Less than 48	28
Less than 42	5	Less than 50	32
Less than 44	9	Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula. **[NCERT]**

Sol. To represent the data in the table graphically, we mark the upper limits of the class interval on x-axis and their corresponding cumulative frequency on y-axis choosing a convenient scale.

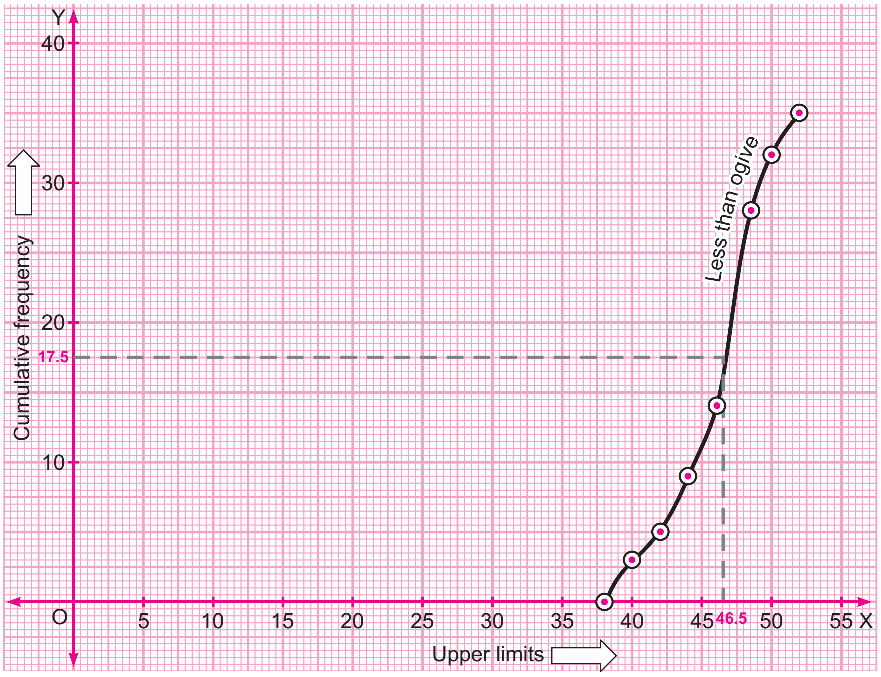


Fig. 6.4

Now, let us plot the points corresponding to the ordered pair given by (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) on a graph paper and join them by a freehand smooth curve.

Thus, the curve obtained is the less than type ogive.

Now, locate $\frac{n}{2} = \frac{35}{2} = 17.5$ on the y-axis,

We draw a line from this point parallel to x-axis cutting the curve at a point. From this point, draw a perpendicular line to the x-axis. The point of intersection of this perpendicular with the x-axis gives the median of the data. Here it is 46.5.

Let us make the following table in order to find median by using formula.

Weight (in kg)	**No. of Students frequency (f_i)**	**Cumulative frequency (cf)**
36 – 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	14
46 – 48	14	28
48 – 50	4	32
50 – 52	3	35
Total	$\Sigma f_i = 35$	

Here, $n = 35$, $\frac{n}{2} = \frac{35}{2} = 17.5$, cumulative frequency greater than $\frac{n}{2} = 17.5$ is 28 and corresponding class is 46–48. So median class is 46–48.

Now, we have $l = 46, \frac{n}{2} = 17.5, cf = 14, f = 14, h = 2$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2 = 46 + \frac{3.5}{14} \times 2 = 46 + \frac{7}{14} = 46 + 0.5 = 46.5$$

Hence, median is verified.

HOTS (Higher Order Thinking Skills)

1. The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class is 20 – 40 and 60 – 80 are missing. Find the missing frequencies.

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	32	f_2	19	120

Sol. Let the assumed mean $A = 50$ and $h = 20$.

Calculation of mean

Class interval	**Mid-values (x_i)**	**Frequency (f_i)**	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 – 20	10	17	– 2	– 34
20 – 40	30	f_1	– 1	$-f_1$
40 – 60	50	32	0	0
60 – 80	70	f_2	1	f_2
80 – 100	90	19	2	38
Total		$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i u_i = 4 - f_1 + f_2$

We have, $\Sigma f_i = 120$ [Given]

$$68 + f_1 + f_2 = 120$$

$$f_1 + f_2 = 52 \quad \text{...(i)}$$

Now, Mean $= 50$

$$\overline{X} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \qquad 50 = 50 + 20 \times \left\{\frac{4 - f_1 + f_2}{120}\right\}$$

$$50 = 50 + \frac{4 - f_1 + f_2}{6} \qquad 0 = \frac{4 - f_1 + f_2}{6}$$

$$f_1 - f_2 = 4 \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we get

$$f_1 + f_2 = 52$$

$$f_1 - f_2 = 4$$

$$2f_1 = 56 \qquad f_1 = 28$$

Putting the value of f_1 in equation (*i*), we get

$$28 + f_2 = 52 \qquad f_2 = 24$$

Hence, the missing frequencies f_1 is 28 and f_2 is 24.

2. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	Total
Frequency	5	x	20	15	y	5	60

Sol. Here, median $= 28.5$ and $n = 60$

Now, we have

Class interval	Frequency (f_i)	Cumulative frequency (cf)
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total	$\Sigma f_i = 60$	

Since the median is given to be 28.5, thus the median class is 20 – 30.

$$\frac{n}{2} = 30, l = 20, h = 10, cf = 5 + x \text{ and } f = 20$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10 \qquad 28.5 = 20 + \frac{25 - x}{20} \times 10$$

$$28.5 = 20 + \frac{25 - x}{2} \qquad 57 = 40 + 25 - x$$

$$57 = 65 - x \qquad x = 65 - 57 = 8$$

Also, $n = \Sigma f_i = 60$

$45 + x + y = 60$ $\qquad 45 + 8 + y = 60$ $\qquad [\because x = 8]$

$y = 60 - 53$ $\qquad y = 7$

Hence, $x = 8$ and $y = 7$.

Exercise

A. Multiple Choice Questions

Write the correct answer for each of the following:

1. The algebraic sum of the deviations of a frequency distribution from its mean is

(*a*) always positive (*b*) always negative (*c*) 0 (*d*) a non-zero number

2. The mean of a discrete frequency distribution $x_i / f_i; i = 1, 2, \ldots - n$ is given by

(*a*) $\dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ (*b*) $\dfrac{1}{n}\sum_{i=1}^{n} f_i x_i$ (*c*) $\dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} x_i}$ (*d*) $\dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} i}$

3. The mode of a frequency distribution can be determined graphically from

(*a*) Histogram (*b*) Frequency polygon (*c*) Frequency curve (*d*) Ogive

4. The median of a given frequency distribution is found graphically with the help of

(*a*) Bar graph (*b*) Histogram (*c*) Frequency polygon (*d*) Ogive

5. If the mean of the following distribution is 2.6, then the value of k is

x	1	2	3	4	5
y	k	5	8	1	2

(*a*) 3 (*b*) 4 (*c*) 2 (*d*) 5

6. If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\sum(f_i x_i - \bar{x})$ is equal to

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2

7. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are deviations from a of

(*a*) lower limits of the classes (*b*) upper limits of the classes

(*c*) mid-points of the classes (*d*) frequencies of the class marks

8. Consider the following distribution:

Marks Obtained	Numbers of students
More than or equal to 0	68
More than or equal to 10	53
More than or equal to 20	50
More than or equal to 30	45
More than or equal to 40	38
More than or equal to 50	25

The number of students having marks more than 29 but less than 40 is

(*a*) 38 (*b*) 45 (*c*) 7 (*d*) 13

9. The heights (in cm) of 100 students of a class is given in the following distribution:

Height (in cm)	150–155	155–160	160–165	165–170	170–175	175–180
Number of students	15	16	28	16	17	8

The number of students having height less than 165 cm is

(*a*) 28 (*b*) 16 (*c*) 75 (*d*) 59

10. For the following distribution:

Income (in ₹)	Number of families
Below 20,000	16
Below 40,000	25
Below 60,000	32
Below 80,000	38
Below 1, 00, 000	45

the modal class is

(*a*) 40,000–60,000 (*b*) 60,000–80,000 (*c*) 0–20,000 (*d*) 20,000–40,000

11. Consider the data:

Class	50–70	70–90	90–110	110–130	130–150	150–170
Frequency	15	21	32	19	8	5

The difference of the upper limit of the median class and the lower limit of the modal class is

(*a*) 0 (*b*) 20 (*c*) 19 (*d*) 21

12. For the following distribution:

Class	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Frequency	16	12	20	18	9	15	10

the sum of lower limits of the median class and modal class is

(*a*) 5 (*b*) 15 (*c*) 25 (*d*) 30

B. Short Answer Questions Type–I

1. Which measure of central tendency is given by the *x*-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive?

2. What is the empirical relation between mean, median and mode?

3. Find the class marks of classes 15–35 and 20–40.

4. Write the median class of the following distribution:

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	14	6	8	20	15	12	9

5. Write the median class of the following distribution:

Classes	100–150	150–200	200–250	250–300	300–350	350–400	400–450	450–500	500–550
Frequency	49	62	33	39	85	45	61	55	24

6. Write the modal class for the following frequency distribution:

Class	15–25	25–35	35–45	45–55	55–65	65–75
Frequency	39	42	26	30	48	22

7. Write the modal class for the following frequency distribution:

Class	1–4	5–8	9–12	13–16	17–20	21–24
Frequency	3	9	1	12	8	9

8. What is the value of the median of the data represented by the following graph of less than Ogive and more than ogive?

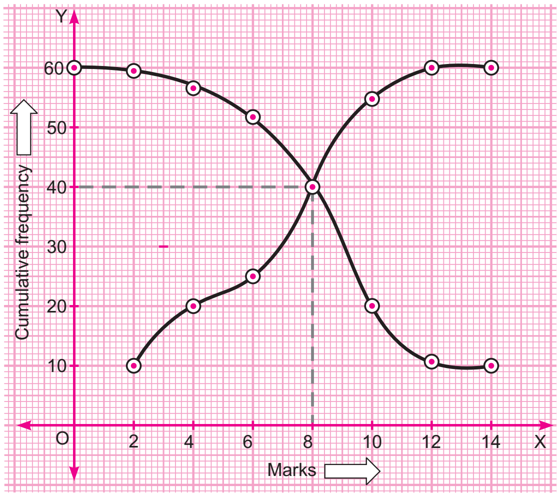


Fig. 6.5

9. The times, in seconds, taken by 150 athletes to run a 110m hurdle race are tabulated below:

Class	13.8–14	14–14.2	14.2–14.4	14.4–14.6	14.6–14.8	14.8–15
Frequency	2	4	5	71	48	20

Find the number of athletes who completed the race is less than 14.6 seconds.

State true or false for the following statements and justify your answer:

10. The median of an ungrouped data and the median calculated when the same data is grouped are always the same.

11. The mean, median and mode of grouped data are always different.

12. The median class and modal class of grouped data can never coincide.

C. Short Answer Questions Type–II

1. If the mean of the following data is 20.6, find the value of p.

x	10	15	p	25	35
f	3	10	25	7	5

2. Find the value of p, if the mean of the following distribution is 20.

x	15	17	19	$20+p$	23
f	2	3	4	5	6

3. The arithmetic mean of the following data is 14. Find the value of k.

x	5	10	15	20	25
f_i	7	k	8	4	5

4. If the mean of the following data is 18.75, find the value of p.

x_i	10	15	p	25	30
f_i	5	10	7	8	2

5. The following table gives the number of children of 250 families in a town:

No. of Children	0	1	2	3	4	5	6
No. of Families	15	24	29	46	54	43	39

Find the average number of children per family.

6. Find the mean age of 100 residents of a town from the following data:

Age equal and above (in years)	0	10	20	30	40	50	60	70
No. of Persons	100	90	75	50	25	15	5	0

7. For the following distribution, calculate mean:

Class	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Frequency	14	22	16	6	5	3	4

8. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency f_1 and f_2.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	5	f_1	10	f_2	7	8

9. If the mean of the following distribution is 27, find the value of p.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	8	p	12	13	10

10. The table below shows the daily expenditure on food of 25 households in a locality.

Daily Expenditure (in ₹)	100–150	150–200	200–250	250–300	300–350
No. of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

11. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table:

Number of seats	100–104	104–108	108–112	112–116	116–120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights.

12. The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below:

Mileage (km/*l*)	10–12	12–14	14–16	16–18
Number of Cars	7	12	18	13

Find the mean mileage. The manufacturer claimed that the mileage of the model was 16 km/l. Do you agree with this claim?

13. The following table shows the cumulative frequency distribution of marks of 800 students in an examination:

Marks	**Number of students**
Below10	10
Below 20	50
Below 30	130
Below 40	270
Below 50	440
Below 60	570
Below 70	670
Below 80	740
Below 90	780
Below 100	800

Construct a frequency distribution table for the data above.

14. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
No. of patients	60	42	55	70	53	20

Form:

(*i*) Less than type cumulative frequency distribution.

(*ii*) More than type cumulative frequency distribution.

15. If the median of the following distribution is 28.5, find the missing frequencies:

Class Interval	0–10	10–20	20–30	30–40	40–50	50–60	Total
Frequency	5	f_1	20	15	f_2	5	60

16. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Age (in years)	0–10	10–20	20–30	30–40	40–50
Number of Persons	5	25	f	18	7

17. Calculate the median from the following data:

Rent (in ₹)	1500 – 2500	2500 – 3500	3500 – 4500	4500 – 5500	5500 – 6500	6500 – 7500	7500 – 8500	8500 – 9500
Number of Tenants	8	10	15	25	40	20	15	7

18. The weight of coffee in 70 packets are shown in the following table:

Weight (in g)	Number of Packets
200–201	12
201–202	26
202–203	20
203–204	9
204–205	2
205–206	1

Determine the modal weight.

19. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0–20	20–40	40–60	60–80	80–100	100–120
No. of components	10	35	52	61	38	29

Determine the modal lifetimes of the components.

20. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5–15	15–25	25–35	35–45	45–55	55–65
No.of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

D. Long Answer Questions

Find the mean, mode and median of the following frequency distribution: (1–4)

1.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	4	4	7	10	12	8	5

2.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	8	8	14	22	30	8	10

3.

Class	0–50	50–100	100–150	150–200	200–250	250–300	300–350
Frequency	2	3	5	6	5	3	1

4.

Class	100–120	120–140	140–160	160–180	180–200
Frequency	12	14	8	6	10

5. Draw 'less than' ogive and 'more than' ogive for the following distribution and hence find its median.

Class	20–30	30–40	40–50	50–60	60–70	70–80	80–90
Frequency	10	8	12	24	6	25	15

6. The following is the frequency distribution of duration for 100 calls made on a mobile phone:

Duration (in seconds)	Number of calls
95–125	14
125–155	22
155–185	28
185–215	21
215–245	15

Calculate the average duration (in sec.) of a call and also find the median from a cumulative frequency curve.

7. 50 students enter for a school javelin throw competition. The distance (in metres) thrown are recorded below:

Distance (in m)	0–20	20–40	40–60	60–80	80–100
No. of students	6	11	17	12	4

(*i*) Construct a cumulative frequency table.

(*ii*) Draw a cumulative frequency (less than type) curve and calculate the median distance thrown by using this curve.

(*iii*) Calculate the median distance by using the formula for median.

(*iv*) Are the median distance calculated in (*ii*) and (*iii*) same?

8. The annual rainfall record of a city of 66 days is given in the following table:

Rainfall (in cm)	0–10	10–20	20–30	30–40	40–50	50–60
Number of days	22	10	8	15	5	6

Calculate the median rainfall using ogive (of more than type and of less than type).

9. Size of agricultural holdings in a survey of 200 families is given in the following table:

Size of agricultural holdings (in ha)	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Number of days	10	15	30	80	40	20	5

Compute median and mode size of the holdings.

10. The annual profits earned by 30 shops of a shopping complex in locality give rise to the following distribution:

Profit (in lakhs in ₹)	**Number of shops (frequency)**
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the above data and hence obtain the median.

11. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weights (in kg)	**Number of students**
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence ,obtain the median weight from the graph and verify the result by using the formula.

12. The following distribution gives the daily income of 50 workers of a factory:

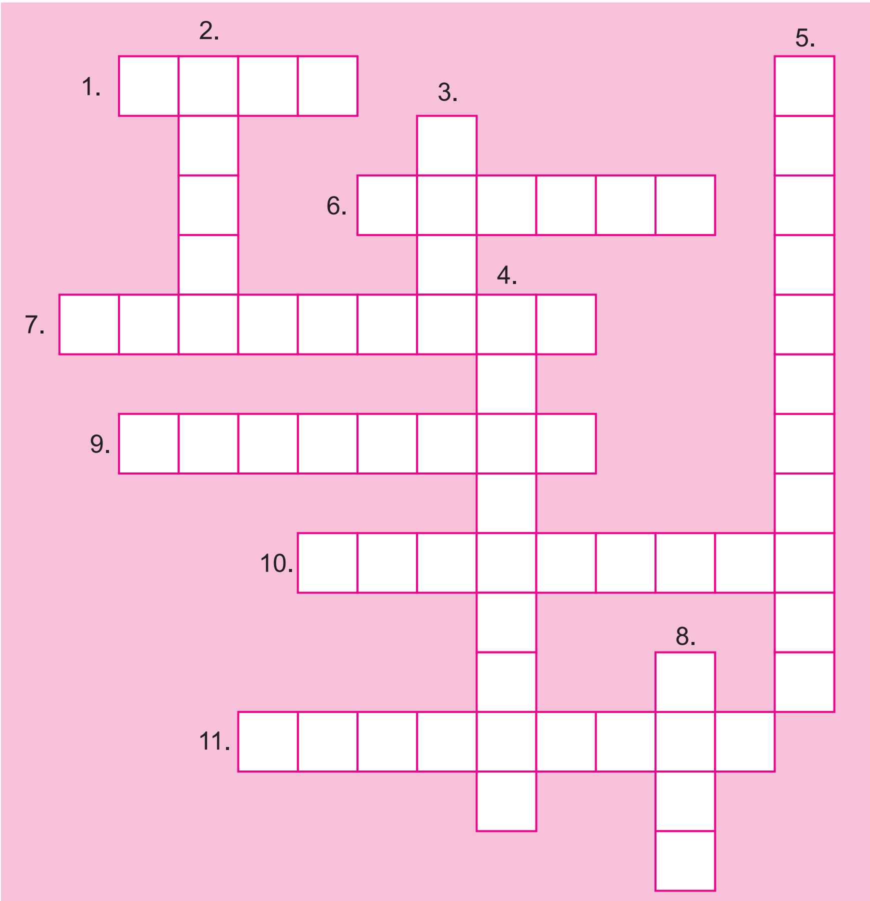
Daily income (in ₹)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive. Find the median from this ogive.

Formative Assessment

Activity: 1

- **Solve the following crossword puzzle, hints are given alongside:**



Across

1. Most frequently occurring observation of data.
6. The positional mid value of the observation in a data.
7. Number of times a particular observation occurs.
9. The group of numbers formed to place observations like 10–20, etc. is called class __________.
10. Difference between the two limits of a class interval.
11. The _________ relation relates mean, median and mode of data.

Down

2. The graphical representation of cumulative frequency distribution.
3. Average of data.
4. Mid value of a class interval.
5. A number which is considered to simplify the calculation of mean after taking deviations.
8. Numerical information collected to make certain studies.

Activity: 2

- **Collect information regarding the number of hours your 25 friends spent in (i) self-study and (ii) watching TV or playing. Prepare a table given below for the information collected.**

Name of friend	Number of hours spent in self-study	Number of hours spent in watching TV or playing
(i)		
(ii)		
(iii)		
(iv)		

Note: The data should be collected at least for 25 children and for a particular age group, say 10–15 years or 12–18 years.

Now present the above data in grouped form and prepare two tables.

I. For self-study	
Number of hours	Number of students
0 – 2	
2 – 4	
4 – 6	
6 – 8	
8 – 10	
10 – 12	

II. For watching TV and playing	
Number of hours	Number of students
0 – 2	
2 – 4	
4 – 6	
6 – 8	
8 – 10	
10 – 12	

Note: You may use different class intervals for the tables. Calculate the mean, median and mode for each table separately.

Suggested Activities

1. Collect the marks obtained by different students of a particular class in Mathematics and repeat the above activity.
2. Collect the daily maximum temperatures recorded for a period of at least 30 days in your city and repeat the above activity.
3. Collect information regarding (*a*) number of children (*b*) number of vehicles used by at least 25 families of your locality or in your relation and repeat the above activity.

Hands on Activity (Math Lab Activity)

Tabular and Graphical Representation of Data

Objective

Analysis of a language text, using graphical and pie chart techniques.

How to Proceed

1. Students should select any paragraph containing approximately 300 words from any source. e.g., newspaper, magazine, textbook, etc.
2. Now read every word and obtain a frequency table for each letter of the alphabet as follows:

Table – 1

Letter	Tally Marks	Frequency
A		
B		
C		
.		
.		
.		
.		
Z		

3. Note down the number of two-letter words, three-letter words, so on and obtain a frequency table as follows:

Table – 2

Number of Words With	Tally Marks	Frequency
2 Letter		
3 Letter		
.		
.		
.		
.		
.		

Investigate the following:

From Table 1

1. What is the most frequently occurring letter?
2. What is the least frequently occurring letter?
3. Compare the frequency of vowels.
4. Which vowel is most commonly used?
5. Which vowel has the least frequency?
6. Make a pie chart of the vowels *a, e, i, o, u* and remaining letters. (The pie chart will thus have 6 sectors.)
7. Compare the percentage of vowels with that of consonants in the given text.

From Table 2

1. Compare the frequency of two letter words, three letter words, ... and so on.
2. Make a pie chart. Note any interesting patterns.

Seminar

Students should make presentations on following topics and discuss them in the class in the presence of teachers.

1. Different types of graphical presentation of data, with examples from daily life (may use news paper cuttings also).
2. Measures of central tendency.
3. Why do we need deviation and step deviation methods?

Multiple Choice Questions

Tick the correct answer for each of the following:

1. While computing mean of a grouped data, we assume that the frequencies are
   (*a*) centered at the lower limits of the classes (*b*) centered at the upper limits of the classes
   (*c*) centered at the class marks of the classes (*d*) evenly distributed over all the classes.
2. The graphical representation of a cumulative frequency distribution is called
   (*a*) Bar graph (*b*) Histogram (*c*) Frequency polygon (*d*) an Ogive

3. Construction of a cumulative frequency table is useful in determining the

(*a*) mean (*b*) median (*c*) mode (*d*) all of the above

4. The class mark of the class 15.5–20.5 is

(*a*) 15.5 (*b*) 20.5 (*c*) 18 (*d*) 5

5. If x_i's are the mid-points of the class intervals of a grouped data f_i' s are the corresponding frequencies and $\bar{x}$ is the mean, then $\Sigma(f_i x_i - \bar{x})$ is equal to

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) 2

6. In the formula, Mode $= l + \left(\frac{f_i - f_o}{2f_1 - f_o - f_2}\right) \times h$, f_2 is

(*a*) frequency of the modal class

(*b*) frequency of the second class

(*c*) frequency of the class preceding the modal class

(*d*) frequency of the class succeeding the modal class

7. Consider the following distribution:

Marks Obtained	Number of Students
Less than 10	5
Less than 20	12
Less than 30	22
Less than 40	29
Less than 50	38
Less than 60	47

The frequency of the class 50–60 is

(*a*) 9 (*b*) 10 (*c*) 38 (*d*) 47

8. For the following distribution:

Class	0–8	8–16	16–24	24–32	32–40
Frequency	12	26	10	9	15

The sum of upper limits of the median class and modal class is

(*a*) 24 (*b*) 40 (*c*) 32 (*d*) 16

9. Consider the following distribution:

Marks	Number of Students
More than or equal to 0	53
More than or equal to 20	51
More than or equal to 40	45
More than or equal to 60	37
More than or equal to 80	25

The modal class is

(*a*) 80–100 (*b*) 60–80 (*c*) 40–60 (*d*) 0–20

10. Consider the following frequency distribution:

Class	0–15	15–30	30–45	45–60	60–75
Frequency	15	12	18	16	9

The difference of the upper limit of the median class and the lower limit of the modal class is

(*a*) 0 (*b*) 15 (*c*) 10 (*d*) 5

11. The runs scored by a batsman in 35 different matches are given below:

Runs Scored	0–15	15–30	30–45	45–60	60–75	75–90
Number of Matches	5	7	4	8	8	3

The number of matches in which the batsman scored less than 60 runs are

(*a*) 16 (*b*) 24 (*c*) 8 (*d*) 19

Rapid Fire Quiz

State which of the following statements are true (T) or false (F).

1. The mean, median and mode of a data can never coincide.

2. The modal class and median class of a data may be different.

3. An ogive is a graphical representation of a grouped frequency distribution.

4. An ogive helps us in determining the median of the data.

5. The median of ungrouped data and the median calculated when the same data is grouped are always the same.

6. The ordinate of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its median.

7. While computing the mean of grouped data, we assume that the frequencies are centered at the class marks of the classes.

8. A cumulative frequency table is useful in determining the mode.

9. The value of the mode of a grouped data is always greater than the mean of the same data.

Match the Columns

Consider the following distribution:

Height (cm)	**Number of Students**
135–140	3
140–145	9
145–150	22
150–155	15
155–160	8
160–165	5
165–170	2

On the basis of the above data, match the following columns:

Column I	Column II
(*i*) Lower limit of median class	(*a*) 12
(*ii*) Upper limit of modal class	(*b*) 57
(*iii*) Number of students with heights less than 160 cm	(*c*) 5
(*iv*) Number of students with heights more than or equal to 150 cm	(*d*) 145
(*v*) Number of students in the median class	(*e*) 150
(*vi*) Cumulative frequency of the class preceding the modal class	(*f*) 15
(*vii*) Class size	(*g*) 30
(*viii*) Number of students in the class succeeding the modal class	(*h*) 22

Group Discussion

Divide the whole class into small groups and ask them to discuss the choice of different measures of central tendency in different situations, *i.e.*, which measure is more appropriate in a given situation.

The situations may include, finding average income, putting shirts of different sizes in a shop, dividing a group in two parts on the basis of the heights of members of group, etc.

(**Note:** The students may discuss it on the basis of the activities done by them.)

Project Work

Objective

To apply the knowledge of statistics in real life.

Form group of students with about 5-8 students in each group. Each group is supposed to work as a team for the completion of project. Some members can take responsibility of gathering required information for the project, other students can work for making a rough draft from the collected information. All members of the group should discuss the draft and give inputs for final presentation. After finalizing, few members can write the report.

Suggested Projects

- Study on the types of works that 20 selected persons do.
- Study on the most popular newspaper in a locality.
- Study on the most popular TV channel in a housing society.
- Effect of advertisements in day-to-day life.

Oral Questions

1. What is the relationship between the mean, median and mode of observations?
2. Can the mean, median and mode of data coincide?
3. What does the abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves represent?

4. What do we call the graphical representation of the cumulative frequency distribution?

5. In the formula, Median $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$, what does the letter '$h$' represent?

6. How do we calculate the mode of a grouped data?

7. How do we calculate the mean of a grouped data by the assumed mean method?

8. Which measure of central tendency should be avoided if the extreme values affect the data?

9. Is the mean of the ungrouped data always same as the mean calculated when the same data is grouped? Give reasons.

10. How will you define the 'median' of a data?

Class Worksheet

1. Tick the correct answer for each of the following:

(*i*) Which of the following is not a measure of central tendency?

(*a*) Class mark (*b*) Mean (*c*) Median (*d*) Mode

(*ii*) In the formula, $\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$ for finding the mean of a grouped data, d_i's are deviations from a of

(*a*) frequencies of the class marks (*b*) lower limits of the classes

(*c*) upper limits of the classes (*d*) mid-points of the classes

(*iii*) An Ogive is useful in determining the

(*a*) mean (*b*) median (*c*) mode (*d*) all of the above

(*iv*) In the following distribution, the frequency of the class 20–40 is

Age (years)	Number of Persons
More than or equal to 0	83
More than or equal to 20	55
More than or equal to 40	32
More than or equal to 60	19
More than or equal to 80	8

(*a*) 23 (*b*) 28 (*c*) 55 (*d*) 32

(*v*) The time, in seconds, taken by 180 athletes to run a 110 m hurdle race are tabulated below:

Class	13.6–13.8	13.8–14	14–14.2	14.2–14.4	14.4–14.6	14.6–14.8
Frequency	8	11	18	20	75	48

The number of athletes who completed the race in less than 14.2 seconds is

(*a*) 20 (*b*) 37 (*c*) 57 (*d*) 38

2. Write true or false for the following statements and justify your answer:

(*i*) The mean, median and mode of a grouped data are always different.

(*ii*) The median of an ungrouped data and the median calculated when the same data is grouped are always the same.

3. In an ungrouped distribution $\Sigma fx = 180$ and $\Sigma f = 9$. Find $\bar{x}$.

4. In the class interval 50-55,

   Lower limit = ______________

   Upper limit = ______________

   Class Mark = ______________

5. If $d_i = x_i - a$, then $\bar{x}$ = ______________

6. If $u_i = (x_i - a)/h$, then $\bar{x}$ = ______________

7. Complete the following table:

Class Interval	Observation x	Frequency f	$u = \frac{(x-350)}{100}$	fu
0 – 100	50	2	–3	–6
100 – 200	–	8	–	–
200 – 300	250	12	–	–
300 – 400	–	20	0	0
400 – 500	–	5	–	–
500 – 600	550	–	–	–
		50		

$\bar{x}$ = ______________

8. Fill in the blanks.

   (*i*) In an ungrouped data, the value which occurs maximum number of times is called ______________ of the data.

   (*ii*) To find the mode of a grouped data, the size of the classes is ______________ (uniform/ non-uniform).

   (*iii*) In a grouped distribution, the class having largest frequency is known as ______________ class.

   (*iv*) The relationship between mean, median and mode is ______________ median = 2 ______________ + ______________.

   (*v*) On an ogive, point *A* whose *y*-coordinate is n/2 (half the total number of entries) has its *x*-coordinate equal to ______________ of the data.

   (*vi*) Two ogives, less than and more than type for the same data intersect at the point *P*. The *y* coordinate of *P* represents ______________.

9. In the given formula: Mode $= l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$

   What does f_2 stand for?

Paper Pen Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Tick the correct answer for each of the following:

   (*i*) The class marks of the class 18–22 is 1

   (*a*) 4 (*b*) 18 (*c*) 22 (*d*) 20

(*ii*) In the formula $\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$ for finding the mean of a grouped frequency distribution, u_i = 1

(*a*) $\frac{x_i + a}{h}$ (*b*) $h(x_i - a)$ (*c*) $\frac{x_i - a}{h}$ (*d*) $\frac{a - x_i}{h}$

(*iii*) If x_i's are the mid-points of the class intervals of a grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\Sigma(f_i x_i - \bar{x})$ is equal to 1

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) 2

(*iv*) The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its 1

(*a*) mean (*b*) median (*c*) mode (*d*) all of these

(*v*) If for any distribution $\Sigma f_i = 18$, $\Sigma f_i x_i = 2p + 24$ and mean is 2, then p is equal to 1

(*a*) 3 (*b*) 4 (*c*) 8 (*d*) 6

(*vi*) Consider the following distribution: 2

Marks Obtained	Number of Students
Below 20	7
Below 40	18
Below 60	33
Below 80	47
Below 100	60

The sum of the lower limits of the median class and modal class is

(*a*) 100 (*b*) 120 (*c*) 20 (*d*) 80

2. Write true or false for the following statements and justify your answer:

(*i*) The median class and modal class of grouped data will always be different.

(*ii*) Consider the distribution: $2 \times 2 = 4$

Weight (kg)	Number of Persons
Less than 20	8
Less than 40	19
Less than 60	32
Less than 80	57
Less than 100	72

The number of persons with weights between 60–80 kg is 32.

3. (*i*) Find the unknown entries a, b, c, d, e, f in the following distribution of heights of students in a class:

Height (cm)	Frequency	Cumulative Frequency
150–155	12	a
155–160	b	25
160–165	10	c
165–170	d	43
170–175	e	48
175–180	2	f
Total	50	

(*ii*) The monthly income of 100 families are given below: $3 \times 2 = 6$

Income (₹)	Number of Families
0–5,000	8
5,000–10,000	26
10,000–15,000	41
15,000–20,000	16
20,000–25,000	3
25,000–30,000	3
30,000–35,000	2
35,000–40,000	1

Calculate the modal income.

4. (*i*) Determine the mean and median of the following distribution:

Marks	Number of Students
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

(*ii*) During the medical check-up of 35 students of a class their weights were recorded as follows: $4 \times 2 = 8$

Weight (in kg)	Number of Students
38 – 40	3
40 – 42	2
42 – 44	4
44 – 46	5
46 – 48	14
48 – 50	4
50 – 52	3

Draw a less than type and a more than type ogive from the given data. Hence, obtain the median weight from the graph.

Design of CBSE Sample Question Paper
Mathematics
Class X
Summative Assessment – I

Type of Question	Marks per Question	Total No. of Questions	Total Marks
M.C.Q.	1	10	10
SA-I	2	8	16
SA-II	3	10	30
LA	4	6	24
Total		**34**	**80**

Blue Print
CBSE Sample Question Paper
Mathematics SA-I
Summative Assessment – I

Topic / Unit	MCQ	SA(I)	SA(II)	LA	Total
Number System	2(2)	1(2)	2(6)	–	5(10)
Algebra	2(2)	2(4)	2(6)	2(8)	8(20)
Geometry	1(1)	2(4)	2(6)	1(4)	6(15)
Trigonometry	4(4)	1(2)	2(6)	2(8)	9(20)
Statistics	1(1)	2(4)	2(6)	1(4)	6(15)
Total	**10(10)**	**8(16)**	**10(30)**	**6(24)**	**34(80)**

Note: Marks are within brackets.

CBSE Sample Question Paper

Mathematics, (Solved) –1

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions:

1. ***All*** *questions are* ***compulsory****.*
2. *The question paper consists of 34 questions divided into 4 sections, A, B, C and D.* ***Section - A*** *comprises of 10 questions of* ***1 mark*** *each.* ***Section - B*** *comprises of 8 questions of* ***2 marks*** *each.* ***Section-C*** *comprises of 10 questions of* ***3 marks*** *each and* ***Section-D*** *comprises of 6 questions of* ***4 marks*** *each.*
3. *Question numbers 1 to 10 in Section-A are multiple choice questions where you are to select one correct option out of the given four.*
4. *There is no overall choice. However, internal choice has been provided in 1 question of* ***two marks,*** *3 questions of* ***three marks*** *each and 2 questions of* ***four marks*** *each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is* ***not*** *permitted.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. Euclid's Division Lemma states that for any two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where, r must satisfy.

(*a*) $1 < r < b$ (*b*) $0 < r < b$

(*c*) $0 \leq r < b$ (*d*) $0 < r \leq b$

2. In Fig. 1, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is

(*a*) 4 (*b*) 1

(*c*) 2 (*d*) 3

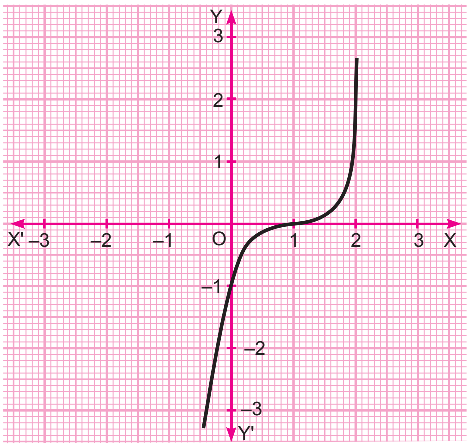


Fig. 1

3. In Fig. 2, if $DE \parallel BC$, then x equals

(*a*) 6 cm (*b*) 8 cm

(*c*) 10 cm (*d*) 12.5 cm

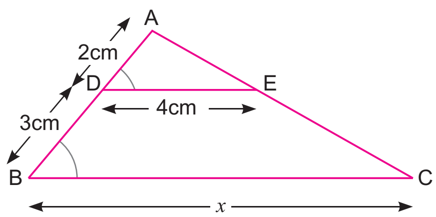


Fig. 2

4. If $\sin 3\theta = \cos(\theta - 6°)$, where (3θ) and $(\theta - 6)°$ are both acute angles, then the value of θ is

(*a*) 18° (*b*) 24°

(*c*) 36° (*d*) 30°

5. Given that $\tan\theta = \frac{1}{\sqrt{3}}$, the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is

(*a*) –1 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) $-\frac{1}{2}$

6. In Fig. 3, $AD = 4$ cm, $BD = 3$ cm and $CB = 12$ cm, then $\cot\theta$ equals

(a) $\frac{3}{4}$ (b) $\frac{5}{12}$

(c) $\frac{4}{3}$ (d) $\frac{12}{5}$

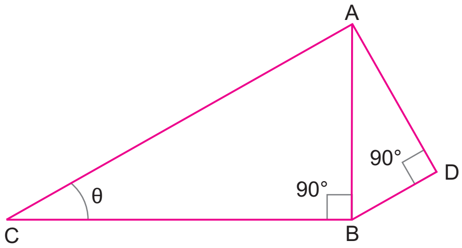


Fig. 3

7. The decimal expansion of $\frac{147}{120}$ will terminate after how many places of decimal?

(a) 1 (b) 2 (c) 3 (d) will not terminate

8. The pair of linear equations $3x + 2y = 5$; $2x - 3y = 7$ has

(a) One solution (b) Two solutions (c) Many solutions (d) No solution

9. If $\sec A = \operatorname{cosec} B = \frac{15}{7}$, then $A + B$ is equal to

(a) zero (b) 90° (c) <90° (d) >90°

10. For a given data with 70 observations, the 'less than ogive' and 'more than ogive' intersect at (20.5, 35). The median of the data is

(a) 20 (b) 35 (c) 70 (d) 20.5

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your answer.

12. Can $(x–2)$ be the remainder on division of a polynomial $p(x)$ by $(2x+ 3)$? Justify your answer.

13. In Fig. 4, $ABCD$ is a rectangle. Find the values of x and y.

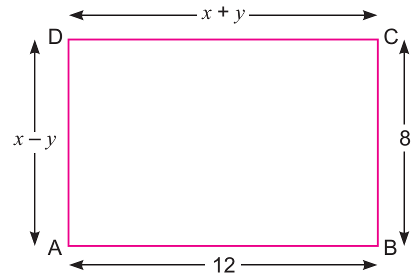


Fig. 4

14. If $7\sin^2\theta + 3\cos^2\theta = 4$, show that $\tan\theta = \frac{1}{\sqrt{3}}$.

OR

If $\cot = \frac{15}{8}$, evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

15. In Fig. 5, $DE || AC$ and $DF || AE$. Prove that $\frac{FE}{BF} = \frac{EC}{BE}$.

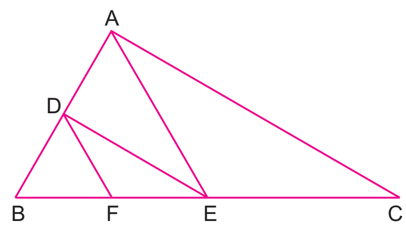


Fig. 5

16. In Fig. 6, $AD \quad BC$ and $BD = \frac{1}{3}CD$. Prove that $2CA^2 = 2AB^2 + BC^2$.

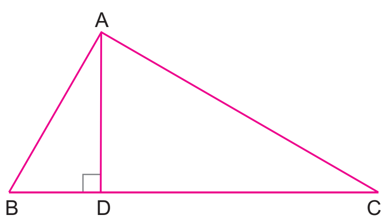


Fig. 6

17. The following distribution gives the daily income of 50 workers of a factory:

Daily income	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Write the above distribution as less than type cumulative frequency distribution.

18. Find the mode of the following distribution of marks obtained by 80 students:

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	6	10	12	32	20

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$ where q is a positive integer.

20. Prove that $\frac{2\sqrt{3}}{5}$ is irrational.

OR

Prove that $(5 - \sqrt{2})$ is irrational.

21. A person can row a boat at the rate of 5 km/hour in still water. He takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

OR

In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for each wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

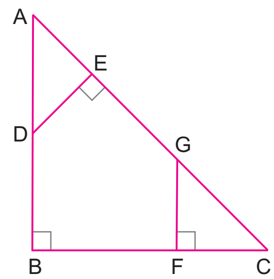


Fig. 7

22. If α, β are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are (2α) and (2β).

23. Prove that $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$.

24. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

25. In Fig. 7, AB BC, FG BC, and DE AC. Prove that $\Delta ADE \sim \Delta GCF$.

26. In Fig. 8, ΔABC and ΔDBC are on the same base BC and on opposite sides of BC and O is the point of intersection of AD and BC.

Prove that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$.

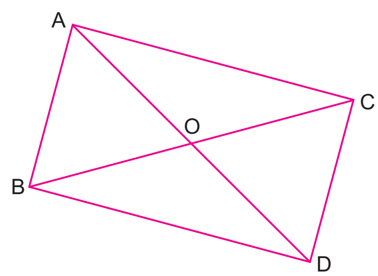


Fig. 8

27. Find mean of the following frequency distribution, using step-deviation method:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	7	12	13	10	8

OR

The mean of the following frequency distribution is 25. Find the value of p.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	2	3	5	3	p

28. Find the median of the following data.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	5	3	4	3	3	4	7	9	7	8

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Find other zeroes of polynomial $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$ if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

30. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

OR

Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

31. Prove that : $\dfrac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{\cos\theta}{1 - \sin\theta}$

OR

Evaluate: $\dfrac{\sec\theta\,\text{cosec}\,(90° - \theta) - \tan\theta\cot(90° - \theta) + \sin^2 55° + \sin^2 35°}{\tan 10°\tan 20°\tan 60°\tan 70°\tan 80°}$

32. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \dfrac{p^2 - 1}{P^2 + 1}$.

33. Draw the graphs of following equations: $2x - y = 1$, $x + 2y = 13$ and

(*i*) find the solution of the equations from the graph.

(*ii*) shade the triangular region formed by the lines and the y-axis.

34. The following table gives the production yield per hectare of wheat of 100 farms of a village:

Production yield in kg/hectare	50–55	55–60	60–65	65–70	70–75	75–80
Number of farms	2	8	12	24	38	16

Change the above distribution to more than type distribution and draw its ogive.

Solutions

Section – A

1.	(*c*)	
2.	(*b*)	
3.	(*c*)	$\because\ DE \parallel BC,\ \Delta ADE \sim \Delta ABC \quad \dfrac{AD}{AB} = \dfrac{DE}{BC}$ or $\dfrac{2}{5} = \dfrac{4}{x}$ or $x = 10$ cm
4.	(*b*)	$\cos(90 - 3\theta) = \cos(\theta - 6) \quad 90 - 3\theta = \theta - 6$ or $4\theta = 96$ or $\theta = 24°$
5.	(*c*)	$\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \dfrac{1 + \cot^2\theta - 1 - \tan^2\theta}{1 + \cot^2\theta + 1 + \tan^2\theta} = \dfrac{(\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{2 + (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{8}{3} \times \dfrac{3}{16} = \dfrac{1}{2}$
6.	(*d*)	$AB = \sqrt{AD^2 + DB^2} = \sqrt{4^2 + 3^2} = 5$ cm $\quad \cot\theta = \dfrac{BC}{AB} = \dfrac{12}{5}$
7.	(*d*)	

8.	(*a*)	$\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{-2}{3}$, $\because$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ it has a unique solution
9.	(*b*)	$\sec A = \operatorname{cosec} B$ $\operatorname{cosec}(90 - A) = \operatorname{cosec} B$ $90 - A = B$ or $A + B = 90°$
10.	(*d*)	

1×10=10

Section – B

11. $7 \times 5 \times 3 \times 2 + 3 = 3\,(7 \times 5 \times 2 + 1)$

$= 3 \times 71$ ...(*i*) **(1)**

By Fundamental Theorem of Arithmetic, every composite number can be expressed as product of primes in a unique way, apart from the order of factors.

(*i*) is a composite number **(1)**

12. In case of division of a polynomial by another polynomial, the degree of remainder (polynomial) is always less than that of divisor. **(1)**

$(x - 2)$ cannot be the remainder when $p(x)$ is divided by $(2x+3)$ as degree is same. **(1)**

13. Opposite sides of a rectangle are equal

$x + y = 12$ ...(*i*) and $x - y = 8$ ...(*ii*) **(1)**

Adding (*i*) and (*ii*), we get $2x = 20$ or $x = 10$ **(½)**

and $y = 2$

$x = 10, y = 2$ **(½)**

14. $7\sin^2\theta + 3\cos^2\theta = 4$ or $3(\sin^2\theta + \cos^2\theta) + 4\sin^2\theta = 4$ **(1)**

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \frac{1}{2}$ $\theta = 30°$ **(½)**

$\tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$ **(½)**

OR

$\cot\theta = \frac{15}{8}$ (given)

Given expression $= \frac{2(1 + \sin\theta)(1 - \sin\theta)}{2(1 + \cos\theta)(1 - \cos\theta)} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$ **(1)**

$= \left(\frac{15}{8}\right)^2 = \frac{225}{64}$ **(1)**

15. $DE \| AC$ $\frac{BE}{EC} = \frac{BD}{DA}$ ...(*i*) (By BPT) **(½)**

Similarly, $DF \| AE$ $\frac{BF}{EF} = \frac{BD}{DA}$ ...(*ii*) **(½)**

From (*i*) and (*ii*), $\frac{BE}{EC} = \frac{BF}{EF}$ or $\frac{CE}{BE} = \frac{FE}{BF}$ **(1)**

16. Let $BD = x$

In right triangle ADC, $CD = 3x$

$$CA^2 = CD^2 + AD^2 \quad ...(i)$$

and in right ΔABD, $AB^2 = AD^2 + BD^2$

$$AD^2 = AB^2 - BD^2 \quad ...(ii)$$ **(½ + ½)**

Substituting (ii) in (i),

$$CA^2 = CD^2 + AB^2 - BD^2$$

$$CA^2 = 9x^2 + AB^2 - x^2$$

or $2CA^2 = 2AB^2 + 2(9x^2 - x^2) = 2AB^2 + BC^2$ $(\because BC = 4x)$ **(1)**

$$2CA^2 = 2AB^2 + BC^2$$

17. **(2)**

Daily Income	**Less than**				
	120	140	160	180	200
Number of workers	12	26	34	40	50

18. Modal class = 30 – 40 **(½)**

$$\text{Mode} = 30 + \frac{32-12}{64-32} \times 10 = 30 + 6.25 = 36.25$$ **(1+½)**

Section – C

19. Let a be a positive odd integer

By Euclid's Division algorithm $a = 4q + r$

where q, r are positive integers and $0 \le r < 4$ **(1)**

$a = 4q$ or $4q+1$ or $4q+2$ or $4q+3$ **(½)**

But $4q$ and $4q+2$ are both even **(½)**

a is of the form $4q+1$ or $4q+3$ **(1)**

20. Let $\frac{2\sqrt{3}}{5} = x$ where x is a rational number

$2\sqrt{3} = 5x$ or $\sqrt{3} = \frac{5x}{2}$ ...(i) **(1)**

As x is a rational number, so is $\frac{5x}{2}$ **(½)**

$\sqrt{3}$ is also rational which is a contradiction as $\sqrt{3}$ is an irrational **(1)**

$\frac{2\sqrt{3}}{5}$ is irrational. **(½)**

OR

Let $5 - \sqrt{2} = y$, where y is a rational number

$5 - y = \sqrt{2}$ ...(i) **1**

As y is a rational number, so is $5 - y$ **(½)**

From (i), $\sqrt{2}$ is also rational which is a contradiction as $\sqrt{2}$ is irrational **1**

$5 - \sqrt{2}$ is irrational **(½)**

21. Let the speed of stream be x km/h

Speed of the boat rowing

upstream = $(5 - x)$ km/hour (½)

downstream = $(5 + x)$ km/h (½)

According to the question,

$$\frac{40}{5-x} = \frac{3 \times 40}{5+x}$$ (1)

$(5 + x) = 3(5 - x)$ (½)

$4x = 10 \qquad x = 2.5$ (½)

Speed of the stream = 2.5 km/h

OR

Let the number of correct answers be x.

Wrong answers are $(120 - x)$ in number. (½)

$$x - \frac{1}{2}(120 - x) = 90$$ (1)

$$\frac{3x}{2} = 150 \qquad x = 100$$ (1)

The number of correctly answered questions = 100 (½)

22. $p(x) = x^2 - 2x - 15$ ....(i)

As α, β are zeroes of (i), $\alpha + \beta = 2$ and $\alpha\beta = -15$ (½)

zeroes of the required polynomial are 2α and 2β (½)

Sum of zeroes = $2(\alpha + \beta) = 4$

Product of zeroes = $(2\alpha)(2\beta) = 4(\alpha\beta) = 4(-15) = -60$ (1)

The required polynomial is $x^2 - 4x - 60$ (1)

23. LHS can be written as $\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$ (½)

$$= \frac{(1 - \sin^2\theta)(1 - \cos^2\theta)}{\sin\theta\cos\theta} = \sin\theta.\cos\theta$$ (1)

$$= \frac{\sin\theta.\cos\theta}{\sin^2\theta + \cos^2\theta} = \frac{1}{\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}}$$ (1)

$$= \frac{1}{\tan\theta + \cot\theta}$$ (½)

24. $\sin\theta + \cos\theta = \sqrt{2}\cos\theta \qquad \sin\theta\,(\sqrt{2} - 1)\cos\theta$ (1)

or $$\sin\theta = \frac{(\sqrt{2} - 1)(\sqrt{2} + 1)}{(\sqrt{2} + 1)}\cos\theta$$ (1)

or $$\sin\theta = \frac{\cos\theta}{\sqrt{2} + 1} \qquad \cos\theta - \sin\theta = \sqrt{2}\sin\theta$$ (1)

25. In right ΔABC, $A + \quad C = 90°$ ...(i)

Also, in right ΔADE $A + \quad 2 = 90°$ ...(ii) (1)

From (i) and (ii), we have

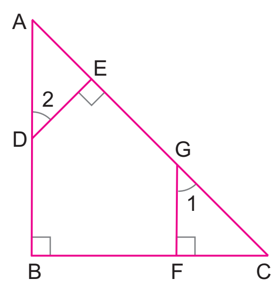


Fig. 9

$A + \quad C = \quad A + \quad 2$ (½)

or $C = \quad 2$ (½)

Now in ΔADE and ΔGCF

$AED = \quad GFC$ (each 90°)

$2 = \quad C$

$\Delta ADE \sim \Delta GCF$ (By AA similarity) (1)

26. Draw $AL \quad BC$ and $DM \quad BC$ (½)

Δ' s AOL and DOM are similar (By AA similarity)

$$\frac{AO}{DO} = \frac{AL}{DM} \quad ...(i)$$ (½)

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BCD)} = \frac{\frac{1}{2}BC.AL}{\frac{1}{2}BC.DM} = \frac{AL}{DM} = \frac{AO}{DO}$$ [using (1)] (2)

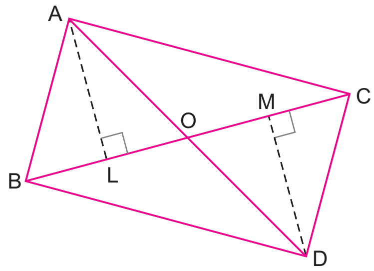


Fig. 10

27.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Class marks (x_i)	5	15	25	35	45
Frequency (f_i)	7	12	13	10	8
$d_i = \frac{x_i - 25}{10}$	–2	–1	0	1	2
$f_i d_i$	–14	–12	0	10	16

(1)

$\Sigma f_i = 50, \Sigma fidi = 0$ (½)

$\bar{x} = \text{A.M} + \frac{\Sigma fidi}{\Sigma fi} \times 10 = 25 + 0 = 25$ (1½)

OR

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency (f_i)	2	3	5	3	p
Class marks (x_i)	5	15	25	35	45
$f_i x_i$	10	45	125	105	45p

(1)

$\Sigma f_i = 13 + p, \quad \Sigma f_i x_i = 285 + 45p$

Mean = 25 (given) (1)

$\frac{285 + 45p}{13 + p} = 25 \qquad 25 \times (13 + p) = 285 + 45p$

$20p = 40 \qquad p = 2$ (1)

28.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	5	3	4	3	3	4	7	9	7	8
Cumulative Frequency	5	8	12	15	18	22	29	38	45	53

(½)

Median Class is 60 – 70 **(½)**

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$ **(½)**

$$= 60 + \left(\frac{26.5 - 22}{7}\right) \times 10 = 60 + \frac{45}{7} = 60 + 6.43 = 66.43$$ **(1+ ½)**

Section – D

29. $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

Since $\sqrt{2}$ and $-\sqrt{2}$ are zeroes of $p(x)$ **(1)**

$(x + \sqrt{2})(x - \sqrt{2})$ or $x^2 - 2$ is a factor of $p(x)$

Now, we divide $p(x)$ by $x^2 - 2$ to obtain other zeroes.

$$\begin{array}{r l}
 & 2x^2 + 7x - 15 \\
x^2 - 2 \,\big) & 2x^4 + 7x^3 - 19x^2 - 14x + 30 \\
 & \underline{\overset{-}{}2x^4 \qquad\quad \overset{+}{-} 4x^2} \\
 & \quad 7x^3 - 15x^2 - 14x + 30 \\
 & \quad \underline{\overset{-}{}7x^3 \qquad\quad \overset{+}{-} 14x} \\
 & \qquad -15x^2 \qquad + 30 \\
 & \qquad \underline{\overset{+}{-}15x^2 \qquad \pm 30} \\
 & \qquad\qquad 0
\end{array}$$ **(1½)**

Now, $2x^2 + 7x - 15 = 2x^2 + 10x - 3x - 15$ **(½)**

$= 2x(x + 5) - 3(x + 5) = (x + 5)(2x - 3)$ **(1)**

other two zeroes of $p(x)$ are $\frac{3}{2}$ and –5

30. Given: Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$

To Prove: $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$

Construction: Draw AM BC and PN QR. **(½)**

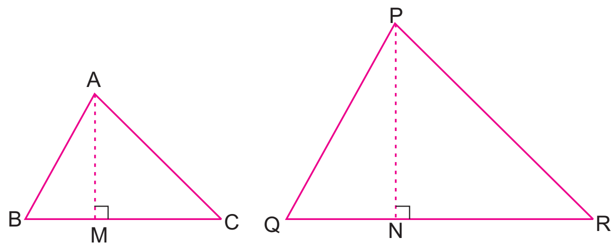


Fig. 11 **(½)**

Proof : $\text{ar}(\Delta ABC) = \frac{1}{2} \times BC \times AM$

and $\text{ar}(\Delta PQR) = \frac{1}{2} \times QR \times PN$

So, $\dfrac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \dfrac{BC \times AM}{QR \times PN}$ ...(*i*) **(1)**

Now, in ΔABM and ΔPQN,

$B = Q$ [As $\Delta ABC \sim \Delta PQR$]

and $AMB = PNQ$ [Each 90°]

So, $\Delta ABM \sim \Delta PQN$ [*AA* similarity criterion]

Therefore, $\dfrac{AM}{PN} = \dfrac{AB}{PQ}$ ...(*ii*) **(1)**

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ...(*iii*) **(½)**

Therefore, $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB}{PQ} \times \dfrac{AM}{PN}$ [From (*i*) and (*iii*)]

$= \dfrac{AB}{PQ} \times \dfrac{AB}{PQ} = \left(\dfrac{AB}{PQ}\right)^2$ [From (*ii*)]

Now using (*iii*), we get $\dfrac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$ **(½)**

OR

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$. **(½)**

To Prove: $B = 90°$. **(½)**

Construction: We construct a ΔPQR right-angled at Q such that $PQ = AB$ and $QR = BC$. **(½)**

Proof: Now, from ΔPQR, we have,

$PR^2 = PQ^2 + QR^2$ [Pythagoras Theorem, as $Q = 90°$]

or, $PR^2 = AB^2 + BC^2$ [By construction] ...(*i*)

But $AC^2 = AB^2 + BC^2$ [Given] ...(*ii*)

So, $AC^2 = PR^2$ [From (*i*) and (*ii*)]

$AC = PR$ ...(*iii*)

Now, in ΔABC and ΔPQR,

$AB = PQ$ [By construction]

$BC = QR$ [By construction]

$AC = PR$ [Proved in (*iii*)]

So, $\Delta ABC \quad \Delta PQR$ [SSS congruency]

Therefore,

$B = Q$ (CPCT)

But $Q = 90°$ [By construction]

So, $B = 90°$ **(2)**

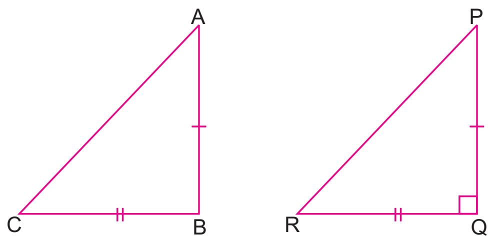


Fig. 12 **(½)**

31. LHS $= \dfrac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{\sec\theta + \tan\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$ **(1)**

$= \dfrac{(\sec\theta + \tan\theta)[1 - \sec\theta + \tan\theta]}{(1 - \sec\theta + \tan\theta)} = \sec\theta + \tan\theta = \dfrac{1 + \sin\theta}{\cos\theta}$ **(1+1)**

$= \dfrac{(1 + \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)\cos\theta} = \dfrac{1 - \sin^2\theta}{(1 - \sin\theta)\cos\theta} = \dfrac{\cos^2\theta}{(1 - \sin\theta)\cos\theta} = \dfrac{\cos\theta}{1 - \sin\theta} = \text{RHS}$ **(1)**

OR

$\because\ \text{cosec}(90° - \theta) = \sec\theta,\ \ \cot\theta(90° - \theta) = \tan\theta,\ \sin 55° = \cos(90° - 55°) = \cos 35°$ **(1)**

and $\tan 80° = \tan(90° - 10°) = \cot 10°,\ \ \tan 70° = \tan(90° - 20°) = \cot 20°,\ \ \tan 60° = \sqrt{3}$ **(1)**

Given expression becomes $\dfrac{(\sec^2\theta - \tan^2\theta) + (\sin^2 35° + \cos^2 35°)}{\tan 10° \cot 10° \tan 20° \cot 20° \sqrt{3}}$ **(1)**

$= \dfrac{1 + 1}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$ **(1)**

32. RHS $= \dfrac{p^2 - 1}{p^2 + 1} = \dfrac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$ **(½)**

$= \dfrac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1} = \dfrac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$ **(1)**

$= \dfrac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)} = \dfrac{\tan\theta}{\sec\theta} = \dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}}$ **(1+1)**

$= \dfrac{\sin\theta\cos\theta}{\cos\theta} = \sin\theta = \text{LHS}$ **(½)**

33. Graph **(2)**

x	0	1	3
$y = 2x - 1$	–1	1	5

x	0	3	13
$y = \dfrac{13 - x}{2}$	6.5	5	0

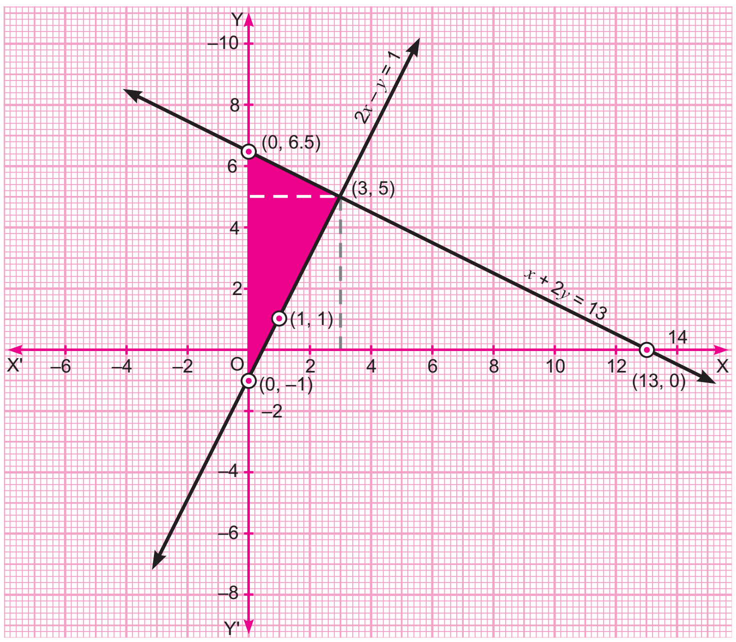


Fig. 13

(*i*) $x = 3, y = 5$ **(1)**

(*ii*) shaded region is shown in figure. **(1)**

34.

Classes	Frequency	Cumulative Frequency	More than type
50–55	2	50 or more than 50	100
55–60	8	55 or more than 55	98
60–65	12	60 or more than 60	90
65–70	24	65 or more than 65	78
70–75	38	70 or more than 70	54
75–80	16	75 or more than 75	16

(1)

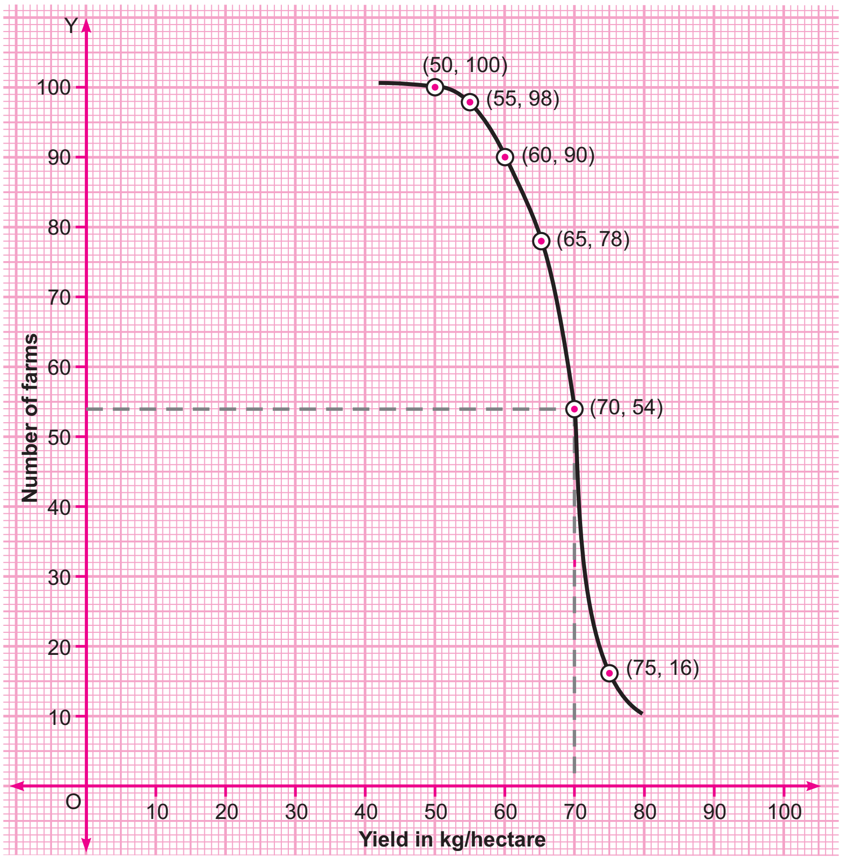


Fig. 14

(3)

Mathematics

Model Question Paper (Solved) –1

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in CBSE Sample Question Paper.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. After how many decimal places will the decimal expansion of the number $\frac{53}{2^2 5^3}$ terminate?

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

2. The largest number which divides 318 and 739 leaving remainder 3 and 4 respectively is

(*a*) 110 (*b*) 7 (*c*) 35 (*d*) 105

3. If one zero of the quadratic polynomial $4x^2 + kx - 1$ is 1, then the value of k is

(*a*) 5 (*b*) –5 (*c*) 3 (*d*) –3

4. The pair of equations $x + 2y + 5 = 0$ and $3x + 6y + 15 = 0$ has

(*a*) a unique solution (*b*) no solution

(*c*) infinitely many solutions (*d*) exactly two solutions

5. If $\Delta ABC \sim \Delta PQR$, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{9}{4}$, $PQ = 8$ cm, then AB is equal to

(*a*) 14 cm (*b*) 8 cm (*c*) 10 cm (*d*) 12 cm

6. If $\cos A = \frac{4}{5}$, then the value of $\sin A$ is

(*a*) $\frac{3}{4}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{5}{4}$

7. The value of $(\tan 10° \tan 15° \tan 75° \tan 80°)$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) $\frac{1}{2}$

8. Given that $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cos\beta = \frac{\sqrt{3}}{2}$, then the value of $(\alpha + \beta)$ is

(*a*) 90° (*b*) 60° (*c*) 75° (*d*) 45°

9. The value of $(\sin 60° + \cos 60°) - (\sin 30° + \cos 30°)$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

10. For the following distributions

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is

(*a*) 15 (*b*) 25 (*c*) 30 (*d*) 35

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Is there any natural number n for which 4^n ends with digit 0? Give reason in support of your answer.

12. Write a quadratic polynomial sum of whose zeroes is $2\sqrt{3}$ and their product is 2.

OR

If α, β are zeroes of the polynomial $3x^2 + 5x + 2$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

13. The line represented by $x = 9$ is parallel to the x-axis. Justify whether the statement is true or false.

14. In Fig. 1, $DE \| AC$ and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that $DC \| AP$.

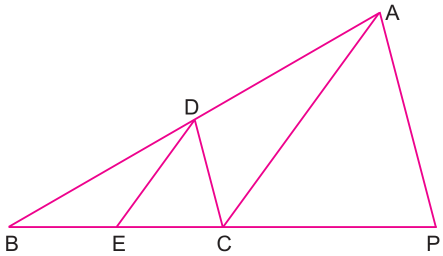


Fig. 1

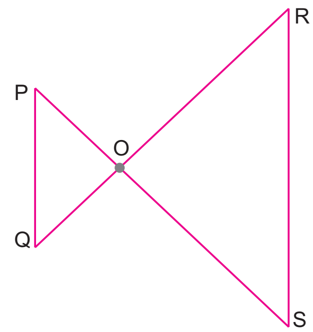


Fig. 2

15. In Fig. 2, if $PQ \| RS$, prove that $\Delta POQ \sim \Delta SOR$.

16. If $3 \cot\theta = 4$, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$.

17. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

18. Find the mean of first five prime numbers.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $\sqrt{3}$ is irrational.

OR

Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

20. Using Euclid's division algorithm, find the HCF of 56, 96 and 404.

21. Find the zeroes of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeroes and the coefficients of the polynomial.

22. Represent the following system of linear equations graphically:

$$3x + y - 5 = 0; \qquad 2x - y - 5 = 0.$$

From the graph, find the points where the lines intersect y-axis.

23. In ΔABC, if AD is the median, show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$.

24. Two triangles ABC and DBC are on the same base BC and on the same side of BC in which $A = \quad D = 90°$. If CA and BD meet each other at E, show that $AE.\ EC = BE.\ ED$.

25. Find the value of sin 45° geometrically.

26. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

OR

If $\angle A$ or $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

27. Calculate the arithmetic mean of the following frequency distribution, using the step-deviation method:

Class interval	0–50	50–100	100–150	150–200	200–250	250–300
Frequency	17	35	43	40	21	24

OR

The arithmetic mean of the following frequency distribution is 25. Determine the value of p.

Class interval	0–10	10–20	20–30	30–40	40–50
Frequency	5	18	15	p	6

28. The weight of coffee in 70 packets are shown in the following table:

Weight (*mg*)	200–201	201–202	202–203	203–204	204–205	205–206
Number of Packets (*f*)	12	26	20	9	2	1

Determine the modal weight.

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Find all zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

30. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

31. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

OR

Prove that, if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, the other two sides are divided in the same ratio.

32. Prove the following:

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2.$$

OR

Prove that: $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta.$

33. Evaluate: $\dfrac{\sec 29°}{\operatorname{cosec} 61°} + 2\cot 8°\cot 17°\cot 45°\cot 73°\cot 82° - 3(\sin^2 38° + \sin^2 52°).$

34. Following distribution shows the marks obtained by 100 students in a class:

Marks	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	10	15	30	32	8	5

Draw a less than ogive for the given data and hence obtain the median marks from the graph.

Solutions

Section – A

1.	(*b*)	$\frac{53}{2^2 5^3} = \frac{106}{(2\times5)^3} = \frac{106}{10^3} = 0.106$
2.	(*d*)	$318 - 3 = 315,\ 739 - 4 = 735;\quad 315 = 3^2 \times 5 \times 7,\quad 735 = 3 \times 5 \times 7^2$ HCF (315, 735) = $3 \times 5 \times 7$
3.	(*d*)	Since 1 is the zero $4(1)^2 + k(1) - 1 = 0 \quad 3 + k = 0$ or $k = -3$
4.	(*c*)	$\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3},\ \frac{c_1}{c_2} = \frac{5}{15} = \frac{1}{3}$ As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$, therefore the system of linear equations has infinitely many solutions
5.	(*d*)	As $\Delta ABC \sim \Delta PQR \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2}$ $\frac{9}{4} = \left(\frac{AB}{PQ}\right)^2 \quad \frac{AB}{PQ} = \frac{3}{2} \quad \frac{AB}{8} = \frac{3}{2} \quad AB = \frac{3}{2}\times 8 = 12$ cm
6.	(*b*)	$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$
7.	(*b*)	$\tan 10°\tan 15°\cot(90°-75°)\cot(90°-80°) = \tan 10°\tan 15°\cot 15°\cot 10° = 1$
8.	(*c*)	$\sin\alpha = \frac{1}{\sqrt{2}} \quad \sin\alpha = \sin 45° \quad \alpha = 45°$ $\cos\beta = \frac{\sqrt{3}}{2} \quad \cos\beta = \cos 30° \quad \beta = 30°$ $\alpha + \beta = 45° + 30° = 75°$
9.	(*b*)	$(\sin 60° + \cos 60°) - (\sin 30° + \cos 30°) = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 0$
10.	(*b*)	(see table below) Median lies in the class 10–15 Mode lies in the class 15–20 Sum of the lower limit of median and modal classes = 10 + 15 = 25

Class	0–5	5–10	10–15	15–20	20–25
Frequency	10	15	12	20	9
Cumulative Frequency	10	25	37	57	66

1×10=10

Section – B

11. No. $4^n = (2^2)^n = 2^{2n}$ **(1)**

The only prime in the factorisation of 4^n is 2

There is no other primes in the factorisation of 4^n

Therefore, 5 does not occur in prime factorisation of 4^n **(1)**

Hence, 4^n does not end with the digit zero for any natural number n.

12. Quadratic polynomial

$= x^2 -$ (sum of the zeroes) x + product of the zeroes $= x^2 - 2\sqrt{3}x + 2$ **(1+1)**

OR

α, β are zeroes of $3x^2 + 5x + 2$

$$\alpha + \beta = -\frac{b}{a} = -\frac{5}{3}, \quad \alpha\beta = \frac{c}{a} = \frac{2}{3}$$ **(1)**

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-5/3}{2/3} = \frac{-5}{2}$$ **(1)**

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-5}{2}$$

13. False, because the line parallel to x-axis is in the form $y = a$ **(1+1)**

14. In ΔABC, $DE \parallel AC$

$$\frac{BD}{DA} = \frac{BE}{EC} \quad \ldots(i)$$ **(1)**

(Using Basic Proportionality Theorem)

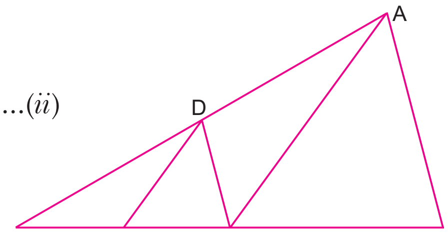


Fig. 3

Now, $\frac{BE}{EC} = \frac{BC}{CP}$ (given) ...(ii)

Using (i) and (ii), we have

$$\frac{BD}{DA} = \frac{BC}{CP}$$

So, in ΔABP

$$\frac{BD}{DA} = \frac{BC}{CP}$$ (from above) **(1)**

$DC \parallel AP$ (Using converse of Basic Proportionality Theorem)

15. In Fig. 4.

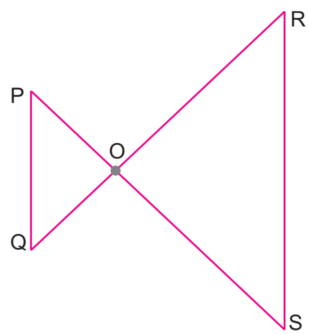


Fig. 4

As $PQ \parallel RS$ (Given)

So, $P = S$ (Alternate angles) **(½)**

$Q = R$ (Alternate angles) **(½)**

Therefore, $\Delta POQ \sim \Delta SOR$ (*AA* similarity criterion) **(1)**

16. Given, $3 \cot\theta = 4$

$$\cot\theta = \frac{4}{3} = \frac{AB}{BC}$$

$$AC = \sqrt{AB^2 + BC^2}$$ **(½)**

$$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$ (½)

$$\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$$

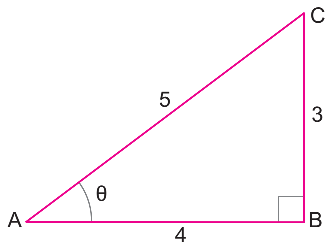


Fig. 5

Now, $$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{5\times\frac{3}{5} - 3\times\frac{4}{5}}{5\times\frac{3}{5} + 3\times\frac{4}{5}} = \frac{\frac{15-12}{5}}{\frac{15+12}{5}} = \frac{3}{27} = \frac{1}{9}.$$ (1)

Alternate Method,

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{\frac{5\sin\theta}{\sin\theta} - \frac{3\cos\theta}{\sin\theta}}{\frac{5\sin\theta}{\sin\theta} + \frac{3\cos\theta}{\sin\theta}}$$ (Dividing numerator and denominator by $\sin\theta$)

$$= \frac{5 - 3\cot\theta}{5 + 3\cot\theta} = \frac{5-4}{5+4} = \frac{1}{9}$$

17. No, it is not always the case. The values of these three measures can be the same. It depends on the type of data. **(1+1)**

18. First five prime numbers are 2, 3, 5, 7, 11 **(1)**

Mean $= \overline{X} = \frac{2+3+5+7+11}{5} = \frac{28}{5} = 5.6.$ **(1)**

Section – C

19. Suppose, $\sqrt{3}$ is rational

$\sqrt{3} = \frac{r}{s}$, where r and s are integers and $s \neq 0$ **(½)**

Let r and s have some common factor other than one, then divide r and s by that common factor and let us get

$\sqrt{3} = \frac{a}{b}$, where a and b are co-prime and $b \neq 0$ **(½)**

$$3 = \frac{a^2}{b^2}$$

$a^2 = 3b^2$ ...(i)

Prime 3 divides a^2,

3 divides a ...(ii) **(½)**

We can write $a = 3k$, where k is some integer.

Put $a = 3k$ in (i), we get

$(3k)^2 = 3b^2$

$9k^2 = 3b^2$ $\quad b^2 = 3k^2$

Prime 3 divides b^2 $\quad$ 3 divides b ...(iii) **(1)**

From (ii) and (iii), we have that a and b have common factor 3 which contradicts the fact that a and b are co-prime.

Therefore, our supposition that $\sqrt{3}$ is rational is wrong and hence $\sqrt{3}$ is irrational. **(½)**

OR

Suppose, $\sqrt{3}+\sqrt{5}$ is a rational number **(½)**

Let $\sqrt{3}+\sqrt{5}=a$, where a is rational number

Therefore, $\sqrt{3}=a-\sqrt{5}$ **(½)**

Squaring on both sides, we get

$$3=a^2+5-2a\sqrt{5}$$

$$2a\sqrt{5}=a^2+2$$

$$\sqrt{5}=\frac{a^2+2}{2a}$$ **(1)**

Which is a contradiction as the right hand side is a rational number while $\sqrt{3}$ is irrational. **(1)**

Hence, $\sqrt{3}+\sqrt{5}$ is irrational.

20. Given integers are 56, 96 and 404.

First we find the HCF of 56 and 96.

Applying Euclid's division algorithm, we get

$$96=56\times 1+40$$

Since the remainder $40\neq 0$, so we apply the division lemma to 56 and 40.

$$56=40\times 1+16$$

Since the remainder $16\neq 0$, so we apply the division lemma to 40 and 16.

$$40=16\times 2+8$$

Since $8\neq 0$, so we apply the division lemma to 16 and 8.

$$16=8\times 2+0$$

Clearly, HCF of 56 and 96 is 8. **(1½)**

Let us find the HCF of 8 and the third number 404 by Euclid's algorithm.

Applying Euclid's division, we get

$$404=50\times 8+4$$

Since the remainder is $4\neq 0$. So, we apply the division lemma to 8 and 4.

$$8=4\times 2+0$$

We observe that the remainder at this stage is zero.

Therefore, the divisor of this stage, *i.e.*, 4 is the HCF of 56, 96 and 404. **(1½)**

$$56\overline{)96}\ (1;\ -56;\ 40) \qquad 40\overline{)56}\ (1;\ -40;\ 16) \qquad 16\overline{)40}\ (2;\ -32;\ 8)$$

$$8\overline{)16}\ (2;\ -16;\ 0) \qquad 8\overline{)404}\ (50;\ -40;\ 4) \qquad 4\overline{)8}\ (2;\ -8;\ 0)$$

21. $5x^2-4-8x=5x^2-8x-4$

$$=5x^2-10x+2x-4=5x(x-2)+2(x-2)$$

$$=(x-2)(5x+2)$$

Zeroes of $5x^2-8x-4$ are $2,-\frac{2}{5}$ **(1)**

Sum of zeroes $=2+\left(-\frac{2}{5}\right)=\frac{8}{5}=\frac{-(-8)}{5}=-\frac{(\text{Coefficient of }x)}{(\text{Coefficient of }x^2)}$ **(1)**

Product of zeroes $=2\left(-\frac{2}{5}\right)=\frac{-4}{5}=\frac{(\text{Constant term})}{\text{Coefficient of }x^2}$. **(1)**

22. Given equation,

$$3x + y - 5 = 0$$

$$y = 5 - 3x \qquad \text{...}(i)$$

x	0	1	2
$y = 5 - 3x$	5	2	– 1

Second equation $2x - y - 5 = 0$

$$y = 2x - 5 \qquad \text{...}(ii)$$

x	0	1	2
$y = 2x - 5$	– 5	– 3	– 1

Equations (*i*) and (*ii*) can be represented graphically as follows:

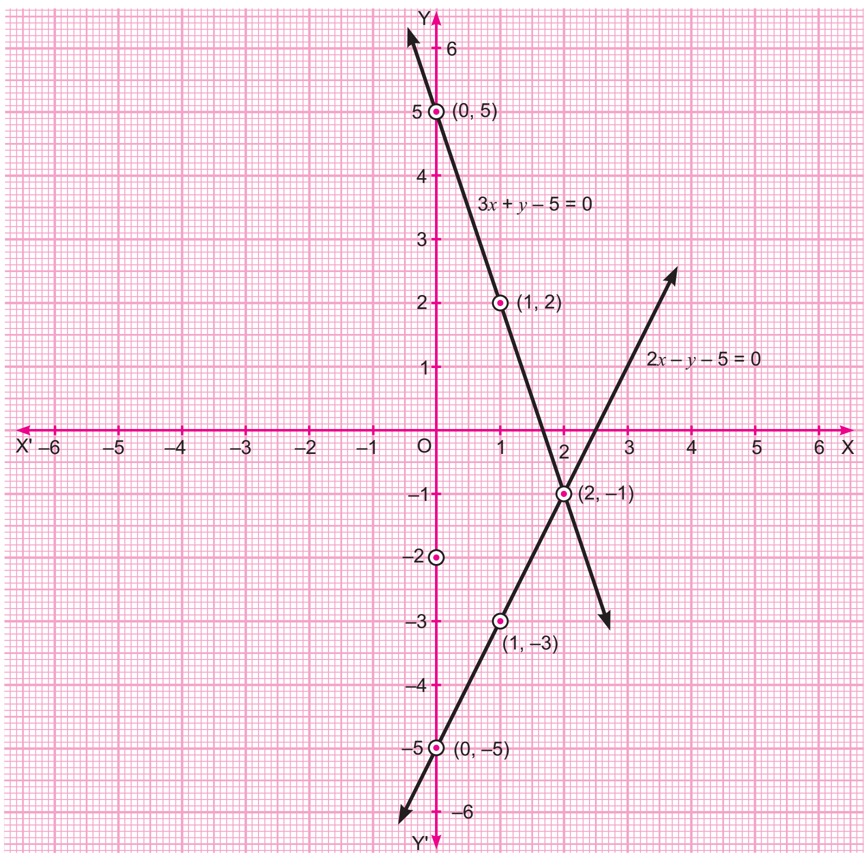


Fig. 6

(2)

Here, $3x + y - 5 = 0$ cuts y-axis at (0, 5), and **(½)**

$2x - y - 5 = 0$ cuts y-axis at (0, –5). **(½)**

23. Draw AE BC

In right-angled ΔAED, $AD^2 = AE^2 + ED^2$ ...(*i*)

In right-angled ΔAEB,

$$AB^2 = AE^2 + BE^2 = AE^2 + (BD - ED)^2$$

$$= AE^2 + BD^2 + ED^2 - 2BD \,.\, ED$$

$$= (AE^2 + ED^2) + BD^2 - 2BD \,.\, ED$$

$$AB^2 = AD^2 + BD^2 - 2BD \,.\, ED \text{ [Using } (i)] \qquad \text{... } (ii)$$

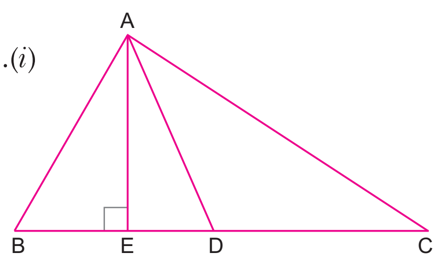


Fig. 7

(1)

In right-angled ΔAEC,

$$AC^2 = AE^2 + EC^2 = AE^2 + (ED + DC)^2$$
$$= (AE^2 + ED^2) + DC^2 + 2ED \,.\, DC$$
$$AC^2 = AD^2 + BD^2 + 2\,ED\,.\,BD \qquad (\because AD \text{ is median}) \ldots(iii)$$ **(1)**

Adding (*ii*) and (*iii*), we get

$$AB^2 + AC^2 = 2AD^2 + 2BD^2$$
$$AB^2 + AC^2 = 2(AD^2 + BD^2).$$ **(1)**

24. In ΔAEB and ΔDEC

$BAC = BDC = 90^o$

$AEB = DEC$ (Vertically opposite angles)

$\Delta AEB \sim \Delta DEC$ (AA similarity) **(1½)**

$$\frac{AE}{ED} = \frac{BE}{EC}$$ **(½)**

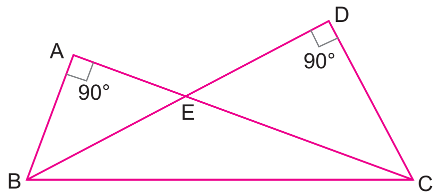


Fig. 8

$AE\,.\,EC = BE\,.\,ED.$ **(1)**

25. In ΔABC, right-angled at B, if one angle is 45°, then other angle is also 45°.

i.e., $A = C = 45°$

So, $BC = AB$ (Sides opposite to equal angles) **(1)**

Let $BC = AB = a$

Then by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$$
$$AC = \sqrt{2}a$$ **(1)**

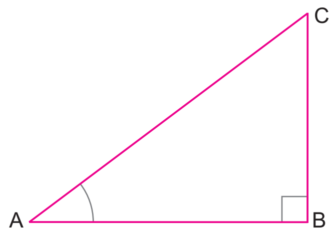


Fig. 9

Now, $$\sin 45° = \frac{\text{side opposite to angle } 45°}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a}.$$

$$\sin 45° = \frac{1}{\sqrt{2}}.$$ **(1)**

26. $\sin\theta + \cos\theta = \sqrt{3}$ (Given)

or $(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$ **(½)**

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$ **(1)**

$$1 + 2\sin\theta\cos\theta = 3 \qquad (\sin^2\theta + \cos^2\theta) = 1$$

$$2\sin\theta\cos\theta = 2$$ **(½)**

$$\sin\theta\cos\theta = 1 \quad \text{or} \quad \sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$$

or $$1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} \quad \text{or} \quad 1 = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$ **(1)**

Therefore, $\tan\theta + \cot\theta = 1.$

OR

In right-angled ΔACB, $C = 90°$, we have **(½)**

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{AB}$$ **(½)**

and $$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AB}$$ **(½)**

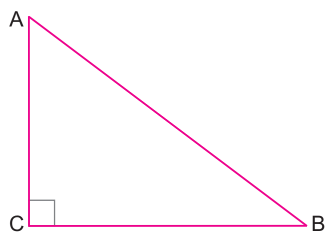


Fig. 10

We have, $\cos A = \cos B$ [given]

$\frac{AC}{AB} = \frac{BC}{AB}$ $AC = BC$ **(½)**

$B = A$ [angles opposite to equal sides are equal]. **(1)**

27. Here, $h = 50$. Let the assumed mean be $A = 125$

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{(x_i - A)}{h}$	$f_i \times u_i$
0–50	17	25	–2	–34
50–100	35	75	–1	–35
100–150	43	125 = A	0	0
150–200	40	175	1	40
200–250	21	225	2	42
250–300	24	275	3	72
	$\Sigma f_i = 180$			$\Sigma(f_i \times u_i) = 85$

(2)

Thus, we have

$A = 125,\ h = 50,\ \Sigma f_i = 180$ and $\Sigma(f_i \times u_i) = 85$

Mean, $\bar{x} = A + \left\{h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right\}$

$= 125 + \left\{50 \times \frac{85}{180}\right\} = (125 + 23.61) = 148.61$ **(1)**

OR

We have,

Class interval	Frequency f_i	Mid-value x_i	$(f_i \times x_i)$
0–10	5	5	25
10–20	18	15	270
20–30	15	25	375
30–40	p	35	$35p$
40–50	6	45	270
	$\Sigma f_i = 44 + p$		$\Sigma(f_i \times x_i) = (940 + 35p)$

(1½)

Mean, $\bar{x} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$ **(½)**

$\frac{(940 + 35p)}{(44 + p)} = 25$

$(940 + 35p) = 25(44 + p)$

$(35p - 25p) = (1100 - 940)$

$10p = 160$ $p = 16$ **(1)**

Hence, $p = 16$

28. We have,

Weight (mg)	**Number of packets (f)**
200–201	12
201–202	26
202–203	20
203–204	9
204–205	2
205–206	1

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$ **(1)**

Here, the modal class is 201–202.

$$l = 201, \; f_1 = 26, \; f_0 = 12, \; f_2 = 20, \; h = 202 - 201 = 1$$

$$\text{Mode} = 201 + \frac{26 - 12}{2 \times 26 - 12 - 20} \times 1$$ **(1)**

$$= 201 + \frac{14}{20} = 201 + 0.7 = 201.7 \text{ g}$$ **(1)**

Section – D

29. Sum of zeroes $= 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Product of zeroes $= (2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$

A polynomial whose zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is given by

$x^2 - 4x + 1$ **(1)**

So, $x^2 - 4x + 1$ is a factor of given polynomial.

On dividing $2x^4 - 9x^3 + 5x^2 + 3x - 1$ by $x^2 - 4x + 1$, we get

$$\begin{array}{r|l} & 2x^2 - x - 1 \\ x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\ & -2x^4 \mp 8x^3 \pm 2x^2 \\ \hline & -x^3 + 3x^2 + 3x \\ & \mp x^3 \pm 4x^2 \mp x \\ \hline & -x^2 + 4x - 1 \\ & -x^2 + 4x - 1 \\ \hline & 0 \end{array}$$ **(2)**

Now, $2x^2 - x - 1 = 2x^2 - 2x + x - 1$

$= 2x(x - 1) + 1(x - 1) = (x - 1)(2x + 1)$ **(½)**

$2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(x - 1)(2x + 1)$

So, the other zeroes are 1 and $-\frac{1}{2}$. **(½)**

Thus, all zeroes of given polynomial are $(2 + \sqrt{3})$, $(2 - \sqrt{3})$, 1 and $-\frac{1}{2}$.

30. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream $= (x - y)$ km/h

Speed downstream $= (x + y)$ km/h

Now, time taken to cover 32 km upstream $= \frac{32}{x - y}$ hours

Time taken to cover 36 km downstream $= \frac{36}{x + y}$ hours

The total time of journey is 7 hours

$$\frac{32}{x-y} + \frac{36}{x+y} = 7 \quad \ldots(i)$$ **(½)**

Time taken to cover 40 km upstream $= \frac{40}{x - y}$

Time taken to cover 48 km downstream $= \frac{48}{x + y}$

In this case, total time of journey is 9 hours.

$$\frac{40}{x-y} + \frac{48}{x+y} = 9 \quad \ldots(ii)$$ **(½)**

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (*i*) and (*ii*), we get **(½)**

$$32u + 36v = 7 \qquad 32u + 36v - 7 = 0 \quad \ldots(iii)$$

$$40u + 48v = 9 \qquad 40u + 48v - 9 = 0 \quad \ldots(iv)$$

By cross-multiplication, we have

$$\frac{u}{36 \times (-9) - 48 \times (-7)} = \frac{-v}{32 \times (-9) - 40 \times (-7)} = \frac{1}{32 \times 48 - 40 \times 36}$$ **(½)**

$$\frac{u}{-324 + 336} = \frac{-v}{-288 + 280} = \frac{1}{1536 - 1440}$$

$$\frac{u}{12} = \frac{-v}{-8} = \frac{1}{96} \qquad \frac{u}{12} = \frac{v}{8} = \frac{1}{96}$$

$$\frac{u}{12} = \frac{1}{96} \quad \text{and} \quad \frac{v}{8} = \frac{1}{96}$$

$$u = \frac{12}{96} \quad \text{and} \quad v = \frac{8}{96}$$

$$u = \frac{1}{8} \quad \text{and} \quad v = \frac{1}{12}$$ **(1)**

We have, $u = \frac{1}{8}$ $\quad \frac{1}{x-y} = \frac{1}{8}$ $\quad x - y = 8$ ...(*v*)

and $v = \frac{1}{12}$ $\quad \frac{1}{x+y} = \frac{1}{12}$ $\quad x + y = 12$ ...(*vi*)

Solving equations (*v*) and (*vi*), we get $x = 10$ and $y = 2$. **(1)**

Hence, speed of the boat in still water is 10 km/h and speed of the stream is 2 km/h.

31. Refer to Q.N. 30 CBSE Sample Question Paper.

OR

Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and then draw DM AC and EN AB. **(½)**

Proof: Area of $\Delta ADE = \left(\frac{1}{2}\text{ base} \times \text{height}\right)$

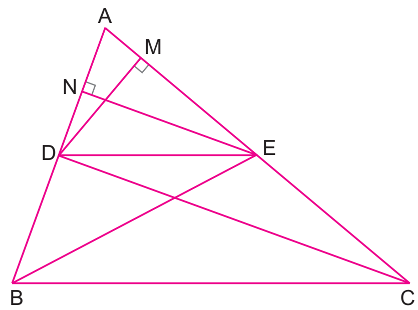


Fig. 11

So, $\quad \text{ar}(\Delta ADE) = \frac{1}{2}AD \times EN$

and $\quad \text{ar}(\Delta BDE) = \frac{1}{2}DB \times EN$

Similarly, $\quad \text{ar}(\Delta ADE) = \frac{1}{2}AE \times DM$

and $\quad \text{ar}(\Delta DEC) = \frac{1}{2}EC \times DM$

Therefore, $\quad \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2}AD \times EN}{\frac{1}{2}DB \times EN} = \frac{AD}{DB} \quad \ldots(i)$ **(1)**

and $\quad \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta DEC)} = \frac{\frac{1}{2}AE \times DM}{\frac{1}{2}EC \times DM} = \frac{AE}{EC} \quad \ldots(ii)$ **(1)**

Now, ΔBDE and $\Delta\, DEC$ are on the same base DE and between the same parallel lines BC and DE.

So, $\quad \text{ar}(\Delta BDE) = \text{ar}(\Delta DEC) \quad \ldots(iii)$ **(1)**

Therefore, from (i), (ii) and (iii) we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **(½)**

32. LHS $= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$ **(½)**

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$

$= -\frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A} \quad [\because (a+b)(a-b) = a^2 - b^2]$ **(1)**

$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A \cos A}$ **(1)**

$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$ **(1)**

$= \frac{2\sin A\cos A}{\sin A\cos A} = 2 = \text{RHS.}$ **(½)**

OR

LHS $= \dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \dfrac{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta} + \dfrac{1}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta} - \dfrac{1}{\sin\theta}}$ **(½)**

$= \dfrac{\cot\theta - 1 + \text{cosec}\,\theta}{\cot\theta + 1 - \text{cosec}\,\theta} = \dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$ **(½)**

$= \dfrac{(\cot\theta + \text{cosec}\,\theta) - (\text{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \text{cosec}\,\theta + 1}$ **(1)**

$= \dfrac{(\cot\theta + \text{cosec}\,\theta) - [(\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)]}{\cot\theta - \text{cosec}\,\theta + 1}$ **(½)**

$= \dfrac{(\text{cosec}\,\theta + \cot\theta)[1 - (\text{cosec}\,\theta - \cot\theta)]}{\cot\theta - \text{cosec}\,\theta + 1}$ **(½)**

$= (\text{cosec}\,\theta + \cot\theta)\dfrac{[\cot\theta - \text{cosec}\,\theta + 1]}{\cot\theta - \text{cosec}\,\theta + 1} = \text{cosec}\,\theta + \cot\theta = \text{RHS}$ **(1)**

33. We have,

$\dfrac{\sec 29°}{\text{cosec } 61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

$= \dfrac{\text{cosec }(90° - 29°)}{\text{cosec } 61°} + 2\tan(90° - 8°)\tan(90° - 17°) \times \cot 45° \times \cot 73° \cot 82°$

$- 3[\sin^2(38°) + \cos^2(90° - 52°)]$ **(1½)**

$= \dfrac{\text{cosec } 61°}{\text{cosec } 61°} + 2\tan 82° \times \tan 73° \times \cot 45° \times \cot 73° \cot 82° - 3[\sin^2 38° + \cos^2 38°]$

$= \dfrac{\text{cosec } 61°}{\text{cosec } 61°} + 2\tan 82° \times \tan 73° \times 1 \times \dfrac{1}{\tan 73°} \times \dfrac{1}{\tan 82°} - 3(1)$ **(1½)**

$= 1 + 2 - 3 = 1 + 2 - 3 = 0.$ **(1)**

34.

Table 1

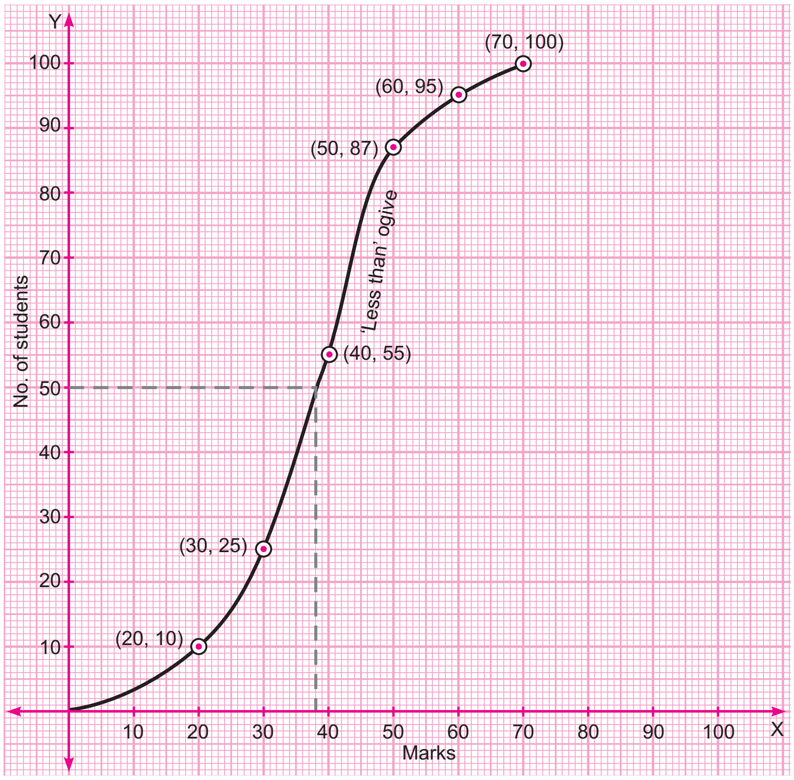
Marks	Number of Students (*f*)
10–20	10
20–30	15
30–40	30
40–50	32
50–60	8
60–70	5
	N = 100

Table 2

Marks	Number of Students (*cf*)
Less than 20	10
Less than 30	25
Less than 40	55
Less than 50	87
Less than 60	95
Less than 70	100

(1)

Now, plot the points (20,10), (30, 25), (40, 55) (50, 87), (60, 95), (70, 100)



(2)

Fig. 12

Median $=$ size of $\left(\frac{N}{2}\right)^{th}$ item $=$ size of $\left(\frac{100}{2}\right)^{th}$ item $=$ size of $(50)^{th}$ items **(1)**

Median $= 38.3$

Mathematics

Model Question Paper (Solved) –2

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in CBSE Sample Question Paper.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. If two positive integers a and b are written as $a = x^5 y^2$ and $b = x^2 y^3$; x, y are prime numbers, then HCF (a, b) is

(*a*) xy (*b*) $x^5 y^3$ (*c*) $x^3 y^3$ (*d*) $x^2 y^2$

2. The product of a non-zero rational and an irrational number is

(*a*) one (*b*) always irrational (*c*) always rational (*d*) rational or irrational

3. If the zeroes of quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then

(*a*) a and c have same sign (*b*) b and c have same sign

(*c*) a and c have opposite sign (*d*) b and c have opposite sign

4. One equation of a pair of dependent linear equations is $3x - 5y = 12$. The second equation can be

(*a*) $-6x + 10y = 24$ (*b*) $9x + 15y = 36$ (*c*) $-\frac{3}{2}x + \frac{5}{2}y = 6$ (*d*) $x - \frac{5}{3}y = 4$

5. If sides of two similar triangles are in the ratio 4 : 9, then areas of these triangles are in the ratio

(*a*) 4 : 9 (*b*) 2 : 3 (*c*) 16 : 81 (*d*) 81: 16

6. If triangle ABC is right angled at C and $\sin A = \frac{\sqrt{3}}{2}$, then the value of $\sec B$ is

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) 2 (*c*) $\frac{1}{\sqrt{3}}$ (*d*) $\frac{2}{\sqrt{3}}$

7. If $\sin A + \sin^2 A = 1$, then the value of expression $(\cos^2 A + \cos^4 A)$ is

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) 3

8. If $\cos\alpha = \sin 2\alpha$ and $2\alpha < 90°$, then the value $\cot 3\alpha$ is

(*a*) 0 (*b*) $\sqrt{3}$ (*c*) 1 (*d*) $\frac{1}{\sqrt{3}}$

9. $\frac{2\tan 30°}{1 - \tan^2 30°}$ is equal to

(*a*) $\sin 30°$ (*b*) $\sin 60°$ (*c*) $\tan 30°$ (*d*) $\tan 60°$

10. If $x_1, x_2, x_3 \ldots\ldots\ldots\ldots, x_n$ are n observations with mean $\bar{x}$, then $\sum_{i=1}^{n}(x_i - \bar{x})$ is

(*a*) = 1 (*b*) > 0 (*c*) = 0 (*d*) < 0

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Using factor tree, determine the prime factorisation of 234.

12. If α, β are the two zeroes of the polynomial $p(y) = y^2 - 8y + a$ and $\alpha^2 + \beta^2 = 40$, find the value of a.

OR

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

13. What type of solution does the pair of equations

$\frac{3}{x} + \frac{8}{y} = -1, \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0$ have?

14. In Fig. 1, $DE \parallel BC$.

If AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm, find AE.

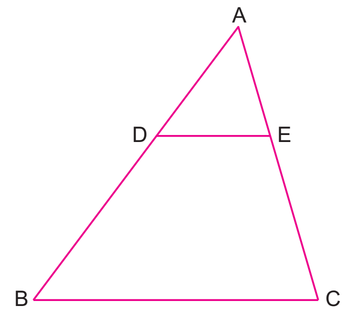


Fig. 1

15. In Fig. 2, PQ = 24 cm, QR = 26 cm, $PAR = 90^o$, PA = 6 cm and AR = 8 cm. Find QPR.

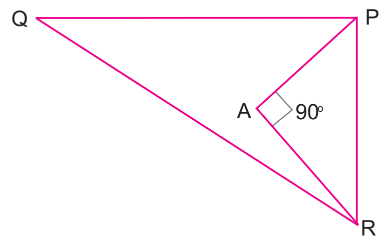


Fig. 2

16. Given that $\tan\theta = \frac{1}{\sqrt{5}}$, what is the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$?

17. Is it correct to say that an ogive is a graphical representation of a frequency distribution? Give reason.

18. In a frequency distribution, the mode and mean are 26.6 and 28.1 respectively. Find out the median.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Using prime factorisation method, find the HCF and LCM of 72, 126 and 168. Also show that HCF × LCM ≠ Product of the three numbers.

20. Prove that $3 + \sqrt{2}$ is an irrational number.

21. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q.

22. Five years ago, Nuri was thrice of Sonu's age. Ten years later, Nuri will be twice of Sonu's age. How old are Nuri and Sonu?

OR

Taxi charges in a city consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a person travels 70 km, he pays ₹ 500 and for travelling 100 km, he pays ₹ 680. Express the above statements with the help of linear equations and hence find the fixed charges and rate per kilometer.

23. In Fig. 3, M is mid-point of side CD of a parallelogram $ABCD$. The line BM is drawn intersecting AC at L and AD produced at E. Prove that $EL = 2\ BL$.

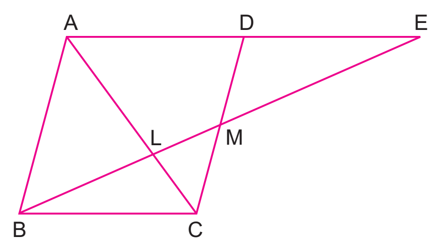


Fig. 3

OR

In Fig. 4, $DEFG$ is a square and $BAC = 90°$. Show that $DE^2 = BD \times EC$.

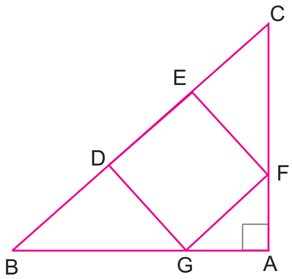


Fig. 4

24. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

25. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2) = 4\sqrt{mn}$.

OR

If $\tan\theta + 1 = \sqrt{2}$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

26. Find the value $\cos 30°$ geometrically.

27. Find the mode for the following data:

Classes	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	4	8	10	12	10	4	2

28. The median of the distribution given below is 14.4. Find the values of x and y, if the total frequency is 20.

Class Interval	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	4	x	5	y	1

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. The remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

30. Draw the graph of the following pair of linear equations

$$x + 3y = 6$$
$$2x - 3y = 12$$

Hence, find the area of the region bounded by the lines $x = 0$, $y = 0$ and $2x - 3y = 12$.

31. State and prove converse of Pythagoras theorem.

OR

Prove that the ratio of areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

32. Prove that: $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$.

OR

Prove that: $\sqrt{\dfrac{\sec A - 1}{\sec A + 1}} + \sqrt{\dfrac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$.

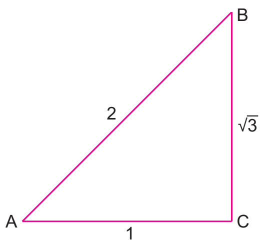
33. Evaluate: $\dfrac{2}{3}\operatorname{cosec}^2 58° - \dfrac{2}{3}\cot 58° \tan 32° - \dfrac{5}{3}\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$.

34. The mean of the following frequency table is 53. But the frequencies f_1 and f_2 in the classes 20–40 and 60–80 are missing. Find the missing frequencies.

Age (in years)	0–20	20–40	40–60	60–80	80–100	Total
Number of people	15	f_1	21	f_2	17	100

Solution

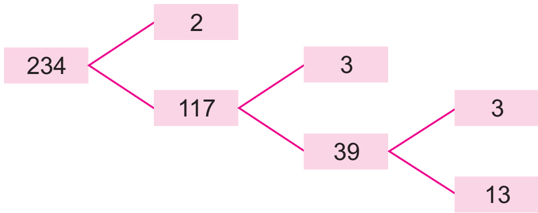
Section – A

1.	(*d*)	
2.	(*b*)	
3.	(*a*)	
4.	(*d*)	$\frac{a_1}{a_2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-5}{-\frac{5}{3}} = 3, \frac{c_1}{c_2} = \frac{12}{4} = 3,$ As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = 3$, therefore the pair of linear equations is dependent.
5.	(*c*)	$\frac{\text{area of first triangle}}{\text{area of second triangle}} = \left(\frac{4}{9}\right)^2 = \frac{16}{81} = 16 : 81$
6.	(*d*)	$\sin A = \frac{BC}{AB} = \frac{\sqrt{3}}{2}$ $AC^2 + BC^2 = AB^2 \qquad AC^2 + (\sqrt{3})^2 = 2^2$ $AC^2 = 4 - 3 = 1 \qquad AC = 1 \qquad \sec B = \frac{AB}{BC} = \frac{2}{\sqrt{3}}$  Fig. 5 **Alternate method** $\sin A = \frac{\sqrt{3}}{2} = \sin 60° \qquad A = 60° \qquad B = 90° - 60° = 30°$ $\sec B = \sec 30° = \frac{2}{\sqrt{3}}$
7.	(*b*)	$\sin A + \sin^2 A = 1 \qquad \sin A = 1 - \sin^2 A = \cos^2 A$ $\cos^2 A + \cos^4 A = \cos^2 A + (\cos^2 A)^2 = \sin A + \sin^2 A = 1 \qquad (\because \cos^2 A = \sin A)$
8.	(*a*)	$\cos\alpha = \sin 2\alpha \qquad \cos\alpha = \cos(90° - 2\alpha)$ $\alpha = 90° - 2\alpha \qquad 3\alpha = 90° \qquad \alpha = 30°$ So, $\cot 3\alpha = \cot 3 \times 30° = \cot 90° = 0$
9.	(*d*)	$\frac{2\tan 30°}{1 - \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60°$
10.	(*c*)	$\Sigma(x_i - \bar{x}) = \Sigma x_i - \Sigma \bar{x} = n\bar{x} - n\bar{x} = 0$

1×10=10

Section – B

11.



(1)

$$234 = 2 \times 3 \times 3 \times 13 = 2 \times 3^2 \times 13$$ **(1)**

12. $p(y) = y^2 - 8y + a$

$\alpha + \beta = -\dfrac{(-8)}{1} = 8$ $\qquad \alpha\beta = \dfrac{a}{1} = a$ **(1)**

$\alpha^2 + \beta^2 = 40$ $\qquad (\alpha + \beta)^2 - 2\alpha\beta = 40$

$8^2 - 2 \times a = 40$ $\qquad -2a = -24 \qquad a = 12$ **(1)**

OR

As we know,

Dividend = Quotient × Divisor + Remainder

So, we have,

$$x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$$ **(½)**

$$x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2)\, g(x)$$

$$x^3 - 3x^2 + 3x - 2 = (x - 2)\, g(x)$$

$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{(x - 2)}$$ **(½)**

Now we divide $x^3 - 3x^2 + 3x - 2$ by $x - 2$.

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & \underline{-\quad +} \\
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & \underline{+\quad -} \\
 & x - 2 \\
 & x - 2 \\
 & \underline{-\quad +} \\
 & 0
\end{array}
$$ **(1)**

Hence, $g(x) = x^2 - x + 1$

13. $\dfrac{3}{x} + \dfrac{8}{y} = -1$ and $\dfrac{1}{x} - \dfrac{2}{y} = 2$

Taking $\dfrac{1}{x} = u$ and $\dfrac{1}{y} = v$, the above equations become

$3u + 8v = -1$ ...(*i*) **(½)**

$u - 2v = 2$ ...(*ii*)

Here, $\frac{a_1}{a_2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{8}{-2} = -4$ **(½)**

$\because \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system of equations have a unique solution. **(1)**

14. In the given Figure,

$AD = 2\ 4$ cm

$DB = 3\ 6$ cm, $AC = 5$ cm [Given]

Let $AE = x$ cm

Then, $EC = (5 - x)$ cm

By B.P.T.

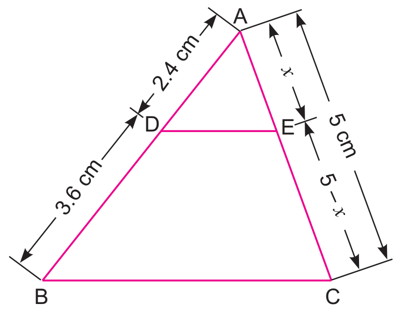


Fig. 6

(½)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

(1)

$$\frac{2\ 4}{3\ 6} = \frac{x}{5 - x} \qquad \frac{2}{3} = \frac{x}{5 - x}$$

$$10 - 2x = 3x \qquad 5x = 10$$

$$x = \frac{10}{5} = 2 \qquad AE = 2\text{ cm}$$ **(½)**

15. In ΔPAR $\quad PR^2 = AP^2 + AR^2$ (By Pythagoras theorem)

$= 6^2 + 8^2 = 36 + 64 = 100$

$PR = 10$ cm **(1)**

Now, $PQ^2 + PR^2 = (24)^2 + (10)^2 = 576 + 100 = 676$

$QR^2 = (26)^2 = 676$

$PQ^2 + PR^2 = QR^2$

ΔPQR is right-angled triangle

$QPR = 90°$. **(1)**

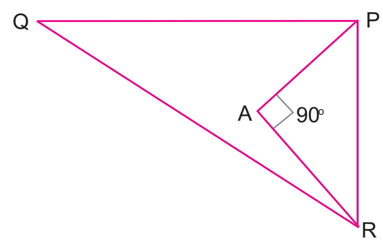


Fig. 7

16. Given, $\tan\theta = \frac{1}{\sqrt{5}}$

$$\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$$

According to Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (1)^2 + (\sqrt{5})^2 = 1 + 5 = 6 \qquad AC = \sqrt{6}$$

$$\text{cosec}\,\theta = \frac{\sqrt{6}}{1}, \sec\theta = \frac{\sqrt{6}}{\sqrt{5}}$$ **(½)**

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{\left(\frac{\sqrt{6}}{1}\right)^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{6}}{1}\right)^2 + \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2} = \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}} = \frac{\frac{30 - 6}{5}}{\frac{30 + 6}{5}} = \frac{24}{36} = \frac{2}{3}.$$

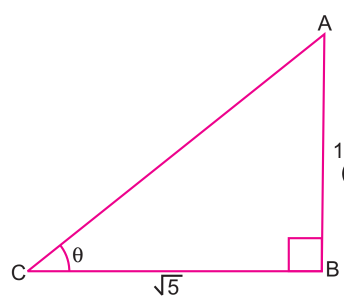


Fig. 8 **(½ + 1)**

Alternate method:

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)} \quad \textbf{(1)}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta+\tan^2\theta} = \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}} = \frac{24}{36} = \frac{2}{3} \quad \textbf{(1)}$$

17. Graphical representation of frequency distribution may not be an ogive. It may be a histogram. An ogive is a graphical representation of cumulative frequency distribution. **(1+1)**

18. Given, Mode = 26.6, Mean =28.1

Mode = 3 Median – 2 Mean   3 Median = Mode + 2 Mean **(1)**

$$\text{Median} = \frac{\text{Mode} + 2\text{Mean}}{3} = \frac{26.6+2\times 28.1}{3} = \frac{26.6+56.2}{3} = \frac{82.8}{3}$$

Median = 27.6 **(1)**

Section – C

19. Given numbers = 72, 126, 168

$72 = 2^3 \times 3^2$

$126 = 3^2 \times 2 \times 7$

$168 = 2^3 \times 3 \times 7$

HCF $= 2 \times 3 = 6$ **(1)**

LCM $= 2^3 \times 3^2 \times 7 = 504$ **(1)**

HCF × LCM $= (2 \times 3) \times (2^3 \times 3^2 \times 7) = 2^4 \times 3^3 \times 7$

Product of numbers $= 2^3 \times 3^2 \times 3^2 \times 2 \times 7 \times 2^3 \times 3 \times 7 = 2^7 \times 3^5 \times 7^2$ **(1)**

Therefore, HCF × LCM ≠ Product of the numbers.

20. Let us assume, to the contrary, that $3+\sqrt{2}$ is rational. **(½)**

That is, we can find co-prime a and b ($b \neq 0$) such that

$$3+\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 3 = \frac{a-3b}{b} \quad \textbf{(1)}$$

As a and b are integers, therefore $\frac{a-3b}{b}$ is rational, so $\sqrt{2}$ is rational. **(1)**

But this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption that $3+\sqrt{2}$ is rational is incorrect and we conclude that $3+\sqrt{2}$ is irrational. **(½)**

21. We know that

Dividend – Remainder is always divisible by the divisor. **(½)**

It is given that

$$x^4 + 2x^3 + 8x^2 + 12x + 18$$

when divided by $x^2 + 5$ leaves the remainder $px + q$.

Therefore, $x^4 + 2x^3 + 8x^2 + 12x + 18 - (px + q)$ is exactly divisible by $x^2 + 5$.

i.e., $x^4 + 2x^3 + 8x^2 + (12 - p)x + 18 - q$ is exactly divisible by $x^2 + 5$.

Now,

$$
\begin{array}{r|l}
 & x^2 + 2x + 3 \\
\hline
x^2 + 5 & x^4 + 2x^3 + 8x^2 + (12 - p)x + 18 - q \\
 & x^4 \qquad\quad + 5x^2 \\
 & \underline{- \qquad\qquad -} \\
 & \quad 2x^3 + 3x^2 + (12 - p)x \\
 & \quad 2x^3 \qquad + 10x \\
 & \quad \underline{- \qquad\qquad -} \\
 & \qquad 3x^2 + (2 - p)x + 18 - q \\
 & \qquad 3x^2 \qquad\qquad + 15 \\
 & \qquad \underline{- \qquad\qquad\quad -} \\
 & \qquad (2 - p)x - 15 + 18 - q \\
 & \quad = (2 - p)x + 3 - q
\end{array}
$$

(1½)

As $x^4 + 2x^3 + 8x^2 + (12 - p)x + 18 - q$ is exactly divisible by $x^2 + 5$

So, remainder = 0

$(2 - p)x + 3 - q = 0$

$2 - p = 0$ and $3 - q = 0$ $\quad p = 2$ and $q = 3$ **(1)**

22. Let the present age of Nuri be x years and present age of Sonu be y years.

Now, five years ago

Nuri was $(x - 5)$ years and Sonu was $(y - 5)$ years old.

According to question, we have

$(x - 5) = 3(y - 5) \qquad x - 5 = 3y - 15$

$x - 3y = -15 + 5 = -10$

$x - 3y = -10 \qquad \ldots(i)$

Again, ten years later,

Nuri will be $(x + 10)$ years and Sonu will be $(y + 10)$ years.

So, according to question, $(x + 10) = 2(y + 10) \qquad x + 10 = 2y + 20$

$x - 2y = 20 - 10$

$x - 2y = 10 \qquad \ldots(ii)$

Thus, we have system of equations (i) and (ii) **(2)**

Subtracting (ii) from (i), we have

$$
\begin{array}{r}
x - 3y = -10 \\
\underline{\underset{-}{}x \underset{+}{-} 2y = \underset{-}{}10} \\
-y = -20
\end{array}
\qquad y = 20
$$

(½)

Putting the value of y in equation (i), we have

$x - 3 \times (20) = -10$

$x - 60 = -10 \qquad x = -10 + 60 = 50$ **(½)**

Hence, present age of Nuri = 50 years.

and present age of Sonu = 20 years.

OR

Let the fixed charges be ₹ x and the remaining charges be ₹ y per km.

According to question

$x + 70y = 500$ ...(*i*) **(1)**

and $x + 100y = 680$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$\begin{array}{r} x + 70y = 500 \\ \underline{-x \pm 100y = -680} \\ -30y = -180 \end{array} \qquad y = \frac{180}{30} = 6$$ **(1)**

Putting $y = 6$ in (*i*), we get

$x + (70 \times 6) = 500 \qquad x = 500 - 420 = 80$

Thus, Fixed charges = ₹ 80, and Rate = ₹ 6 per km. **(1)**

23. In ΔBMC and ΔEMD, we have

$CM = DM$ (M is the mid-point of CD)

$CMB = DME$ (vertically opposite S)

$MBC = MED$ (alternate angles)

$\Delta BMC \quad \Delta EMD$ (AAS congruence criterion)

$BC = DE$ (CPCT) ...(*i*) **(1)**

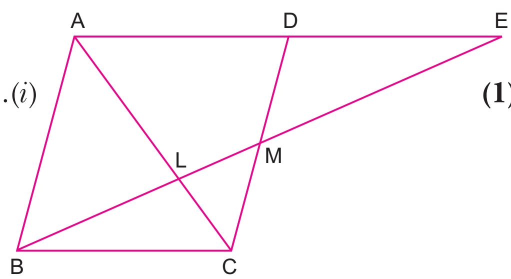


Fig. 9

Now, in ΔAEL and ΔCBL, we have

$ALE = CLB$ (vertically opposite S)

$EAL = BCL$ (alternate angles)

$\Delta AEL \sim \Delta CBL$ (AA similarity criterion)

$\frac{EL}{BL} = \frac{AE}{BC}$ ...(*ii*) **(1)**

Also, $AD = BC$ (opposite sides of $a \| gm$) ...(*iii*)

Now, $AE = AD + DE = BC + BC$ (using (*i*) and (*iii*))

$AE = 2BC$

From (*ii*), $\frac{EL}{BL} = \frac{2BC}{BC}$

$\frac{EL}{BL} = 2 \qquad EL = 2BL$ **(1)**

OR

To prove: $DE^2 = BD \times EC$

In ΔBAC, $1 + 2 + BAC = 180°$

$1 + 2 + 90° = 180°$ [$\because$ $BAC = 90°$]

$1 + 2 = 180° – 90°$

$1 + 2 = 90°$ ...(*i*)

In ΔGDB, $2 + 4 + 5 = 180°$

$2 + 90° + 5 = 180°$

$2 + 5 = 90°$ ... (*ii*)

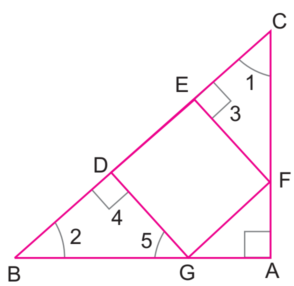


Fig. 10

From (*i*) and (*ii*)

$1 + 2 = 2 + 5 \qquad 1 = 5$ **(1)**

Now in ΔGDB and ΔCEF

$1 = 5$ [Proved above]

$4 = 3$ [each 90°]

$\Delta GDB \sim \Delta CEF$ [*AA* similarity criterion]

$\frac{DG}{EC} = \frac{BD}{EF}$ ... (*iii*) **(1)**

Also $DG = DE = EF$ (sides of a square) ... (*iv*)

From (*iii*) and (*iv*)

$\frac{DE}{EC} = \frac{BD}{DE}$

$DE^2 = BD \times EC$ **(1)**

24. **Given:** A quadrilateral *ABCD*. Its diagonals AC and BD meet at the point E such that $\frac{AE}{EC} = \frac{BE}{ED}$.

To prove: Quadrilateral *ABCD* is trapezium.

Construction: Draw *FG* parallel to *DC* passing through *E*.

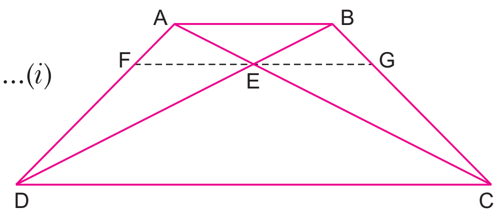


Fig. 11

Proof: $\frac{AE}{EC} = \frac{BE}{ED}$ (Given) ...(*i*)

In triangle *BDC*,

$EG \parallel DC$ ($\because$ $FG \parallel DC$)

$\frac{BE}{ED} = \frac{BG}{GC}$ (Using Thale's theorem) ...(*ii*) **(1)**

From (*i*) and (*ii*),

$\frac{AE}{EC} = \frac{BG}{GC}$

$EG \parallel AB$ (Using converse of basic proportionality theorem in ΔCBA) **(1)**

$FG \parallel AB$

But *FG* is drawn parallel to *DC*

So, $AB \parallel DC$

(Two lines parallel to the same line are parallel to each other)

ABCD is a trapezium. **(1)**

25. We are given $\tan\theta + \sin\theta = m$, and $\tan\theta - \sin\theta = n$, then

LHS $= (m^2 - n^2) = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$ **(1)**

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta = 4\sqrt{\tan^2\theta\sin^2\theta} = 4\sqrt{\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$ **(1)**

$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)} = 4\sqrt{mn} =$ RHS **(1)**

OR

We have, $\tan\theta + 1 = \sqrt{2}$

$\frac{\sin\theta}{\cos\theta} + 1 = \sqrt{2}$ $\frac{\sin\theta + \cos\theta}{\cos\theta} = \sqrt{2}$

$$\sin\theta + \cos\theta = \sqrt{2}\cos\theta \qquad ...(i) \qquad \textbf{(1)}$$

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$\cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$$

$$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta.\cos\theta$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\sin\theta + \cos\theta}$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta.\cos\theta}{\sqrt{2}\cos\theta} \qquad [\because \text{ using } (i)] \qquad \textbf{(1)}$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta \qquad \textbf{(1)}$$

26. Consider an equilateral triangle ABC with each side of length $2a$. As each angle of an equilateral triangle is 60° therefore each angle of triangle ABC is 60°.

Draw $AD \quad BC$. As ΔABC is equilateral, therefore, AD is the bisector of $\quad A$ and D is the mid-point of BC.

$$BD = DC = a \text{ and } \quad BAD = 30° \qquad \textbf{(1)}$$

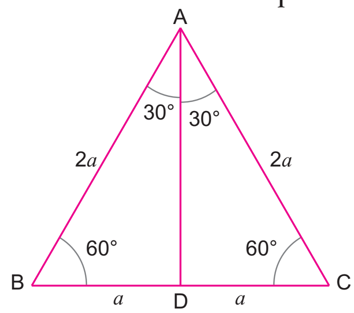


Fig. 12

By Pythagoras theorem, we have

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + a^2 = (2a)^2 \qquad AD^2 = 3a^2$$

$$AD = \sqrt{3}\,a \qquad \textbf{(1)}$$

In right triangle ADB, we have

$$\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \textbf{(1)}$$

27.

Classes	Frequency
10 –20	4
20 – 30	8
30 – 40	10 (f_0)
40 – 50	12 (f_1)
50 – 60	10 (f_2)
60 – 70	4
70 – 80	2

Here, Modal class = 40 – 50, and

$$l = 40,\ h = 10,\ f_1 = 12,\ f_0 = 10,\ f_2 = 10 \qquad \textbf{(1)}$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h \qquad \textbf{(1)}$$

$$= 40 + \left(\frac{12 - 10}{2 \times 12 - 10 - 10}\right) \times 10$$

$$= 40 + \frac{20}{4} = 40 + 5 = 45 \qquad \textbf{(1)}$$

28.

Class Interval	Frequency	Cumulative Frequency
0 – 6	4	4
6 – 12	x	$4 + x$
12 – 18	5	$9 + x$
18 – 24	y	$9 + x + y$
24 – 30	1	$10 + x + y$

(1)

It is given that $n = 20$

So, $10 + x + y = 20$, *i.e.*, $x + y = 10$ ...(*i*)

It is also given that median = 14.4

which lies in the class interval 12 – 18.

So, $l = 12, f = 5, cf = 4 + x, h = 6$ **(1)**

Using the formula

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right)h$$

We get, $14.4 = 12 + \left(\frac{10 - (4 + x)}{5}\right)6$

or $14.4 = 12 + \left(\frac{6 - x}{5}\right)6$ or $x = 4$ ...(*ii*) **(1)**

From (*i*) and (*ii*), $y = 6$

Section – D

29. Let $p(x) = x^3 + 2x^2 + kx + 3$

Then, $p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

$27 + 18 + 3k + 3 = 21 \qquad 3k = -27 \qquad k = -9$ **(1)**

Hence, the given polynomial will become $x^3 + 2x^2 - 9x + 3$

Now,

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x - 3 & x^3 + 2x^2 - 9x + 3 \\ & \underline{-x^3 \mp 3x^2} \\ & 5x^2 - 9x + 3 \\ & \underline{-5x^2 \mp 15x} \\ & 6x + 3 \\ & \underline{-6x \mp 18} \\ & 21 \end{array}$$

So, $x^3 + 2x^2 - 9x + 3 = (x^2 + 5x + 6)(x - 3) + 21$ **(1)**

Also, $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x(x + 3) + 2(x + 3) = (x + 3)(x + 2)$

$x^3 + 2x^2 - 9x + 3 = (x - 3)(x + 2)(x + 3) + 21$ **(1)**

$x^3 + 2x^2 - 9x - 18 = (x - 3)(x + 2)(x + 3)$

So, the zeroes of $x^3 + 2x^2 - 9x - 18$ are 3, –2, –3. **(1)**

30. We have,

$x + 3y = 6$ ...(*i*)

$2x - 3y = 12$ ...(*ii*)

From equation (*i*), we have

$x = 6 - 3y$

x	3	0	6
y	1	2	0

(½)

From equation (*ii*), we have

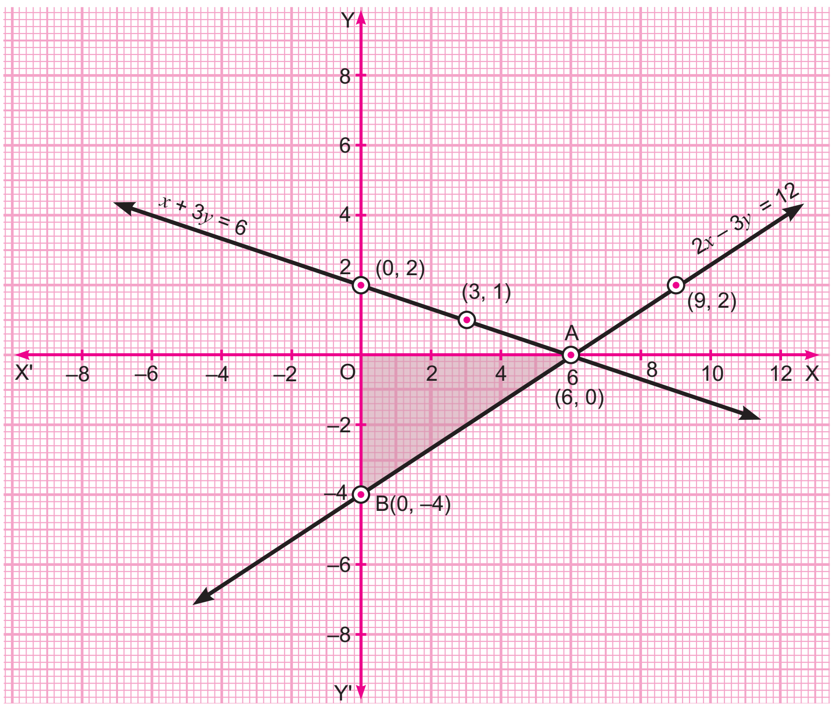
$2x = 12 + 3y$

$$x = \frac{12 + 3y}{2}$$

x	6	9	0
y	0	2	– 4

(½)

Plotting the points (3, 1), (0, 2), (6, 0), (9, 2) and (0, – 4) on the graph paper with a suitable scale and drawing lines joining them equation wise, we obtain the graph of the lines represented by the equations $x + 3y = 6$ and $2x - 3y = 12$ as shown in figure.



(2)

Fig. 13

It is evident from the graph that the two lines intersect at point (6, 0).

Area of the region bounded by $x = 0$, $y = 0$ and $2x - 3y = 12$.

$$= \text{Area of } \Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units.} \qquad \textbf{(1)}$$

31. **Statement:** In a triangle, if square of one side is equal to sum of the squares of the other two sides, then the angle opposite to the first side is a right angle. **(1)**

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$.

To Prove: $B = 90°$.

Construction: We construct a ΔPQR right-angled at Q such that $PQ = AB$ and $QR = BC$ **(½)**

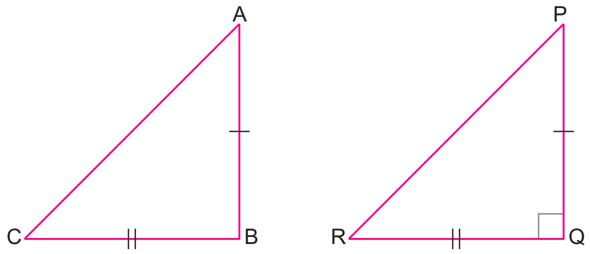


Fig. 14

Proof: Now, from ΔPQR, we have,

$PR^2 = PQ^2 + QR^2$ [Pythagoras theorem, as $Q = 90°$]

or, $PR^2 = AB^2 + BC^2$ [By construction] ...(*i*)

But $AC^2 = AB^2 + BC^2$ [Given] ...(*ii*)

So, $AC^2 = PR^2$ [From (*i*) and (*ii*)] ...(*iii*)

$AC = PR$ **(1)**

Now, in ΔABC and ΔPQR,

$AB = PQ$ [By construction]

$BC = QR$ [By construction]

$AC = PR$ [Proved in (*iii*)]

So, $\Delta ABC \quad \Delta PQR$ [SSS congruency] **(1)**

Therefore, $B = Q$ [CPCT]

But $Q = 90°$ [By construction]

So, $B = 90°$ **(½)**

OR

Refer to CBSE Sample Question Paper Q. N. 30.

32. $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

L.H.S. $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}}$ **(½)**

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}} = \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta}$ **(1)**

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)} = \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\sin\theta(\sin\theta - \cos\theta)} \quad \textbf{(1)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{\cos\theta.\sin\theta(\sin\theta - \cos\theta)} \quad \textbf{(½)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\cos\theta.\sin\theta} = \frac{1 + \sin\theta\cos\theta}{\cos\theta.\sin\theta} \quad \textbf{(½)}$$

$$= \frac{1}{\cos\theta.\sin\theta} + \frac{\sin\theta\cos\theta}{\sin\theta.\cos\theta} = 1 + \sec\theta \text{ cosec}\,\theta = \text{RHS} \quad \textbf{(½)}$$

OR

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \text{ cosec } A$$

LHS
$$= \sqrt{\frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}} + \sqrt{\frac{\frac{1}{\cos A} + 1}{\frac{1}{\cos A} - 1}} = \sqrt{\frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}}} + \sqrt{\frac{\frac{1 + \cos A}{\cos A}}{\frac{1 - \cos A}{\cos A}}} \quad \textbf{(1)}$$

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A}} \quad \textbf{(½)}$$

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A}} \quad \text{(On rationalising)} \quad \textbf{(½)}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} + \sqrt{\frac{(1 + \cos A)^2}{1 - \cos^2 A}} = \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1 + \cos A)^2}{\sin^2 A}} \quad \textbf{(1)}$$

$$= \frac{1 - \cos A}{\sin A} + \frac{1 + \cos A}{\sin A} = \frac{1 - \cos A + 1 + \cos A}{\sin A} = \frac{2}{\sin A} = 2 \text{ cosec } A = \text{RHS} \quad \textbf{(1)}$$

Alternate Method:

LHS $\dfrac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \dfrac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$

$$= \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec A + 1}\sqrt{\sec A - 1}} = \frac{2\sec A}{\sqrt{(\sec A + 1)(\sec A - 1)}} \quad \textbf{(1+1)}$$

$$= \frac{2\sec A}{\sqrt{\sec^2 A - 1}} = \frac{2\sec A}{\tan A} = \frac{2}{\sin A} = 2 \text{ cosec} A = \text{RHS} \quad \textbf{(1+1)}$$

33. $\frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \tan 32° - \frac{5}{3}\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$

$\tan 32° = \cot(90 - 32)° = \cot 58°$ and $\tan 45° = 1$

$\tan 53° = \cot(90 - 53)° = \cot 37° = \dfrac{1}{\tan 37°}$

$\tan 77° = \cot(90 - 77)° = \cot 13° = \dfrac{1}{\tan 13°}$ **(2)**

Putting these values in the given expression, we get

$$\frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \cot 58° - \frac{5}{3}\tan 13° \tan 37° \times 1 \times \frac{1}{\tan 37°}\,\frac{1}{\tan 13°}$$

$$= \frac{2}{3}(\text{cosec}^2 58 - \cot^2 58) - \frac{5}{3} \times 1 \quad \textbf{(1)}$$

$$= \frac{2}{3} \times 1 - \frac{5}{3} \qquad (\because \ \text{cosec}^2\theta - \cot^2\theta = 1)$$

$$= \frac{2-5}{3} = \frac{-3}{3} = -1. \quad \textbf{(1)}$$

34. Here $\overline{X} = 53$

Age	Frequency (f)	Mid-point (x)	fx
0–20	15	10	150
20–40	f_1	30	$30f_1$
40–60	21	50	1050
60–80	f_2	70	$70f_2$
80–100	17	90	1530
Total	100		$\Sigma fx = 2730 + 30f_1 + 70f_2$

(1)

$$\text{Mean } (\overline{X}) = \frac{\Sigma fx}{\Sigma f}$$

$$53 = \frac{2730 + 30f_1 + 70f_2}{100}$$

$$5300 = 2730 + 30f_1 + 70f_2$$

$$30f_1 + 70f_2 = 5300 - 2730$$

$$30f_1 + 70f_2 = 2570$$

$$3f_1 + 7f_2 = 257 \quad \dots (i) \quad \textbf{(1)}$$

Also,
$$f_1 + f_2 = 100 - 15 - 21 - 17$$

$$f_1 + f_2 = 100 - 53$$

$$f_1 + f_2 = 47 \quad \dots (ii)$$

Multiply equation (*ii*) by 3.

$$3f_1 + 3f_2 = 141 \quad \dots (iii) \quad \textbf{(1)}$$

Subtracting equation (*iii*) from equation (*i*), we get

$$3f_1 + 7f_2 = 257$$

$$\underline{\ _-3f_1 \underset{-}{+} 3f_2 = _-141\ }$$

$$4f_2 = 116$$

$$f_2 = 29$$

Putting $f_2 = 29$ in (*ii*), we get

$$f_1 = 18$$

$$f_1 = 18 \text{ and } f_2 = 29. \quad \textbf{(1)}$$

Mathematics

Model Question Paper (Solved) – 3

Summative Assessment – I

Time: 3 to 3½ hours | **Maximum Marks: 80**

General Instructions: *As given in CBSE Sample Question Paper.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. Which of the following will have a terminating decimal expansion?

(*a*) $\frac{47}{18}$ (*b*) $\frac{41}{28}$ (*c*) $\frac{125}{441}$ (*d*) $\frac{37}{128}$

2. If the HCF of 65 and 117 is expressible in the form 65 m – 117, then the value of m is

(*a*) 4 (*b*) 2

(*c*) 1 (*d*) 3

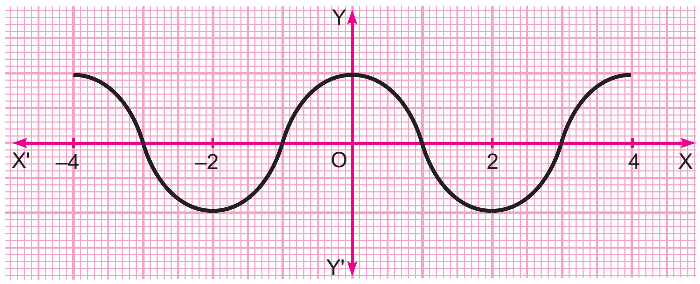


Fig. 1

3. The number of zeroes lying between –2 to 2 of the polynomial $f(x)$, whose graph in Fig. 1 is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 1

4. A father is thrice of his son's age. After twelve years, his age will be twice of his son. The present ages, in years of the son and the father are, respectively

(*a*) 14, 42 (*b*) 11, 33

(*c*) 12, 36 (*d*) 16, 48

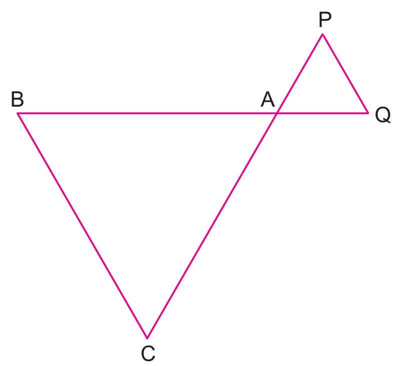


Fig. 2

5. In Fig. 2, $\Delta ACB \sim \Delta APQ$ if $BA = 6$ cm, $BC = 8$ cm, $PQ = 4$ cm. Then AQ is equal to

(*a*) 2 cm (*b*) 2.5 cm

(*c*) 3 cm (*d*) 3.5 cm

6. If $\sin\theta = \frac{1}{3}$, then the value of $(9\cot^2\theta + 9)$ is

(*a*) $\frac{1}{8}$ (*b*) 1 (*c*) 9 (*d*) 81

7. The value of $(\tan 1°\tan 2°\tan 3°\ldots\ldots\tan 89°)$ is

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 2

8. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(*a*) $\frac{\sqrt{b^2 - a^2}}{b}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{b}{\sqrt{b^2 - a^2}}$ (*d*) $\frac{a}{\sqrt{b^2 - a^2}}$

9. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

10. Which of the following cannot be determined graphically?

(*a*) Mode (*b*) Mean (*c*) Median (*d*) None of these

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Given that HCF (54, 336) = 6, find LCM (54, 336).

12. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$. Find the values of a and b.

13. Without drawing the graphs, state whether the following pair of linear equations will represent intersecting lines, coincident lines or parallel lines:

$$6x - 3y + 10 = 0$$
$$2x - y + 9 = 0$$

Justify your answer.

OR

Determine the values of a and b for which the following system of linear equations has infinite solutions:

$$2x - (a - 4)y = 2b + 1$$
$$4x - (a - 1)y = 5b - 1$$

14. In Fig. 3, find the value of x for which $DE \parallel AB$.

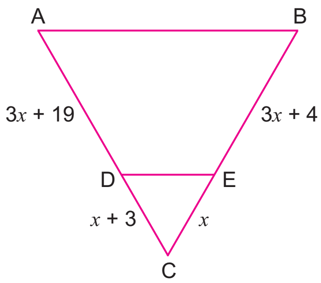


Fig. 3

15. In Fig. 4, if ΔABE ΔACD, show that $\Delta ADE \sim \Delta ABC$.

16. If $\sin(A + B) = \sin A \cos B + \cos A \sin B$, then find the value of $\sin 75°$.

17. Calculate mode when arithmetic mean is 146 and median is 130.

18. Show that $(X_1 - \overline{X}) + (X_2 - \overline{X}) + (X_3 - \overline{X}) + \ldots\ldots\ldots\ldots + (X_n - \overline{X}) = 0$.

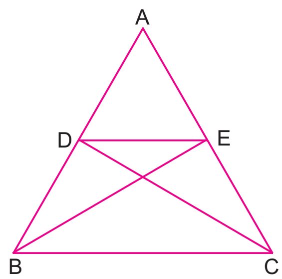


Fig. 4

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

20. If p is a prime number, prove that $\sqrt{p}$ is irrational.

21. Find the zeroes of polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

OR

If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 6x + 4$, find the value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

22. Check graphically whether the pair of equations

$$x + 3y = 6; \quad 2x - 3y = 12$$

is consistent. If so, solve them graphically.

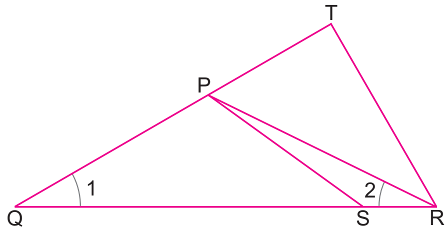


Fig. 5

23. In Fig. 5, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.

24. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

25. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

OR

If $\sec\theta + \tan\theta = m$, show that $\left(\frac{m^2 - 1}{m^2 + 1}\right) = \sin\theta$.

26. Find the value of tan 60°, geometrically.

27. Find the mean for the following data:

Class	Frequency
0–10	8
10–20	16
20–30	36
30–40	34
40–50	6
Total	**100**

28. Find the median of the following frequency distribution:

Marks	Frequency
0 – 100	2
100 – 200	5
200 – 300	9
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	15
700 – 800	9
800 – 900	7
900 – 1000	4

OR

Calculate the missing frequency for the following frequency distribution, it being given that the median of the distribution is 24.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	25	?	18	7

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

OR

It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately?

30. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

31. State and prove Pythagoras theorem.

OR

State and prove Basic Proportionality Theorem.

32. Prove that: $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$.

33. Without using trigonometric tables, evaluate the following:

$$\frac{\operatorname{cosec}^2 65° - \tan^2 25°}{\sin^2 17° + \sin^2 73°} + \frac{1}{\sqrt{3}}(\tan 10° \tan 30° \tan 80°)$$

34. The following table gives production yield per hectare of wheat of 100 farms of a village.

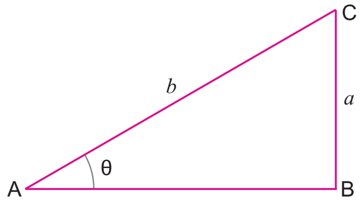
Production yield (in kg/hec)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

Solution

Section – A

1.	(*d*)	As prime factorisation of 128 (*q*) is of the form $2^7 \times 5°$
2.	(*b*)	$65 = 13 \times 5;\ 117 = 3 \times 3 \times 13$ HCF (65, 117) = 13 $65 \times m - 117 = 13$ $\quad 65m = 130 \quad m = 2$
3.	(*a*)	
4.	(*c*)	

5.	(*c*)	As $\Delta ACB \sim \Delta APQ$ So, $\frac{AB}{AQ} = \frac{BC}{PQ}$ $\quad \frac{6}{AQ} = \frac{8}{4}$ $\quad \frac{6}{AQ} = 2$ $\quad AQ = 3$ cm
6.	(*d*)	$9\cot^2\theta + 9 = 9(\cot^2\theta + 1) = 9\,\text{cosec}^2\theta$ Now, $\sin\theta = \frac{1}{3}$ $\quad 3 = \frac{1}{\sin\theta}$ $\quad \text{cosec}\,\theta = 3$ $9\cot^2\theta + 9 = 9\,\text{cosec}^2\theta = 9 \times (3)^2 = 81$
7.	(*c*)	$\tan 1°\tan 2° \ldots \tan 45° \ldots \tan 88° \tan 89° = \tan 1° \tan 2° \ldots \tan 45° \ldots \cot 2° \cot 1°$ $= \left(\tan 1° \times \frac{1}{\tan 1°}\right)\left(\tan 2° \times \frac{1}{\tan 2°}\right) \ldots\ldots \tan 45°$ $= 1 \times 1 \times \ldots \times 1 = 1$
8.	(*a*)	$\sin\theta = \frac{BC}{AC} = \frac{a}{b}$ $AB^2 + BC^2 = AC^2$ $\quad AB^2 + a^2 = b^2$ $AB = \sqrt{b^2 - a^2}$ $\cos\theta = \frac{AB}{AC} = \frac{\sqrt{b^2 - a^2}}{b}$  **Fig. 6**
9.	(*c*)	$\sin\theta - \cos\theta = 0$ $\quad \sin\theta = \cos\theta$ $\quad \theta = 45°$ $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45° = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$
10.	(*b*)	

1×10=10

Section – B

11. LCM (54, 336) × HCF (54, 336) = 54 × 336 **(1)**

$$\text{LCM}(54, 336) = \frac{54 \times 336}{HCF(54, 336)} = \frac{54 \times 336}{6} = 3024$$ **(1)**

12.

$$\begin{array}{r|l} & 2x^2 + 5 \\ \hline 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & \underline{-6x^4 \pm 8x^3 \pm 2x^2} \\ & 15x^2 + 21x + 7 \\ & \underline{-15x^2 \pm 20x \pm 5} \\ & x + 2 \end{array}$$

(1½)

Comparing $(x + 2)$ with the given remainder $ax + b$, we get $a = 1$, $b = 2$. **(½)**

13. The given system of equations is

$$6x - 3y + 10 = 0 \text{ and } 2x - y + 9 = 0$$

Here, $a_1 = 6, b_1 = -3, c_1 = 10;\quad a_2 = 2, b_2 = -1, c_2 = 9$

We have, $\quad \frac{a_1}{a_2} = \frac{6}{2} = 3;\quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3;\quad \frac{c_1}{c_2} = \frac{10}{9}$ **(1)**

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will represent parallel lines. **(1)**

OR

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{a-4}{a-1}$, $\frac{c_1}{c_2} = \frac{2b+1}{5b-1}$ **(½)**

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ **(½)**

$$\frac{1}{2} = \frac{a-4}{a-1} = \frac{2b+1}{5b-1}$$

$\frac{a-4}{a-1} = \frac{1}{2}$ $\qquad 2a - 8 = a - 1 \qquad a = 7$ **(½)**

$\frac{2b+1}{5b-1} = \frac{1}{2}$ $\qquad 4b + 2 = 5b - 1 \qquad b = 3$ **(½)**

14. $DE \| AB$, if $\frac{AD}{DC} = \frac{BE}{EC}$ (By converse of Thales theorem) **(1)**

$\frac{3x+19}{x+3} = \frac{3x+4}{x}$ $\qquad 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$6x = 12 \qquad x = 2$ **(1)**

15. We have

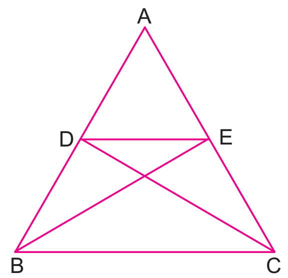


Fig. 7

$\Delta ABE \quad \Delta ACD$

$AB = AC$ (CPCT)

and $AE = AD$ (CPCT)

or $\frac{AB}{AC} = 1$ ...(*i*) **(½)**

and $\frac{AE}{AD} = 1$

$\frac{AD}{AE} = 1$ ...(*ii*) **(½)**

From (*i*) and (*ii*), we have

$\frac{AB}{AC} = \frac{AD}{AE}$ or $\frac{AB}{AD} = \frac{AC}{AE}$

and $A = A$ (Common)

$\Delta ADE \quad \Delta ABC$ (*SAS* criterion of similarity) **(1)**

16. $\sin 75° = \sin (45° + 30°)$

As $\sin (A + B) = \sin A \cos B + \cos A \sin B$ **(½)**

$$\sin (45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$ **(1½)**

17. Mode = 3 median – 2 mean **(1)**

$= 3 \times 130 - 2 \times 146 = 390 - 292 = 98$ **(1)**

18. LHS $= (X_1 - \overline{X}) + (X_2 - \overline{X}) + (X_3 - \overline{X}) + \ldots\ldots\ldots\ldots + (X_n - \overline{X})$

$= (X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots\ldots + X_n) - (\overline{X} + \overline{X} + \overline{X} + \overline{X} + \ldots\ldots\ldots\ldots\ldots)$ **(1)**

$= n\,\overline{X} - n\,\overline{X} = 0 = $ RHS **(1)**

Section – C

19. Let a be any positive integer. Then it is of the form $3q$, $3q + 1$ or $3q + 2$. So, we have the following cases:

Case (*i*) When $a = 3q$

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$ where $m = 3q^3$ **(1)**

Case (*ii*) When $a = 3q + 1$

$a^3 = (3q + 1)^3 = (3q)^3 + 3(3q)^2 .1 + 3(3q).1^2 + 1^3$

$= 27q^3 + 27q^2 + 9q + 1 = 9q(3q^2 + 3q + 1) + 1$

$= 9m + 1$, where $m = q(3q^2 + 3q + 1)$ **(1)**

Case (*iii*) When $a = 3q + 2$

$a^3 = (3q + 2)^3 = (3q)^3 + 3(3q)^2 .2 + 3(3q).2^2 + 2^3$

$= 27q^3 + 54q^2 + 36q + 8 = 9q(3q^2 + 6q + 4) + 8$

$= 9m + 8$, where $m = q(3q^2 + 6q + 4)$ **(1)**

Hence, a^3 is either of the form $9m$ or $9m + 1$ or $9m + 8$

20. Let us assume, to the contrary, that $\sqrt{p}$ is rational. **(½)**

So, we can find integers r and s ($\neq 0$) such that

$$\sqrt{p} = \frac{r}{s} \quad \ldots(i)$$

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{p} = \frac{a}{b}$, where a and b are co-prime

So, $b\sqrt{p} = a$ **(½)**

Squaring both the sides and rearranging,

We get, $pb^2 = a^2$

p divides a^2 [$\because$ p divides pb^2]

p divides a [$\because$ p is prime and p divides a^2 p divides a] **(1)**

So, we can write $a = pc$, for some integer c,

Substituting for a in (*i*), we get

$pb^2 = (pc)^2$, that is $b^2 = pc^2$ p divides b^2, So p divides b

Therefore, a and b have at least p as a common factor.

But this contradicts the fact that a and b have no common factors other than 1. **(1)**

So, our assumption is wrong and we conclude that $\sqrt{p}$ is irrational.

21. $x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(6x^2 + x - 12)$

$= \frac{1}{6}[6x^2 + (9 - 8)x - 12] = \frac{1}{6}[6x^2 + 9x - 8x - 12]$

$= \frac{1}{6}[3x(2x + 3) - 4(2x + 3)] = \frac{1}{6}(3x - 4)(2x + 3)$ **(1)**

Hence, $\frac{4}{3}$ and $-\frac{3}{2}$ are the zeroes of the given polynomial.

The given polynomial is $x^2 + \frac{1}{6}x - 2$.

The sum of zeroes $= \frac{4}{3} + \left(-\frac{3}{2}\right) = \frac{-1}{6} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$ **(1)**

and the product of zeroes $= \frac{4}{3} \times \left(\frac{-3}{2}\right) = -2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$ **(1)**

OR

$$f(x) = 3x^2 - 6x + 4$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-6)}{3} = 2 \text{ and } \alpha . \beta = \frac{c}{a} = \frac{4}{3}$$ **(1)**

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$ **(1)**

$$= \frac{2^2 - 2 \times \frac{4}{3}}{\frac{4}{3}} + \frac{2 \times 2}{\frac{4}{3}} + 3 \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} + 3 + 4 = 1 + 7 = 8$$ **(1)**

22. Here, $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{-3} = -1$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so given system of equations is consistent. **(½)**

x	0	3	6
$y = \frac{6 - x}{3}$	2	1	0

x	0	3	6
$y = \frac{2x - 12}{3}$	– 4	– 2	0

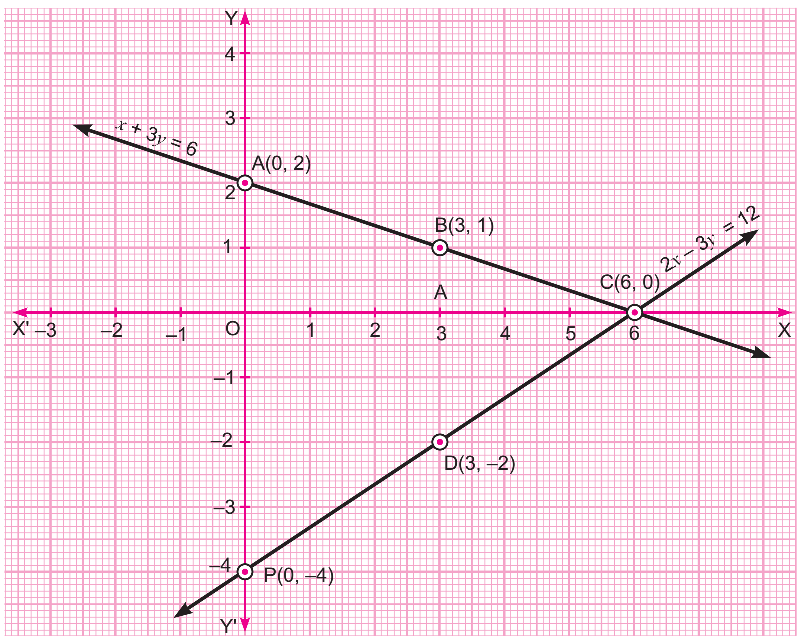


Fig. 8 **(2)**

As the lines representing the pair of equations intersect each other at the point C(6, 0) therefore, the given pair of equations is consistent.

Solution: $x = 6$, $y = 0$ **(½)**

23. We have, $\frac{QR}{QS} = \frac{QT}{PR}$ (Given) ...(*i*)

As $1 = \quad 2$ (Given)

$PR = PQ$ (Sides opposite to equal angles are equal) ...(*ii*) **(1)**

From (*i*) and (*ii*), we have

$\frac{QR}{QS} = \frac{QT}{PQ}$ $\quad\quad$ $\frac{PQ}{QT} = \frac{QS}{QR}$ **(½)**

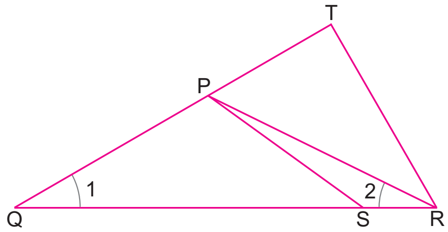


Fig. 9

Now in ΔPQS and ΔTQR, we have

$$\frac{PQ}{QT} = \frac{QS}{QR}$$

and $\quad PQS = \quad TQR = \quad Q$ (Common)

$\Delta PQS \sim \Delta TQR$ by SAS criterion of similarity. **(1½)**

24. Let ABC be an equilateral triangle and let D be a point on BC such that $BD = \frac{1}{3} BC$

To Prove: $9AD^2 = 7AB^2$

Construction: Draw $AE \quad BC$. Join AD. **(½)**

Proof: ΔABC is an equilateral triangle and $AE \quad DC$

$$BE = EC$$

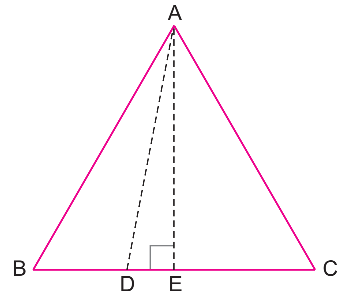


Fig. 10

Thus, we have

$$BD = \frac{1}{3} BC; \quad DC = \frac{2}{3} BC \text{ and } BE = EC = \frac{1}{2} BC$$ **(½)**

In ΔAEB

$AE^2 + BE^2 = AB^2$ (By Pythagoras Theorem) **(½)**

$AE^2 = AB^2 - BE^2$

$AD^2 - DE^2 = AB^2 - BE^2$ $\quad$ [$\because$ In ΔAED, $AD^2 = AE^2 + DE^2$]

$AD^2 = AB^2 - BE^2 + DE^2$

$$AD^2 = AB^2 - \left(\frac{1}{2} BC\right)^2 + (BE - BD)^2$$

$$AD^2 = AB^2 - \frac{1}{4} BC^2 + \left(\frac{1}{2} BC - \frac{1}{3} BC\right)^2$$

$$AD^2 = AB^2 - \frac{1}{4} BC^2 + \left(\frac{BC}{6}\right)^2$$ **(½)**

$$AD^2 = AB^2 - BC^2\left(\frac{1}{4} - \frac{1}{36}\right) \quad\quad AD^2 = AB^2 - BC^2\left(\frac{8}{36}\right)$$

$9AD^2 = 9AB^2 - 2BC^2 \quad\quad 9AD^2 = 7AB^2 \quad (\because AB = AC)$ **(1)**

25. Given that,

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\sqrt{2}\cos\theta - \cos\theta = \sin\theta \quad\quad (\sqrt{2} - 1)\cos\theta = \sin\theta$

$$\cos\theta = \frac{\sin\theta}{(\sqrt{2}-1)} \quad \textbf{(1)}$$

$$\cos\theta = \frac{\sin\theta}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} \quad \textbf{(½)}$$

$$= \frac{(\sqrt{2}+1)\sin\theta}{2-1} = \sqrt{2}\sin\theta + \sin\theta \quad \textbf{(½)}$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta \quad \textbf{(1)}$$

OR

We have,

$$\frac{m^2-1}{m^2+1} = \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1} \quad \textbf{(½)}$$

$$= \frac{\sec^2 + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1} \quad \textbf{(½)}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta + \tan\theta} \qquad [\because\ \sec^2\theta - 1 = \tan^2\theta,\ \tan^2\theta + 1 = \sec^2\theta] \quad \textbf{(1)}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)}$$

$$= \tan\theta \times \frac{1}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta \quad \textbf{(1)}$$

Hence, $\dfrac{m^2-1}{m^2+1} = \sin\theta$

26. Consider an equilateral ΔABC. Let each side be $2a$. Since each angle in equilateral is 60°, therefore,

$$A = B = C = 60°$$

Draw AD BC

In ΔADB and ΔADC

$ADB = ADC$	(Each 90°)
$AB = AC$	(Sides of equilateral Δ)
$AD = AD$	(Common)
ΔADB ΔADC	(RHS congruency) **(1)**
$BD = DC$	(CPCT)

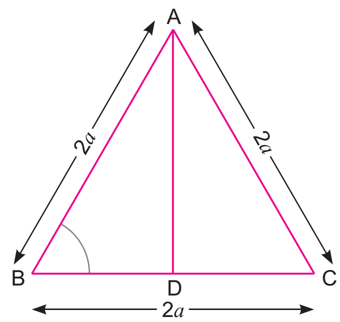


Fig. 11

$$BD = \frac{BC}{2} = \frac{2a}{2} = a \quad \textbf{(½)}$$

In right ΔADB

$$AD^2 + BD^2 = AB^2 \qquad \text{(Pythagoras theorem)}$$

$$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$$

$$AD^2 = 3a^2$$

$$AD = \sqrt{3}a \quad \textbf{(½)}$$

Now, in $\Delta ABD \tan B = \tan 60° = \dfrac{AD}{BD} = \dfrac{\sqrt{3}a}{a} = \sqrt{3}$ **(1)**

27.

Class	f_i	x_i	$u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 – 10	8	5	– 2	– 16
10 – 20	16	15	– 1	– 16
20 – 30	**36**	**25 = A**	0	0
30 – 40	34	35	1	34
40 – 50	6	45	2	12
Total	$n = \Sigma f_i = 100$		$\Sigma u_i = 0$	$\Sigma f_i u_i = -32 + 46 = 14$

(2)

$$\text{Mean}(\bar{X}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 25 + \frac{14}{100} \times 10 = 25 + 1.4 = 26.4$$ **(1)**

28.

Age	Frequency (f)	Cumulative frequency (cf)
0 – 100	2	2
100 – 200	5	7
200 – 300	9	16
300 – 400	12	28
400 – 500	17	45
500 – 600	20	65
600 – 700	15	80
700 – 800	9	89
800 – 900	7	96
900 – 1000	4	100

(1)

We have, $\left(\frac{n}{2}\right) = \frac{100}{2} = 50$

So, median lies in the class interval 500 – 600 **(½)**

Here, $l = 500,\ h = 100,\ f = 20,\ cf = 45,\ \frac{n}{2} = 50$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$ **(½)**

$$= 500 + \frac{50 - 45}{20} \times 100 = 500 + \frac{5}{20} \times 100$$

$$= 500 + 25 = 525$$ **(1)**

OR

Class interval	Frequency (f_1)	Cumulative frequency (*cf*)
0 – 10	5	5
10 – 20	25	30
20 – 30	f_1	$30 + f_1$
30 – 40	18	$48 + f_1$
40 – 50	7	$55 + f_1$
	$N = 55 + f_1$	

(1½)

Let f_1 be the frequency of class interval 20-30. Median is 24, which lies in 20-30, so median class is 20-30.

Now, median $= l + \left(\dfrac{\frac{N}{2} - cf}{f}\right) \times h$ **(½)**

$$24 = 20 + \dfrac{\left(\frac{55 + f_1}{2} - 30\right)}{f_1} \times 10$$

$$4 = \dfrac{(55 + f_1 - 60)}{2f_1} \times 10$$

$4f_1 = 5f_1 - 25 \qquad f_1 = 25$ **(1)**

Section – D

29. Let total number of rows be y and number of students in each row be x.

Total number of students $= xy$ **(½)**

Case I: If one student is extra in a row, there would be two rows less.

Now, number of rows $= (y - 2)$

Number of students in each row $= (x + 1)$

Total number of students = Number of rows × Number of students in each row

$$xy = (y - 2)(x + 1)$$

$$xy = xy + y - 2x - 2$$

$$xy - xy - y + 2x = -2$$

$$2x - y = -2 \qquad \ldots(i)$$ **(1)**

Case II: If one student is less in a row, there would be three rows more.

Now, Number of rows $= (y + 3)$

Number of students in each row $= (x - 1)$

Total number of students = Number of rows × Number of students in each row

$$xy = (y + 3)(x - 1)$$

$$xy = xy - y + 3x - 3$$

$$xy - xy + y - 3x = -3$$

$$-3x + y = -3 \quad \text{...}(ii) \quad \textbf{(1)}$$

On adding equation (*i*) and (*ii*), we have

$$\begin{aligned} 2x - y &= -2 \\ -3x + y &= -3 \\ \hline -x &= -5 \end{aligned}$$

$$x = 5 \quad \textbf{(½)}$$

Putting the value of x in equation (*i*), we get

$$2(5) - y = -2$$

$$10 - y = -2$$

$$-y = -2 - 10$$

$$-y = -12$$

or $$y = 12 \quad \textbf{(½)}$$

Total number of students in the class $= 5 \times 12 = 60$. **(½)**

OR

Let the time taken by the pipe of larger diameter to fill the pool be x hours and that taken by the pipe of smaller diameter pipe alone be y hours.

In x hours, the pipe of larger diameter fills the pool.

So, in 1 hour the pipe of larger diameter fills $\frac{1}{x}$ part of the pool, and so, in 4 hours, the pipe of larger diameter fills $\frac{4}{x}$ parts of the pool.

Similarly, in 9 hours, the pipe of smaller diameter fills $\frac{9}{y}$ parts of the pool.

According to the question,

$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad \text{...}(i) \quad \textbf{(1)}$$

Also, using both the pipes, the pool is filled in 12 hours.

So, $$\frac{12}{x} + \frac{12}{y} = 1 \quad \text{...}(ii) \quad \textbf{(1)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then equations (*i*) and (*ii*) become

$$4u + 9v = \frac{1}{2}$$

or $$8u + 18v = 1 \quad \text{...}(iii) \quad \textbf{(½)}$$

and $$12u + 12v = 1 \quad \text{...}(iv)$$

Multiplying equation (*iii*) by 3 and (*iv*) by 2 and subtracting, we get

$$\begin{aligned} 24u + 54v &= 3 \\ \underset{-}{}24u \underset{-}{+} 24v &= \underset{-}{}2 \\ \hline 30v &= 1 \end{aligned} \qquad v = \frac{1}{30} \quad \textbf{(½)}$$

Putting v in equation (iv), we get

$$12u + 12 \times \frac{1}{30} = 1 \qquad 12u + \frac{2}{5} = 1 \qquad 12u = 1 - \frac{2}{5} = \frac{3}{5}$$

$$12u = \frac{3}{5} \times \frac{1}{12} = \frac{1}{20}$$ **(½)**

So, $\frac{1}{x} = \frac{1}{20}, \frac{1}{y} = \frac{1}{30}$ or $x = 20, y = 30$ **(½)**

So, the pipe of larger diameter alone can fill the pool in 20 hours and the pipe of smaller diameter alone can fill the pool in 30 hours.

30. We know that if $x = \alpha$ is a zero of a polynomial, then $x - \alpha$ is a factor of $f(x)$. **(½)**

Since $\sqrt{2}$ and $-\sqrt{2}$ are zeroes of $f(x)$. Therefore, $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of $f(x)$. **(½)**

Now, we divide $f(x) = x^4 + 3x^3 - 20x^2 - 6x + 36$ by $g(x) = x^2 - 2$ to find the other zeroes of $f(x)$.

We have,

$$\begin{array}{r|l} & x^2 + 3x - 18 \\ x^2 - 2 & x^4 + 3x^3 - 20x^2 - 6x + 36 \\ & \pm x^4 \quad \mp 2x^2 \\ \hline & +3x^3 - 18x^2 - 6x \\ & \pm 3x^3 \quad \mp 6x \\ \hline & -18x^2 + 36 \\ & \mp 18x^2 \pm 36 \\ \hline & 0 \end{array}$$ **(1½)**

By division algorithm, we have

$$\begin{aligned} x^4 + 3x^3 - 20x^2 - 6x + 36 &= (x^2 - 2)(x^2 + 3x - 18) \\ &= [x^2 + (\sqrt{2})^2](x^2 + 6x - 3x - 18) \\ &= (x + \sqrt{2})(x - \sqrt{2})\{x(x + 6) - 3(x + 6)\} \\ &= (x + \sqrt{2})(x - \sqrt{2})(x - 3)(x + 6) \end{aligned}$$ **(1)**

Hence, the other zeroes of the polynomial are 3 and -6. **(½)**

31. Given: A right triangle ABC right-angled at B.

To Prove: $AC^2 = AB^2 + BC^2$ **(½)**

Construction: Draw BD AC

Proof: In ΔADB and ΔABC

$A = A$ (Common)

$ADB = ABC$ (Both 90°)

$\Delta ADB \sim \Delta ABC$ (AA similarity criterion)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides of similar triangles are proportional)

or $AD . AC = AB^2$ ...(i) **(1)**

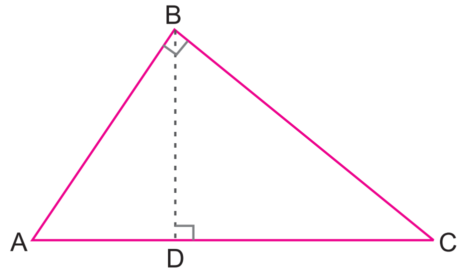


Fig. 12

In ΔBDC and ΔABC

$$C = C \qquad \text{(Common)}$$

$$BDC = ABC \qquad \text{(Each } 90^\circ\text{)}$$

$$\Delta BDC \sim \Delta ABC \qquad \text{(AA similarity)}$$

So, $$\frac{CD}{BC} = \frac{BC}{AC}$$

or, $$CD.AC = BC^2 \qquad ...(ii)$$ **(1)**

Adding (*i*) and (*ii*), we get

$$AD.AC + CD.AC = AB^2 + BC^2$$

or, $$AC\ (AD + CD) = AB^2 + BC^2$$ **(½)**

or, $$AC\ .AC = AB^2 + BC^2$$

or, $$AC^2 = AB^2 + BC^2$$ **(1)**

OR

Refer to Model Question Paper (Solved) – 1.

32. $$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = (\sec\theta + \tan\theta)$$

LHS $$= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing numerator and denominator by $\cos\theta$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$ **(1)**

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{(\tan\theta - \sec\theta + 1)} \qquad (\because \sec^2\theta - \tan^2\theta = 1)$$ **(1)**

$$= \frac{(\sec\theta + \tan\theta) - [(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)]}{(\tan\theta - \sec\theta + 1)}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{(\tan\theta - \sec\theta + 1)}$$ **(1)**

$$= \frac{(\sec\theta + \tan\theta)(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)} = \sec\theta + \tan\theta = \text{RHS}$$ **(1)**

33. We have

$$\frac{\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ}{\sin^2 17^\circ + \sin^2 73^\circ} + \frac{1}{\sqrt{3}}(\tan 10^\circ \tan 30^\circ \tan 80^\circ)$$

$$= \frac{\sec^2(90^\circ - 65^\circ) - \tan^2 25^\circ}{\cos^2(90^\circ - 17^\circ) + \sin^2 73^\circ} + \frac{1}{\sqrt{3}}[\cot(90^\circ - 10^\circ)\tan 30^\circ \tan 80^\circ]$$ **(1)**

$$= \frac{\sec^2 25^\circ - \tan^2 25^\circ}{\cos^2 73^\circ + \sin^2 73^\circ} + \frac{1}{\sqrt{3}}[\cot 80^\circ \tan 30^\circ \tan 80^\circ]$$ **(1)**

$$= \frac{1}{1} + \frac{1}{\sqrt{3}}\left[\frac{1}{\tan 80^\circ} \times \frac{1}{\sqrt{3}} \times \tan 80^\circ\right] = 1 + \frac{1}{3} = \frac{4}{3}$$ **(2)**

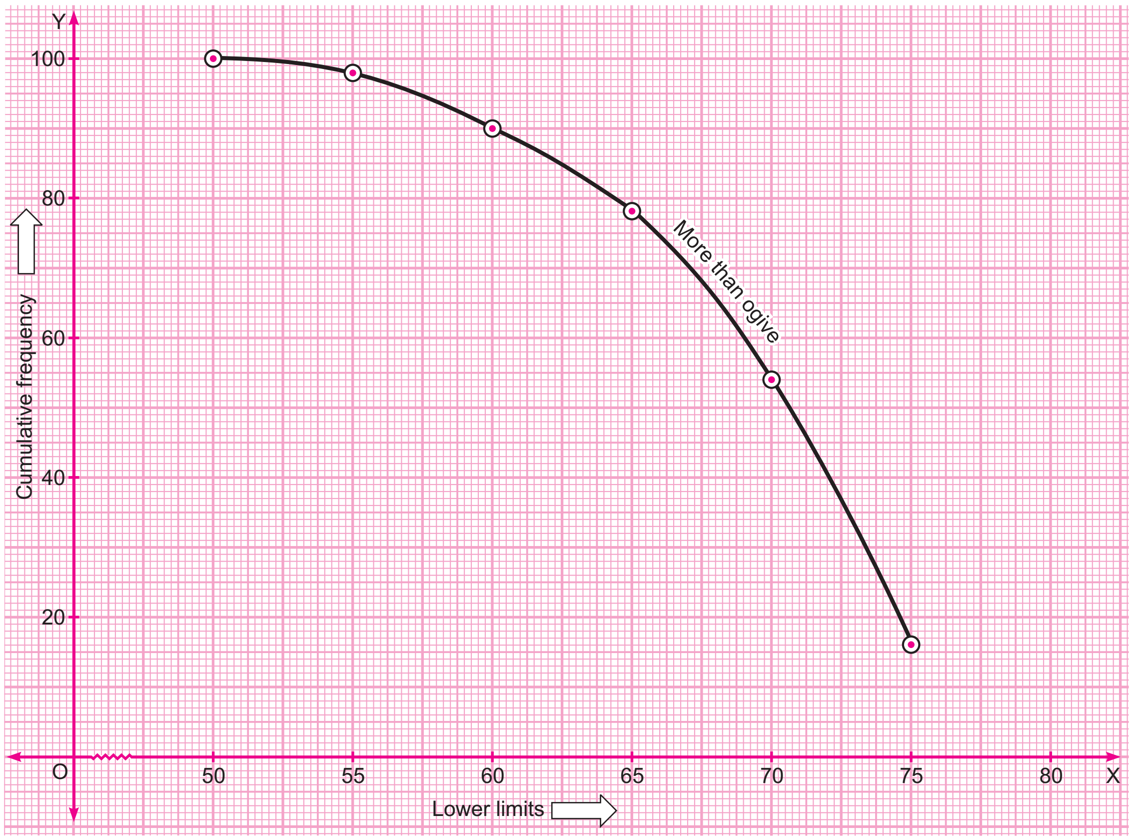
34. We convert the given distribution to a more than type distribution.

We have,

Production yield (kg/hec)	Cumulative frequency (*cf*)
More than or equal to 50	100
More than or equal to 55	100 – 2 = 98
More than or equal to 60	98 – 8 = 90
More than or equal to 65	90 – 12 = 78
More than or equal to 70	78 – 24 = 54
More than or equal to 75	54 – 38 = 16

(2)

Now, we draw the ogive by plotting the points (50, 100), (55, 98), (60, 90) (65, 78), (70, 54), (75, 16) on the graph paper and join them by a freehand smooth curve.



(2)

Fig. 13

Mathematics

Model Question Paper (Unsolved) –1
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions:

1. ***All*** *questions are* ***compulsory****.*
2. *The question paper consists of 34 questions divided into 4 sections, A, B, C and D.* ***Section - A*** *comprises of 10 questions of* **1 mark** *each.* ***Section - B*** *comprises of 8 questions of* **2 marks** *each.* ***Section-C*** *comprises of 10 questions of* **3 marks** *each and* ***Section-D*** *comprises of 6 questions of* **4 marks** *each.*
3. *Question numbers 1 to 10 in Section-A are multiple choice questions where you are to select one correct option out of the given four.*
4. *There is no overall choice. However, internal choice has been provided in 1 question of* ***two marks,*** *3 questions of* ***three marks*** *each and 2 questions of* ***four marks*** *each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is* ***not*** *permitted.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. The decimal expansion of rational number $\frac{67}{2^3 \times 5^2}$ will terminate after

(*a*) one decimal place (*b*) two decimal places
(*c*) three decimal places (*d*) more than three decimal places

2. The least number that is divisible by all the even numbers less than or equal to 10 is

(*a*) 60 (*b*) 80 (*c*) 120 (*d*) 160

3. If one of the zeroes of the polynomial $y^3 - 2y^2 + 4y + k$ is 1, then the value of k is

(*a*) 4 (*b*) 3 (*c*) –2 (*d*) –3

4. Graphically, the pair of equations

$$5x - 3y + 8 = 0 \text{ and } 10x - 6y + 16 = 0$$

represent two straight lines which are

(*a*) intersecting at exactly one point (*b*) parallel
(*c*) intersecting at exactly two points (*d*) coincident

5. If in triangles ABC and PQR, $\frac{AB}{PQ} = \frac{BC}{PR}$, then they will be similar when

(*a*) $B = P$ (*b*) $B = Q$ (*c*) $A = P$ (*d*) $A = R$

6. If $\tan A = \frac{3}{4}$, then the value of $\cos A$ is

(*a*) $\frac{3}{5}$ (*b*) $\frac{5}{3}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{5}{4}$

7. $\frac{\cos\theta}{1+\sin\theta}$ is equal to

(*a*) $\frac{1+\sin\theta}{\cos\theta}$ (*b*) $\frac{1-\sin\theta}{\cos\theta}$ (*c*) $\frac{1-\sin\theta}{\sin\theta}$ (*d*) $\frac{1-\cos\theta}{\sin\theta}$

8. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{\sqrt{3}}{2}$, then the value of $\alpha+\beta$ is

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

9. $6\sec^2 A - 6\tan^2 A$ is equal to

(*a*) 1 (*b*) 6 (*c*) 0 (*d*) 12

10. The mode of a frequency distribution can be determined graphically from

(*a*) ogive (*b*) histogram (*c*) frequency polygon (*d*) bar diagram

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.

12. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder obtained are $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

OR

If the sum of the zeroes of the polynomial $px^2 + 5x + 8p$ is equal to the product of zeroes, find the value of p.

13. For which value of k will the following pair of linear equations have no solution.

$$3x + y = 1; \quad (2k-1)x + (k-1)y = 2k+1.$$

14. In Fig. 1, $\frac{OA}{OC} = \frac{OD}{OB}$. Prove that $A =$ C.

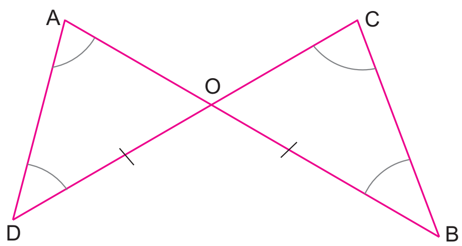


Fig. 1

15. In the trapezium $ABCD$ [Fig. 2], $AB \parallel CD$ and $AB = 2CD$. If area of $\Delta AOB = 84$ cm^2, find the area of ΔCOD.

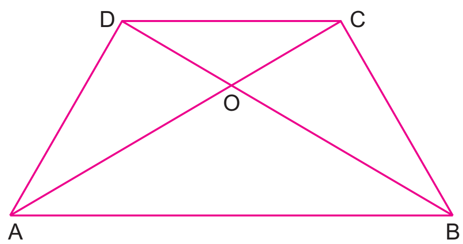


Fig. 2

16. If $4\tan\theta = 3$, then find the value of $\frac{\cos\theta - \sin\theta}{\cos\theta + 2\sin\theta}$

17. The following distribution gives the marks obtained out of 100, by 53 students in a certain examination.

Marks	Number of students
0 – 10	5
10 – 20	3
20 – 30	4
30 – 40	3
40 – 50	3
50 – 60	4
60 – 70	7
70 – 80	9
80 – 90	7
90 – 100	8

Write above distribution as less than type cumulative frequency distribution.

18. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Find the largest number which divides 398, 436 and 542 leaving remainder 7, 11 and 15 respectively.

20. Show that square of an odd integer can be of the form $6q+1$ or $6q+3$ for some integer q.

21. Find the zeroes of the polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relation between the coefficients and the zeroes of the polynomials.

OR

Find a quadratic polynomial whose zeroes are 1 and –3. Verify the relation between the coefficients and zeroes of the polynomial.

22. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

OR

Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B?

23. D is a point on the side BC of ΔABC such that $ADC = BAC$. Prove that $CA^2 = CB \times CD$.

24. Any point X inside ΔDEF is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meeting XE at Q and QR is drawn parallel to EF meeting XF in R. Prove that $PR || DF$.

25. Find the value of $\tan 30°$ geometrically.

26. If $m = \frac{\cos A}{\cos B}$ and $n = \frac{\cos A}{\sin B}$, show that $(m^2 + n^2)\cos^2 B = n^2$

OR

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

27. If the mean of the following distribution is 54, find the value of P.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	7	P	10	9	13

28. The monthly income of 100 families are given below:

Income (in ₹)	Number of families
0 – 5,000	8
5,000 – 10,000	26
10,000 – 15,000	41
15,000 – 20,000	16
20,000 – 25,000	3
25,000 –30,000	3
30,000 – 35,000	2
35,000 – 40,000	1

Calculate the modal income.

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

30. Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines and the x-axis.

31. State and prove Pythagoras theorem.

OR

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

32. Prove that: $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{1 + \cos A}{\sin A}$.

OR

Prove that: $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$

33. Evaluate: $\dfrac{\sec\theta \operatorname{cosec}\theta (90 - \theta) - \tan\theta \cot (90 - \theta) + \sin^2 55° + \sin^2 35°}{\tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}$

34. The median of the following data is 50. Find the values of p and q, if the sum of all frequencies is 90.

Marks	Frequency
20–30	p
30–40	15
40–50	25
50–60	20
60–70	q
70–80	8
80–90	10

Mathematics

Model Question Paper (Unsolved) – 2

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. Which of the following will have a terminating decimal expansion?

(*a*) $\frac{17}{90}$ (*b*) $\frac{53}{343}$ (*c*) $\frac{33}{50}$ (*d*) $\frac{11}{30}$

2. The largest number which divides 88 and 95, leaving remainder 4 and 5 respectively, is

(*a*) 18 (*b*) 12 (*c*) 8 (*d*) 6

3. The number of zeroes of the polynomial $p(x)$, whose graph is given below is

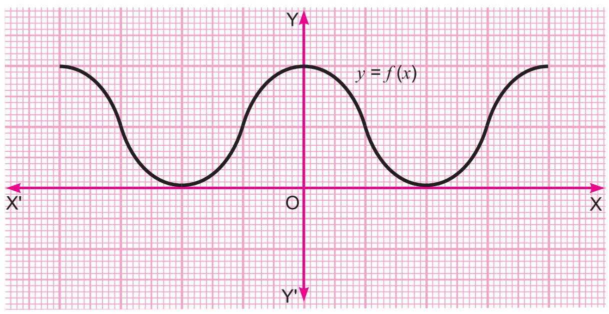


Fig. 1

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) none of these

4. If $x = a$, $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 10$, then the values of a and b are respectively

(*a*) 4 and 2 (*b*) 8 and 2 (*c*) 6 and 4 (*d*) 5 and 3

5. The areas of two similar triangles ABC and DEF are 16 cm^2 and 25 cm^2 respectively. The ratio of their corresponding side is

(*a*) 5 : 4 (*b*) 4 : 5 (*c*) 2 : 5 (*d*) 5 : 2

6. If $\cos A = \frac{15}{17}$, then tan A is equal to

(*a*) $\frac{15}{8}$ (*b*) $\frac{8}{17}$ (*c*) $\frac{17}{8}$ (*d*) $\frac{8}{15}$

7. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(*a*) $\sin\alpha$ (*b*) $\sin 2\alpha$ (*c*) $\cos\beta$ (*d*) $\cos 2\beta$

8. $2\operatorname{cosec}^2 A - 2\cot^2 A$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

9. The value of $\dfrac{2\tan 30°}{1+\tan^2 30°}$ is equal to

(*a*) $\dfrac{1}{\sqrt{3}}$ (*b*) $2\sqrt{3}$ (*c*) $\dfrac{\sqrt{3}}{2}$ (*d*) $\dfrac{2}{\sqrt{3}}$

10. The value of median in the following graph of less than ogive is

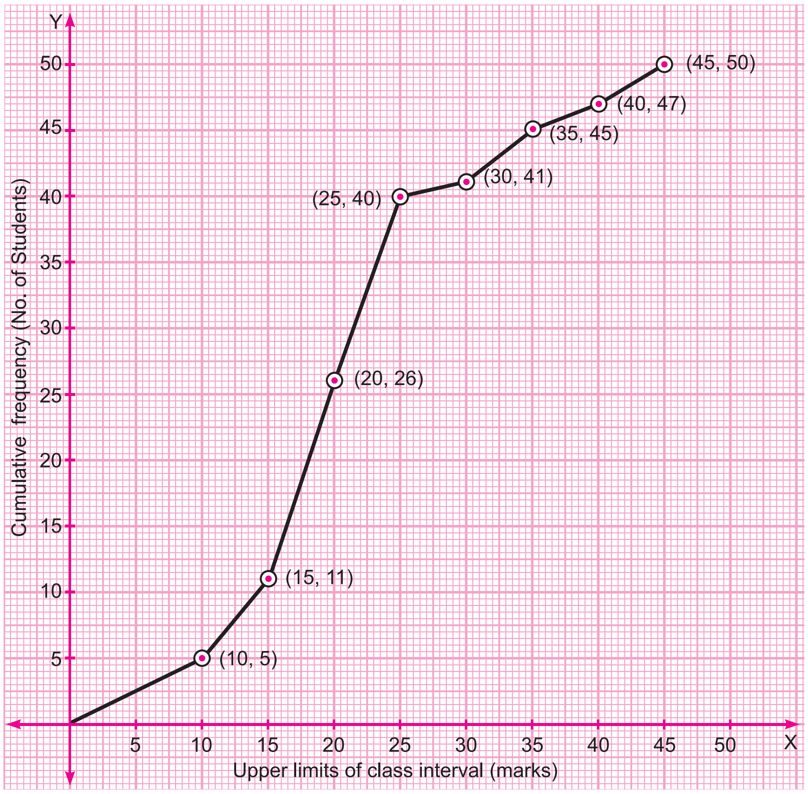


Fig. 2

(*a*) 20 (*b*) 25 (*c*) 40 (*d*) 15

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Given that LCM (26, 169) = 338. Write HCF (26, 169).

12. If one zero of the quadratic polynomial $p(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

OR

If α, β are the zeroes of the polynomial $p(x) = x^2 + x - 6$, then find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$.

13. For which value of k will the pair of linear equations

$kx + 3y = k - 3$ and $12x + ky = k$ have no solution?

14. In Fig. 3, $CAB = 90°$ and AD BC, if $AC = 75$ cm, $AB = 1$ m, and $BC = 1.25$ m, find AD.

15. In ΔABC, AB = 24 cm, BC=10 cm and AC = 26 cm. Is this triangle a right triangle? Give reasons for your answer.

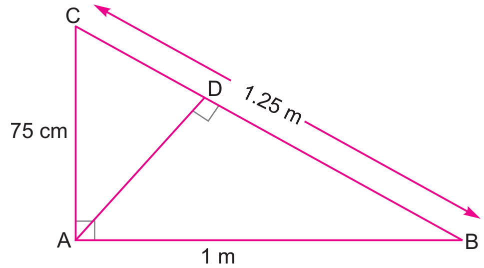


Fig. 3

16. If A, B and C are angles of a ΔABC, then prove that $\sin\frac{B+C}{2} = \cos\frac{A}{2}$.

17. The age of 94 patients are given below:

Age (in years)	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Number of Patients	6	11	18	24	17	13	5

Calculate the modal age.

18. Calculate the difference of the upper limit of the median class and the lower limit of modal class for the data given below.

Class	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Frequency	4	5	13	20	14	7	4

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $\sqrt{3}$ is irrational.

OR

Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

20. Use Euclid's division algorithm to find the HCF of 441, 567 and 693.

21. If the remainder on division of x^3+2x^2+kx+3 by $x-3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3+2x^2+kx-18$.

22. Find a quadratic polynomial the sum and product of whose zeroes are $\frac{-8}{3}$ and $\frac{4}{3}$ respectively. Also, find the zeroes of the polynomial by factorisation.

OR

There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in two halls.

23. ABC is an isosceles triangle with $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. Prove that $BD = BC$.

24. Prove that the sum of the square of the sides of a rhombus is equal to the sum of the squares of its diagonals.

25. Find the value of cos 45° geometrically.

26. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$.

OR

If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\cos\beta}=n$ show that $(m^2+n^2)\cos^2\beta=n^2$.

27. Find the mean of the following frequency distribution:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	15	18	21	29	17

28. The median of the following frequency distribution is 24. Find the missing frequency.

Age in years	0 –10	10 – 20	20 – 30	30 – 40	40 – 50
Number of persons	5	25	x	18	7

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Find all the zeroes of $2x^4 - 3x^3 - 5x^2 + 9x - 3$, if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

30. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are

$$y = x, \quad 3y = x, \quad x + y = 8$$

31. State and prove basic proportionality theorem.

OR

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

32. Prove that: $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1 + \cos\theta}{\sin\theta}$

OR

Prove that: $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta$

33. Without using tables, evaluate the following:

$$\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \times \sec^2 52° - \sin^2 45°.$$

34. The weights of tea in 70 packets are shown in the following table:

Weight (in gram)	**Number of Packets**
200 – 201	13
201 – 202	27
202 – 203	18
203 – 204	10
204 – 205	1
205 – 206	1

Draw the less than type ogive for this data and use it to find the median weight.

Mathematics

Model Question Paper (Unsolved) – 3
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. 5.6212121 .... is

(*a*) an integer (*b*) an irrational number (*c*) a rational number (*d*) none of these

2. The least number that is divisible by all the odd numbers, less than 11 is

(*a*) 35 (*b*) 105 (*c*) 315 (*d*) 630

3. If the zeroes of quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(*a*) c and a have same sign (*b*) c and b have same sign

(*c*) c and a have opposite sign (*d*) c and b have opposite sign

4. If the lines given by $2x + ky = 5$ and $6x + 9y = 10$ are parallel,then the value of k is

(*a*) 2 (*b*) 3

(*c*) –3 (*d*) 4

5. In ΔABC, Fig. 1, $DE \| BC$ such that AD= 1.7 cm, AB = 6.8 cm and AC = 10 cm. Then, AE is

(*a*) 3.6 cm (*b*) 4.5 cm

(*c*) 2.3 cm (*d*) 2.5 cm

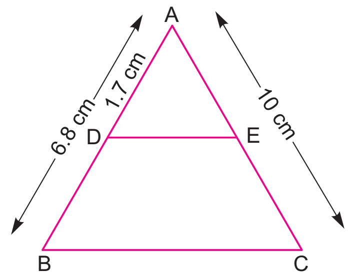


Fig. 1

6. If $\sin A = \frac{12}{13}$, then $\sec A$ is equal to

(*a*) $\frac{5}{12}$ (*b*) $\frac{12}{5}$ (*c*) $\frac{5}{13}$ (*d*) $\frac{13}{5}$

7. If $\cot 9\alpha = \tan \alpha$ and $9\alpha < 90°$, then the value of $\sin 5\alpha$ is

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) 0 (*c*) 1 (*d*) $\sqrt{2}$

8. $\sin(45 + \theta) - \cos(45 - \theta)$ is equal to

(*a*) $2\cos\theta$ (*b*) 0 (*c*) $2\sin\theta$ (*d*) 1

9. $\frac{5\sec^4\theta - 5\tan^2\theta}{\sec^2\theta + \tan^4\theta}$ is equal to

(*a*) 0 (*b*) –5 (*c*) 1 (*d*) 5

10. The median profit of 30 shops of a shopping complex for which a cumulative frequency curve is given below is

(*a*) 15

(*b*) 30

(*c*) 20

(*d*) 40

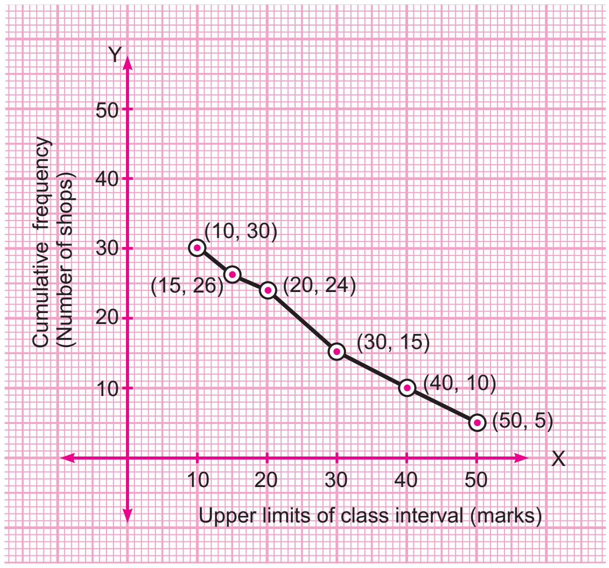


Fig. 2

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other number.

12. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer k>1? Justify your answer.

13. Solve for x and y

$$x + y = 8$$
$$2x - 3y = 1$$

OR

Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

14. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

15. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ΔABC is right triangle.

16. If $A = 30°$, verify that $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$.

17. The mode and mean are 26.6 and 28.1 respectively in a distribution. Find out the median.

18. For the following distributions:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

What is the modal class?

Section – C

Question numbers 19 to 29 carry 3 marks each.

19. Show that $\frac{1}{\sqrt{5}}$ is irrational.

OR

Show that every positive odd integer is of the form $6q+1$ or $6q+3$ or $6q+5$ for some integer q.

20. Using prime factorisation method, find the HCF and LCM of 10224 and 1608.

21. Solve the following system of linear equations graphically.

$$3x + y - 11 = 0, \qquad x - y - 1 = 0$$

Shade the region bounded by these lines and the y-axis.

Find the coordinates of the points where the lines cut the y-axis.

22. Find the zeroes of $x^2 + 4\sqrt{3}x - 15$ by factorization method and verify the relations between the zeroes and the coefficients of the polynomial.

OR

If α and β and the zeroes of the polynomial $p(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

23. If the areas of two similar triangles are equal, prove that they are congruent.

24. In Fig. 3, ΔFEC ΔGDB and ADE = AED. Prove that $\Delta ADE \sim \Delta ABC$.

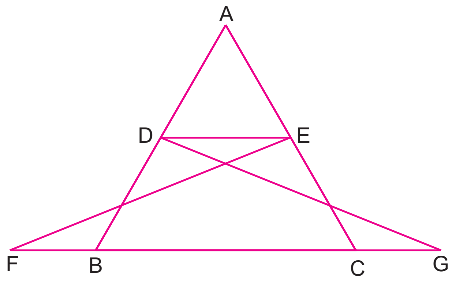


Fig. 3

25. Find the value of $\tan 30°$ geometrically.

26. Prove that: $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \text{cosec}\,\theta$

OR

Prove that: $\frac{1 - \cos\theta}{1 + \cos\theta} = (\text{cosec}\,\theta - \cot\theta)^2$

27. The mean of the following distribution is 18. The frequency f in the class 19 – 21 is missing. Determine f.

Class interval	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

28. A survey regarding the heights (in cm) of 50 girls of a class was conducted and the following data was obtained.

Height (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170	Total
Frequency	2	8	12	20	8	50

Find the mode of the above data.

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Find all the zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

30. A two digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

OR

A man travels 370 km, partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

31. State and prove converse of Pythagoras Theorem.

OR

State and prove Thales theorem.

32. Without using trigonometric tables, evaluate:

$$2\left(\frac{\cos^2 20° + \cos^2 70°}{\operatorname{cosec}^2 25° - \tan^2 65°}\right) - \tan 45° + \tan 13° \tan 23° \tan 30° \tan 67° \tan 77°$$

33. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, then show that $l^2 m^2 (l^2 + m^2 + 3) = 1$.

34. The annual rainfall record of a city for 66 days is given in the following table.

Rainfall (in cm)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of days	22	10	8	15	5	6

Calculate the median rainfall using a more than type ogive.

Mathematics

Model Question Paper (Unsolved) – 4
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. π is

(*a*) a rational number (*b*) an irrational number (*c*) an integer (*d*) none of these

2. If two positive integers a and b are written as $a = x^3 y^4$ and $b = x^5 y^2$; x, y are prime numbers, then HCF (a, b) is

(*a*) xy (*b*) $x^2 y^2$ (*c*) $x^3 y^2$ (*d*) $x^5 y^4$

3. Zeroes of $p(x) = x^2 - 2x - 15$ are

(*a*) –5 and 3 (*b*) –5 and –3 (*c*) 5 and –3 (*d*) 5 and 3

4. For what value of k, the system of equations $x + 2y = 3$, $5x + ky = 7$ is inconsistent?

(*a*) $k \neq 10$ (*b*) $k = 10$ (*c*) $k = \frac{3}{7}$ (*d*) $k = \frac{-3}{7}$

5. In ΔABC, if $AB = 12$cm, $BC = 6$ cm and $AC = 6\sqrt{3}$, then C is equal to

(*a*) 60° (*b*) 45° (*c*) 90° (*d*) none of these

6. If $\tan A = \frac{4}{3}$, then the value of cosec A is

(*a*) $\frac{4}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{5}{3}$ (*d*) $\frac{5}{4}$

7. If $\sin\theta = \frac{1}{3}$, then the value of $(9\cot^2\theta + 9)$ is

(*a*) $\frac{1}{81}$ (*b*) 1 (*c*) 9 (*d*) 81

8. If for some angle θ, $\tan 2\theta = \frac{1}{\sqrt{3}}$, then the value of $\cos 4\theta$, where $2\theta \leq 90°$ is

(*a*) 0 (*b*) $\frac{\sqrt{3}}{2}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{2}}$

9. The value of the expression $\tan(75° + \theta) - \cot(15° - \theta) - \sec(65° + \theta) + \text{cosec}(25° - \theta)$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

10. Construction of a cumulative frequency table is useful in determining the

(*a*) mean (*b*) median

(*c*) mode (*d*) all of the above three measures

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Write whether the square of any positive integer can be of the form $3m+2$, where m is a natural number. Justify your answer.

12. If one zero of the polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, find the value of a.

OR

Find a quadratic polynomial whose zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

13. Solve for x and y

$$2x+5y=1 \quad \text{and} \quad 2x+3y=3$$

14. In Fig. 1, $AB \| DC$ and diagonals AC and BD intersect at O. If $OA=3x-1$ cm and $OB=2x+1$ cm, $OC=5x-3$ cm and $OD=6x-5$ cm, then find x.

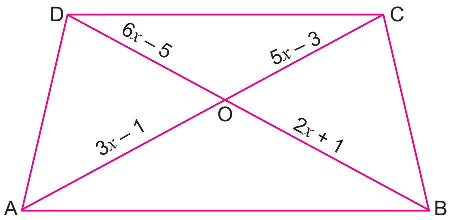


Fig. 1

15. The areas of two similar triangles ABC and PQR are 25 cm^2 and 49 cm^2 respectively. If $QR = 9.8$ cm, find BC.

16. Taking $\theta=30°$, verify that: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

17. Numbers 29, 32, 48, 50, x, x, $x+2$, 72, 78, 84, 95 are written in ascending order. If the median of the data is 63, find the value of x.

18. The mean of ungrouped data and the mean calculated when the same data is grouped are always the same. Do you agree with the statement? Give reason for your answer.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $\sqrt{3}+\sqrt{5}$ is irrational.

20. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respectively.

21. Find the zeroes of the polynomial $4x^2+5\sqrt{2}x-3$ and verify the relation between the coefficients and the zeroes of the polynomial.

22. By the graphical method, find whether the pair of linear equations $2x-3y=5$, $6y-4x=3$ is consistent or inconsistent.

OR

Divide $3x^2-x^3-3x+5$ by $x-1-x^2$, and verify the division algorithm.

23. BL and CM are medians of a ΔABC, right-angled at A. Prove that $4(BL^2+CM^2)=5BC^2$.

24. The diagonal BD of a parallelogram $ABCD$ intersects the segment AE at point F, where E is any point on the side BC. Prove that $DF \times EF = FB \times FA$.

25. If $\sin(A-B)=\dfrac{1}{2}$ and $\cos(A+B)=\dfrac{1}{2}$, $0° < A+B \le 90°$, $A < B$ find A and B.

OR

Given that $\alpha+\beta=90°$, show that $\sqrt{\cos\alpha\ \text{cosec}\beta - \cos\alpha\ \sin\beta} = \sin\alpha$.

26. Find the value of $\sin 30°$ geometrically.

27. Find the mean of following frequency distribution using step deviation method.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	12	18	27	20	17	6

OR

The mode of following frequency distribution is 36. Find the missing frequency.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	...	16	12	6	7

28. Given below is the distribution of I.Q. of 100 students. Find the median I.Q.

I.Q.	75–84	85–94	95–104	105–114	115–124	125–134	135–144
Frequency	8	11	26	31	18	4	2

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Given that $x-\sqrt{5}$ is a factor of the cubic polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$, find all the zeroes of the polynomial.

30. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

OR

8 men and 12 women can finish a piece of work in 5 days, while 6 men and 8 women can finish it in 7 days. Find the time taken by 1 man alone and that by 1 woman alone to finish the work.

31. State and prove Basic Proportionality Theorem.

OR

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

32. Prove that: $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$.

33. Without using trigonometric tables, evaluate:

$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2+\left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2+2\tan 15^\circ\tan 37^\circ\tan 53^\circ\tan 60^\circ\tan 75^\circ.$$

34. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in ₹)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive and hence obtain the median daily income.

Mathematics

Model Question Paper (Unsolved) – 5
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. Decimal expansion of $\sqrt{5}$ is

(*a*) terminating (*b*) non-terminating repeating
(*c*) non-terminating non-repeating (*d*) none of these

2. If the HCF of 126 and 132 is expressible in the form of $5P-9$, then the value of P is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

3. If the sum of the zeroes of the polynomial $f(x)=2x^3-3kx^2+4x-5$ is 6, then the value of k is

(*a*) 1 (*b*) –1 (*c*) 4 (*d*) – 4

4. If a pair of linear equations is consistent, then the lines represented by those equations will be

(*a*) parallel (*b*) always coincident
(*c*) intersecting or coincident (*d*) always intersecting

5. In ΔABC (Fig. 1), $DE \| BC$ and $\frac{AD}{DB}=\frac{3}{5}$. *If* $AC = 5.6$, then AE is equal to

(*a*) 1.2 (*b*) 2.1
(*c*) 2.5 (*d*) 3.2

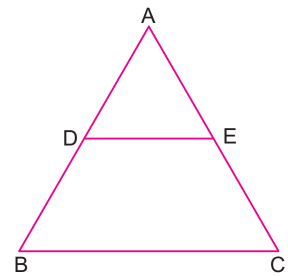


Fig. 1

6. If $\tan\theta=\frac{3}{4}$, then $\cos^2\theta-\sin^2\theta=$

(*a*) $\frac{7}{25}$ (*b*) 1 (*c*) $\frac{-7}{25}$ (*d*) $\frac{4}{25}$

7. If $\cos A+\cos^2 A=1$, then $\sin^2 A+\sin^4 A=$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) none of these

8. $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to

(*a*) $\sec^2 A$ (*b*) –1 (*c*) $\cot^2 A$ (*d*) $\tan^2 A$

9. If θ is an acute angle such that $\cos\theta=\frac{3}{5}$, then $\frac{\sin\theta.\tan\theta-1}{2\tan^2\theta}=$

(*a*) $\frac{16}{625}$ (*b*) $\frac{1}{36}$ (*c*) $\frac{3}{160}$ (*d*) $\frac{160}{3}$

10. What is the value of median of the data represented by the graph in Fig. 2, of less than ogive and more than ogive?

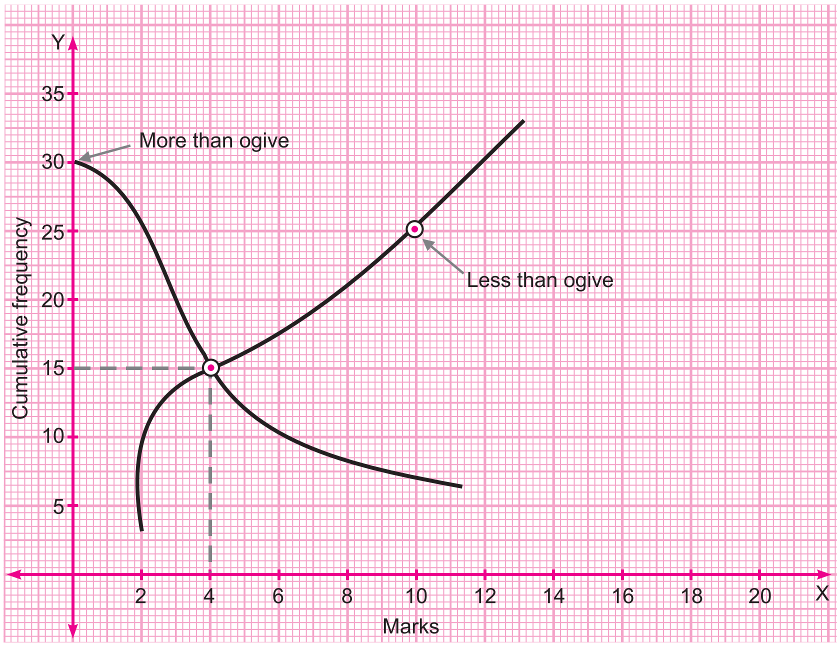


Fig. 2

(*a*) 15 (*b*) 30 (*c*) 4 (*d*) None of these

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Complete the missing entries in the following factor tree.

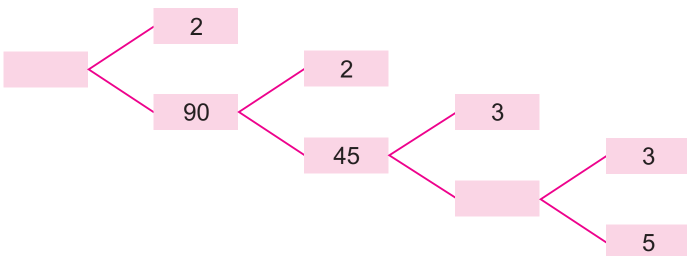


Fig. 3

12. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

13. For what value of k will the pair of linear equations?

$$3x + 5y - (k - 5) = 0$$

$$6x + 10y - k = 0$$

have infinitely many solutions?

OR

Find the condition for which the following system of equations will be inconsistent.

$$(p - q)x = (p + q)y \quad \text{and} \quad px - 2y = r$$

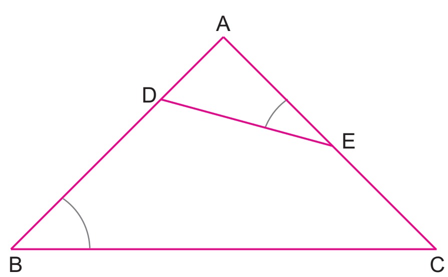


Fig. 4

14. In Fig. 4, D and E are points on sides AB and CA of ΔABC such that $B = \quad AED$. Show that $\Delta ABC \sim \Delta AED$.

15. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$. Prove that ΔABC is a right triangle.

16. If $A = 30°$, verify that $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$.

17. The mean of 5 observations is 7. Later on, it was found that two observations 4 and 8 were wrongly taken instead of 5 and 9. Find the correct mean.

18. In a distribution, the arithmetic mean and median are 30 and 32 respectively. Calculate the mode.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $3+2\sqrt{3}$ is irrational.

OR

Show that any positive odd integer is of the form $6q+1$ or $6q+3$ or $6q+5$ for some integer q.

20. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

21. Find all the zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2+\sqrt{3}$ and $2-\sqrt{3}$

22. Solve the following system of equation graphically:

$$x + 2y = 5, \qquad 2x - 3y = -4$$

Also find the points where the lines meet the x-axis.

OR

A man has only 20 paise coins and 25 paise coins in his purse. If he has 50 coins in all totalling ₹ 11.25, how many coins of each class will then he have?

23. In a ΔABC, the angles at B and C are acute. If BE and CF are drawn perpendicular on AC and AB respectively,

prove that: $BC^2 = AB \times BF + AC \times CE$.

24. In Fig. 5, $AB||DE$ and $BD||EF$. Prove that $DC^2 = CF \times AC$.

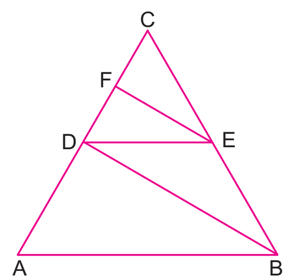


Fig. 5

25. Without using trigonometric tables, evaluate:

$$\frac{-\tan\theta \cot(90°-\theta) + \sec\theta \operatorname{cosec}(90°-\theta) + \sin^2 35° + \sin^2 55°}{\tan 10° \tan 20° \tan 30° \tan 70° \tan 80°}.$$

26. If $\tan(A-B) = \frac{1}{\sqrt{3}}$ and $\tan(A+B) = \sqrt{3}$, $0° < A+B \le 90°$, $A > B$, find A and B.

27. Find the value of median from the following data:

Class interval	10–19	20–29	30–39	40–49	50–59	60–69	70–79
Frequency	2	4	8	9	4	2	1

OR

If the mean of the following distribution is 6, find the value of p.

x	2	4	6	10	P + 5
f	3	2	3	1	2

28. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Complete the missing frequency f_1 and f_2.

Class interval	0–20	20–40	40–60	60–80	80–100	100–120	Total
Frequency	5	f_1	10	f_2	7	8	50

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

30. Find all the zeroes of the polynomial $x^4 - 11x^2 + 28$, if two of the zeroes are $\sqrt{7}$ and $-\sqrt{7}$.

31. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

OR

Prove that in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

32. Find the value of cosec $30°$ geometrically.

OR

If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$. Prove that $x^2 - y^2 = a^2 - b^2$

33. Prove that: $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec\theta.$

34. Find the mean, mode and median for the following data:

Class	Frequency
0–10	8
10–20	16
20–30	36
30–40	34
40–50	6
Total	100

Mathematics

Model Question Paper (Unsolved) – 6
Summative Assessment – I

Time: 3 to 3½ hours | **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. The decimal expansion of the rational number $\frac{23}{2^2.5}$ will terminate after

(*a*) one decimal place (*b*) two decimal places
(*c*) three decimal places (*d*) more than three decimal places

2. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number (*c*) an odd integer (*d*) an even integer

3. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is – 3, then the value of k is

(*a*) $-\frac{4}{3}$ (*b*) $\frac{4}{3}$ (*c*) $\frac{2}{3}$ (*d*) $-\frac{2}{3}$

4. The lines representing the linear equations $2x - y = 3$ and $4x - y = 5$

(*a*) intersect at a point (*b*) are parallel
(*c*) are coincident (*d*) intersect at exactly two points

5. In Fig. 1, if D is mid-point of BC, the value of $\frac{\tan x°}{\tan y°}$ is

(*a*) $\frac{1}{3}$ (*b*) 1 (*c*) 2 (*d*) $\frac{1}{2}$

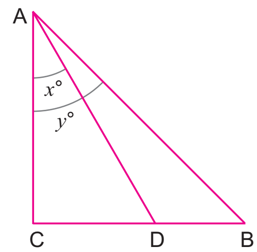


Fig. 1

6. Construction of a cumulative frequency table is useful in determining the

(*a*) mean (*b*) median
(*c*) mode (*d*) all the above three measures

7. If $x = 3\sec^2\theta - 1, y = \tan^2\theta - 2$, then $x - 3y$ is equal to

(*a*) 3 (*b*) 4 (*c*) 8 (*d*) 5

8. If $\cos\theta + \cos^2\theta = 1$, the value of $(\sin^2\theta + \sin^4\theta)$ is

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2

9. If $\Delta ABC \quad \Delta RQP$, $\ A = 80°$, $\ B = 60°$, the value of $\ P$ is

(*a*) 60° (*b*) 50° (*c*) 40° (*d*) 30°

10. In Fig. 2, $\ ACB = 90°$, $\ BDC = 90°$, $CD = 4$ cm, $BD = 3$ cm, $AC = 12$ cm. $\cos A - \sin A$ is equal to

(*a*) $\frac{5}{12}$ (*b*) $\frac{5}{13}$ (*c*) $\frac{7}{12}$ (*d*) $\frac{7}{13}$

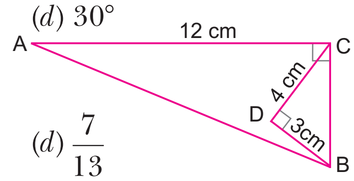


Fig. 2

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Use Euclid's division algorithm to find H.C.F. of 870 and 225.

12. Solve: $37x + 43y = 123,\ 43x + 37y = 117$.

OR

Solve: $x + \frac{6}{y} = 6,\ 3x - \frac{8}{y} = 5$.

13. α, β are the roots of the quadratic polynomial $p(x) = x^2 - (k+6)x + 2(2k-1)$.

Find the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

14. If $\cot\theta = \frac{7}{8}$, find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

15. Find the median class and the modal class for the following distribution.

Class interval	135-140	140-145	145-150	150-155	155-160	160-165
Frequency	4	7	18	11	6	5

16. Write the following distribution as more than type cumulative frequency distribution:

Class interval	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	2	6	8	14	15	5

17. Two poles of height 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$ m, find the distance between their tops.

18. In Fig. 3, $AB \quad BC$, $DE \quad AC$ and $GF \quad BC$. Prove that $\Delta ADE \sim \Delta GCF$.

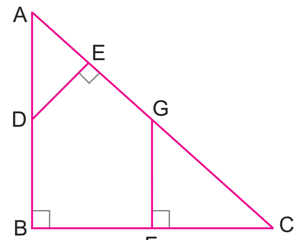


Fig. 3

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Show that $5 + \sqrt{2}$ is an irrational number.

OR

Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

20. Show that 5^n can't end with the digit 2 for any natural number n.

21. If α, β are the two zeroes of the polynomial $21y^2 - y - 2$, find a quadratic polynomial whose zeroes are 2α and 2β.

22. If A, B, C are interior angles of ΔABC, show that $\sec^2\left(\frac{B+C}{2}\right) - 1 = \cot^2\frac{A}{2}$

OR

Prove that: $\frac{\cos(90°-\theta)}{1+\sin(90°-\theta)} + \frac{1+\sin(90°-\theta)}{\cos(90°-\theta)} = 2\operatorname{cosec}\theta$

23. In Fig. 4, ABC is a triangle right-angled at B, $AB = 5$ cm, $\quad ACB = 30°$. Find the length of BC and AC.

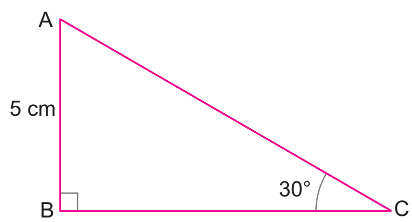


Fig. 4

24. The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class interval	0–10	10–20	20–30	30–40	40–50
Frequency	8	x	10	11	9

25. Find the mode of the following frequency distribution:

Class interval	5–15	15–25	25–35	35–45	45–55	55–65	65–75
Frequency	2	3	5	7	4	2	2

26. Nine times a two-digit number is the same as twice the number obtained by interchanging the digits of the number. If one digit of the number exceeds the other number by 7, find the number.

OR

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

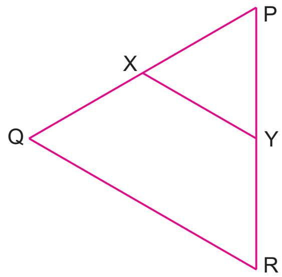


Fig. 5

27. In Fig. 5, $XY||QR, \frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm. Find YR.

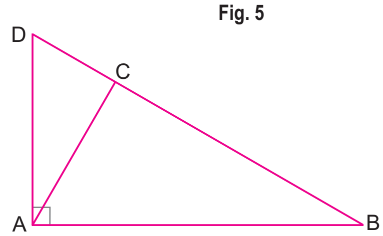


Fig. 6

28. In Fig. 6, ABD is a triangle in which $DAB = 90°$ and AC BD. Prove that $AC^2 = BC \times DC$.

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Solve the following system of equations graphically and find the vertices of the triangle formed by these lines and the x-axis.

$$4x - 3y + 4 = 0, \quad 4x + 3y - 20 = 0$$

30. Draw 'less than ogive' for the following frequency distribution and hence obtain the median.

Marks obtained	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of students	3	4	3	3	4	7	9

31. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

OR

Prove that in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

32. Find all the zeroes of the polynomial $x^4 - 5x^3 + 2x^2 + 10x - 8$, if two of its zeroes are $\sqrt{2}, -\sqrt{2}$.

33. Prove that: $\dfrac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta} = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$

OR

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2)^2 = 16\,mn$

34. Prove that: $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

Mathematics

Model Question Paper (Unsolved) –7

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

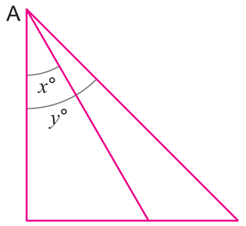


Fig. 1

1. In Fig. 1, if D is mid-point of BC, the value of $\frac{\cot y°}{\cot x°}$ is:

(*a*) 2 (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{2}$

2. If $\operatorname{cosec}\theta = \frac{3}{2}$, then $2(\operatorname{cosec}^2\theta + \cot^2\theta)$ is:

(*a*) 3 (*b*) 7 (*c*) 9 (*d*) 5

3. If p, q are two consecutive natural numbers, then HCF (p, q) is:

(*a*) q (*b*) p (*c*) 1 (*d*) pq

4. If $d = LCM(36, 198)$, then the value of d is:

(*a*) 396 (*b*) 198 (*c*) 36 (*d*) 1

5. In Fig. 2, the number of zeroes of $y = f(x)$ are:

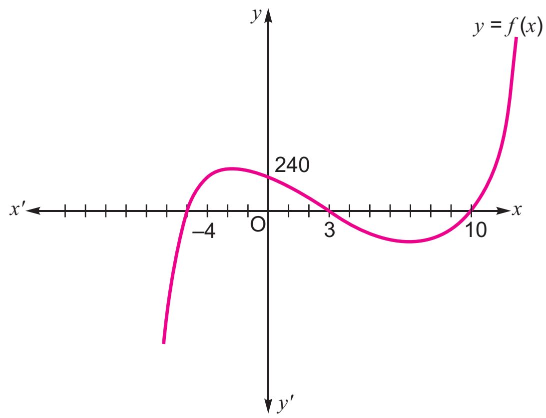


Fig. 2

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

6. The measure of central tendency which take into account all data items is:

(*a*) mode (*b*) mean (*c*) median (*d*) none of these

7. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then the ratio of their corresponding altitude is:

(*a*) 3 : 2 (*b*) 16 : 81 (*c*) 4 : 9 (*d*) 2 : 3

8. If $\sin\theta + \sin^2\theta = 1$, the value of $(\cos^2\theta + \cos^4\theta)$ is:

(*a*) 3 (*b*) 2 (*c*) 1 (*d*) 0

9. If a pair of linear equations is consistent, then the lines will be:

(*a*) parallel (*b*) always coincident

(*c*) intersecting or coincident (*d*) always intersecting

10. In Fig. 3, if $PS = 14$ cm, the value of tan *a* is equal to:

(*a*) $\frac{4}{3}$ (*b*) $\frac{14}{3}$

(*c*) $\frac{5}{3}$ (*d*) $\frac{13}{3}$

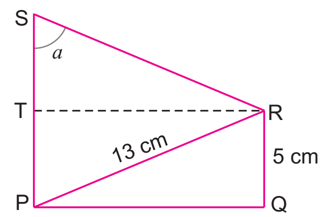


Fig. 3

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. In the adjoining factor tree, find the numbers *m*, *n*:

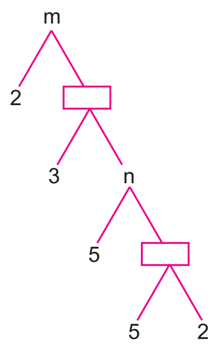


Fig. 4

12. Write the following distribution as less than type cumulative frequency distribution:

Class Interval	140–145	145–150	150–155	155–160	160–165	165–170
Frequency	10	8	20	12	6	4

13. Find the modal class and the median class for the following distribution:

Class Interval	0–10	10–20	20–30	30–40	40–50
Frequency	6	10	12	8	7

14. In ΔABC, AD BC such that $AD^2 = BD \times CD$. Prove that ΔABC is right-angled at A.

15. From the given Fig. 5, find MLN.

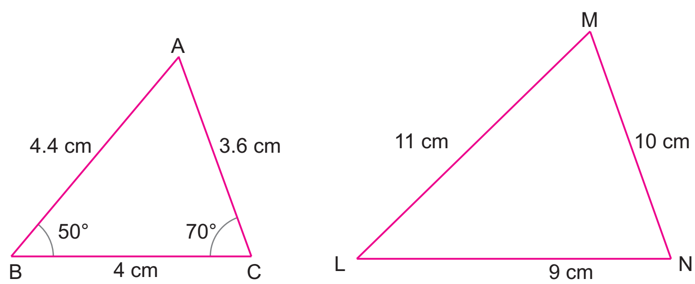


Fig. 5

16. Solve: $47x + 31y = 63, \quad 31x + 47y = 15$

OR

Solve: $\frac{3}{x} - 5y + 1 = 0, \frac{2}{x} - y + 3 = 0$

17. α, β are the roots of the quadratic polynomial $p(x) = x^2 - (k-6)x + (2k+1)$. Find the value of k, if $\alpha + \beta = \alpha\beta$.

18. Simplify: $\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{\cos\theta}\right)$

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Find the mean of the given frequency distribution table:

Class interval	15–25	25–35	35–45	45–55	55–65	65–75	75–85
Frequency	6	11	7	4	4	2	1

20. Find the median of the following frequency distribution:

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	5	8	15	20	14	8	5

21. Find the zeroes of $4\sqrt{5}x^2 - 17x - 3\sqrt{5}$ and verify the relation between the zeroes and coefficient of the polynomial.

22. If $\sin(A+B) = \frac{\sqrt{3}}{2}$ and $\cos(A-B) = 1$, $0° < (A+B) < 90°$, $A \geq B$, find A and B.

23. Show that $5 - \sqrt{3}$ is irrational.

OR

Show that $\sqrt{2} + \sqrt{3}$ is irrational.

24. Check whether 6^n can end with the digit zero for any natural number n.

25. If A, B, C are interior angles of ΔABC, show that:

$$\operatorname{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1$$

OR

Show that: $\sec^2\theta + \cot^2(90° - \theta) = 2\operatorname{cosec}^2(90° - \theta) - 1$.

26. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of the food per day.

OR

The sum of a two-digit number and the number obtained by reversing the digit is 66. If the digits of a number differ by 2, find the number.

27. In Fig. 6, $QPR = 90°$, $PMR = 90°$, $QR = 25$ cm, $PM = 8$ cm, $MR = 6$ cm. Find area (ΔPQR).

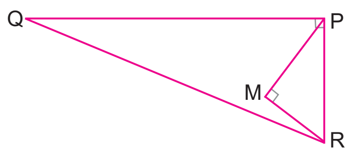


Fig. 6

28. In ΔABC Fig. 7, D and E are two points lying on side AB such that $AD = BE$. If $DP || BC$ and $EQ || AC$, then prove that $PQ || AB$.

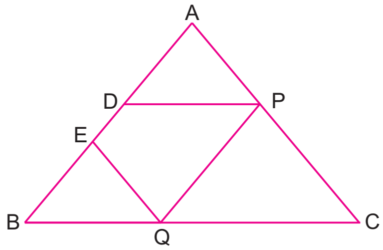


Fig. 7

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Solve the following system of equations graphically and find the vertices of the triangle bounded by these lines and y-axis.

$x - y + 1 = 0, 3x + 2y - 12 = 0.$

30. Prove that $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta.$

31. If $x = r\sin A\cos C$, $y = r\sin A\sin C$, $z = r\cos A$, prove that $r^2 = x^2 + y^2 + z^2$.

OR

Prove that $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A.$

32. Prove the following:

The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

OR

Prove that in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

33. Draw 'more than ogive' for the following frequency distribution and hence obtain the median

Class interval	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	2	12	2	4	3	4	3

34. Find all the zeroes of the polynomial $x^4 + x^3 - 9x^2 - 3x + 18$, if two of its zeroes are $\sqrt{3}, -\sqrt{3}$.

Mathematics

Model Question Paper (Unsolved) – 8
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. If $d = LCM(54, 336)$, then the value of d is:

(*a*) 3024 (*b*) 2024 (*c*) 3025 (*d*) 3020

2. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number (*c*) an odd integer (*d*) an even integer

3. If $\cos A = \frac{1}{\sqrt{2}}$, the value of cot A is

(*a*) $\sqrt{2}$ (*b*) 1 (*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{\sqrt{3}}$

4. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = \frac{1}{2}$. The value of $(\alpha - \beta)$ is

(*a*) 0° (*b*) 60° (*c*) 90° (*d*) 120°

5. $\frac{1+\tan^2 A}{1+\cot^2 A} =$

(*a*) $\sec^2 A$ (*b*) -1 (*c*) $\cot^2 A$ (*d*) $\tan^2 A$

6. $\cot 85° + \cos 75°$ in terms of trigonometric ratios of angles between 0° and 45° is:

(*a*) tan10° + sin 15° (*b*) tan 5° + sin 15° (*c*) tan 15° + sin 10° (*d*) tan 15° + sin 5°

7. If one root of the polynomial $f(x) = 5x^2 + 13x + k$ is reciprocal of the other, then the value of k is

(*a*) 0 (*b*) 5 (*c*) $\frac{1}{6}$ (*d*) 6

8. The value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7$ has no solution is

(*a*) 10 (*b*) 5 (*c*) $\frac{1}{6}$ (*d*) 6

9.

Marks obtained	Number of students
Less than 10	5
Less than 20	12
Less than 30	22

Less than 40	29
Less than 50	38
Less than 60	47

the frequency of class 50-60 is

(*a*) 9 (*b*) 10 (*c*) 38 (*d*) 47

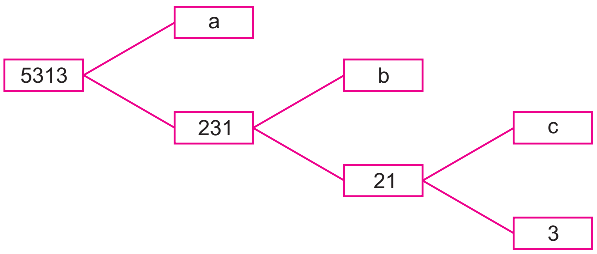
10. In ΔABC, if $AB = 6\sqrt{3}$, $AC = 12$ cm and $BC = 6$ cm, then B is

(*a*) 120° (*b*) 60° (*c*) 90° (*d*) 45°

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. In the adjoining factor tree, find the number a, b, c.



12. Find whether the following pair of equations are consistent or not by graphical method.

$$4x + 7y = -11$$
$$5x - y + 4 = 0$$

13. Solve:

$$2x + 3y + 5 = 0$$
$$3x - 2y - 12 = 0$$

14. Daily wages of 110 workers, obtained in a survey, are tabulated below:

Daily wages (in ₹)	Number of workers
100 – 120	10
120 – 140	15
140 – 160	20
160 – 180	22
180 – 200	18
200–220	12
220–240	13

Compute the mean daily wages of these workers.

15. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

16. It is given that $\Delta FED \sim \Delta STU$. Is it true to say that $\frac{DE}{ST} = \frac{EF}{TU}$? Why?

17. D is a point on side QR of ΔPQR such that PD QR. Will it be correct to say that $\Delta PQD \sim \Delta RPD$? Why?

18. Prove that: $(\sec^4\theta - \sec^2\theta) = (\tan^2\theta + \tan^4\theta)$

OR

If $4\tan\theta = 3$, then find the value of $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Use Euclid's division Lemma to show that the square of any positive integer is either of the form $3m$ or $3m+1$ for some integer m.

20. If a is rational and $\sqrt{b}$ is irrational, then prove that $(a+\sqrt{b})$ is irrational.

21. If the two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes.

OR

If α, β are zeroes of a quadratic polynomial $x^2 + px + 45$ and $\alpha^2 + \beta^2 = 234$, find the value of p.

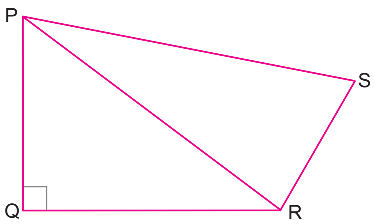


Fig. 1

22. In a two digit number, the ten's digit is three times the unit digit. When the number is decreased by 54, the digits are reversed. Find the number.

23. In a quadrilateral $PQRS$ (Fig. 1), $Q = 90°$. If $PQ^2 + QR^2 + RS^2 = PS^2$, then prove that $PRS = 90°$.

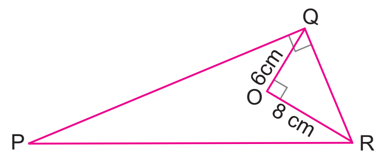


Fig. 2

24. In the given Fig. 2, $PQR = QOR = 90°$. If $PR = 26$ cm, $OQ = 6$ cm, $OR = 8$ cm, find PQ.

OR

If ABC is an equilateral triangle of side $2a$, then prove that altitude $AD = a\sqrt{3}$.

25. Without using tables, evaluate the following:

$$3\cos 68°\operatorname{cosec} 22° - \frac{1}{2}\tan 43°.\tan 47°, \tan 12°, \tan 60° \tan 78°$$

OR

Without using trigonometric tables, evaluate the following:

$$\frac{\cot(90° - \theta).\sin(90° - \theta)}{\sin\theta} + \frac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$$

26. Evaluate:

$$\left(\frac{\sin 47°}{\cos 43°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 4\cos^2 45°$$

27. Find the median of the following frequency distribution.

Class interval	0–20	20–40	40–60	60–80	80–100
Frequency	20	16	28	20	5

28. Find the mean of the following distribution by assumed mean method.

Class interval	10–25	25–40	40–55	55–70	70–85	85–100
Frequency	2	3	7	6	6	6

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Show graphically $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ has unique solution. Also, find the area of triangle formed by these lines with x-axis and y-axis.

OR

Draw the graph of $5x - y = 7$ and $x - y + 1 = 0$. Also find the coordinates of the points where these lines intersect the y-axis.

30. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

31. Prove: $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\operatorname{cosec} A$

32. Prove that $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$

33. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

OR

Prove that in a triangle, if square of one side is equal to the sum of the squares of the other sides, then angle opposite the first side is a right angle.

34. Find the missing frequency in the following frequency distribution table, if $N = 100$ and median is 32.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	Total
Number of students	10	f_1	25	30	f_2	10	100

Mathematics

Model Question Paper (Unsolved) – 9
Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. If d = HCF (92,510), the value of d is:

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 6

2. The decimal expansion of the rational number $\frac{19}{2^2 \times 5}$ will terminate after:

(*a*) one decimal place (*b*) two decimal places
(*c*) three decimal places (*d*) more than three decimal places

3. If ΔABC is right-angled at A, then $\cos(B + C)$ is

(*a*) 1 (*b*) $\frac{1}{\sqrt{2}}$ (*c*) $\frac{1}{2}$ (*d*) 0

4. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(*a*) $\cos\beta$ (*b*) $\cos 2\beta$ (*c*) $\sin\alpha$ (*d*) $\sin 2\alpha$

5. Given that $\sin\theta = \frac{x}{y}$, then $\cos\theta$ is equal to

(*a*) $\frac{y}{\sqrt{y^2 - x^2}}$ (*b*) $\frac{y}{x}$ (*c*) $\frac{\sqrt{y^2 - x^2}}{y}$ (*d*) $\frac{x}{\sqrt{y^2 - x^2}}$

6. In Fig. 1, $CDB = 90°$ and $ACB = 90°$, then $\sin A + \cos A$ is equal to

(*a*) $\frac{5}{12}$ (*b*) $\frac{7}{12}$
(*c*) $\frac{17}{13}$ (*d*) $\frac{7}{13}$

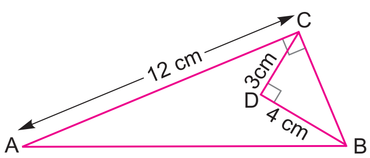


Fig. 1

7. In the formula $\bar{x} = a + \frac{\sum f_i u_i}{\sum f}.h$, for finding the mean of grouped frequency distribution, u_i =

(*a*) $(x_i + a)/h$ (*b*) $h(x_i - a)$ (*c*) $(x_i - a)/h$ (*d*) $(a - x_i)/h$

8. The zeroes of the quadratic polynomial $x^2 + ax + ba, b > 0$ are

(*a*) both positive (*b*) both negative
(*c*) one positive one negative (*d*) can't say

9. If a pair of linear equations has infinitely many solutions, then the lines representing them will be:

(*a*) parallel (*b*) intersecting or coincident (*c*) always intersecting (*d*) always coincident

10. For the following distribution

Class interval	0–8	8–16	16–24	24–32	32–40
Frequency	12	26	10	9	15

The sum of upper limits of the median class and modal class is

(*a*) 24 (*b*) 40 (*c*) 32 (*d*) 16

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Use Euclid's division algorithm to find HCF of 306 and 657.

12. If α and β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

OR

Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$ and then verify the relationship between the zeroes and the coefficients.

13. Is the pair of equations $x - y = 5$ and $2y - x = 10$ inconsistent? Justify your answer.

14. Find the mode of the following distribution of marks obtained by 20 students:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

15. ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$

16. In Fig. 2, $PQ > PR$. QS and RS are the bisectors of $\angle Q$ and $\angle R$ respectively. Prove that $SQ > SR$.

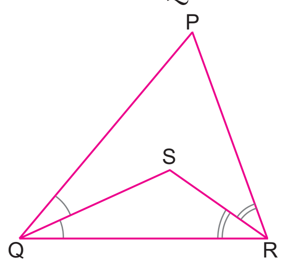


Fig. 2

17. Find the median for the following data:

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	10	20	7	8	5

18. If $5 \cot\theta = 3$, find the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Using Euclid's division algorithm, show that the square of any positive integer is either of the form $3q$ or $3q + 1$ for some integer q.

20. Show that $(2+\sqrt{3})$ is an irrational number.

21. If the polynomial $p(x) = 3x^3 - 4x^2 - 17x + k$ is exactly divisible by $(3x - 1)$, find the value of k.

22. Find the condition which must be satisfied by the coefficients of the polynomial $f(x) = x^3 - px^2 + qx - r$ when the sum of its two zeroes is zero.

OR

Find a two-digit number such that product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number.

23. Prove that $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

OR

Prove that: $\left(1 + \dfrac{1}{\tan^2 A}\right)\left(1 + \dfrac{1}{\cot^2 A}\right) = \dfrac{1}{\sin^2 A - \sin^4 A}$

24. Prove that: $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

25. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	5	8	x	12	7	8

OR

The following table gives the literacy rate (in %) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45–55	55–65	65–75	75–85	85–95
Number of cities	3	10	11	8	3

26. The length of 40 leaves of a plant are measured correct to the nearest millimetre and the data obtained is represented in the table given below. Find the mode of the data.

Length (in mm)	118–126	127–135	136–144	145–153	154–162	163–171	172–180
No. of leaves	3	5	9	12	5	4	2

27. In Fig. 3, $DE \| OQ$ and $DF \| OR$, show that $EF \| QR$.

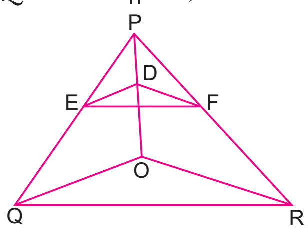


Fig. 3

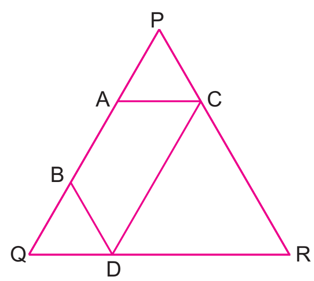


Fig. 4

28. In Fig. 4, $PA = QB$, $AC \| QR$ and $BD \| PR$. Prove that $CD \| PQ$.

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. State and prove the Pythagoras Theorem. Using this theorem, prove that in a triangle ABC, if AD is perpendicular to BC, then $AB^2 + CD^2 = AC^2 + BD^2$.

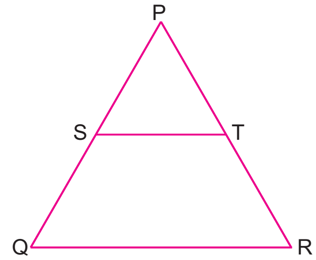


Fig. 5

OR

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. Using the above result, do the following.

In Fig. 5, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $PST = PRQ$. Prove that PQR is an isosceles triangle.

30. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

31. Solve the following system of linear equations graphically:

$$3x + y - 12 = 0$$
$$x - 3y + 6 = 0$$

Shade the region bounded by these lines and the x-axis. Also find the ratio of areas of triangles formed by given lines with x-axis and the y-axis.

32. Prove that: $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$

33. Prove that: $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\text{cosec } A - 1}{\text{cosec } A + 1}$

34. Draw the more than cumulative frequency curve for the following. Also find the median from the graph.

Weight (Kg)	40–44	44–48	48–52	52–56	56–60	60–64	64–68
No. of students	7	12	33	47	20	11	5

OR

Draw a less than ogive from the following distribution:

Class interval	100–150	150–200	200–250	250–300	300–350
Frequency	4	6	13	5	2

Find the median from the graph

Mathematics

Model Question Paper (Unsolved) – 10

Summative Assessment – I

Time: 3 to 3½ hours **Maximum Marks: 80**

General Instructions: *As given in Model Question Paper (Unsolved) – 1.*

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. The rational number between $\sqrt{3}$ and $\sqrt{5}$ is:

(*a*) $\frac{7}{5}$ (*b*) $\frac{9}{5}$ (*c*) $\frac{5}{9}$ (*d*) None of these

2. The decimal expression of the rational number $\frac{44}{2^3 \times 5}$ will terminate after:

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) more than three decimal places

3. The value sec 30° is

(*a*) $\frac{2}{\sqrt{3}}$ (*b*) $\frac{\sqrt{3}}{2}$ (*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{2}$

4. If $\sec\theta = \frac{13}{12}$, then the value of $\tan\theta$ is

(*a*) $\frac{4}{12}$ (*b*) $\frac{7}{12}$ (*c*) $\frac{5}{12}$ (*d*) $\frac{12}{5}$

5. If $\sin 2\theta = \cos 3\theta$, where 2θ and 3θ are acute angles, the value of θ is

(*a*) 17° (*b*) 19° (*c*) 18° (*d*) 20°

6. $\operatorname{cosec}^2\theta - \cot^2\theta$ is equal to:

(*a*) $\tan^2\theta$ (*b*) -1

(*c*) $\cot^2\theta$ (*d*) 1

7. A quadratic polynomial with 3 and 2 as the sum and product of its zeroes respectively is

(*a*) $x^2 + 3x - 2$ (*b*) $x^2 - 3x + 2$

(*c*) $x^2 - 2x + 3$ (*d*) $x^2 - 2x - 3$

8. The number of zeroes of the polynomial represented in Fig. 1, is:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 0

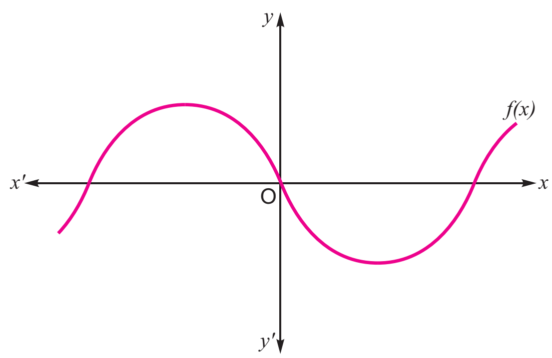


Fig. 1

9. In the given Fig. 2, $PQR = QOR = 90°$. If $PR = 26$ cm, $OQ = 6$ cm, $OR = 8$ cm, then PQ is

(*a*) 25 cm (*b*) 27 cm

(*c*) 24 cm (*d*) 30 cm

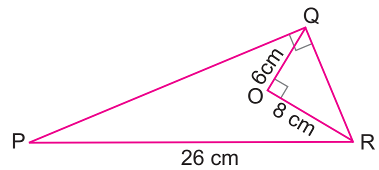


Fig. 2

10. If the mean of the following distribution is 7.5, then value of p is

x	3	5	7	9	11	13
f	6	8	15	p	8	4

(*a*) 3 (*b*) 4 (*c*) 2 (*d*) 5

Section – B

Question numbers 11 to 18 carry 2 marks each.

11. Prove that the sum of a rational and an irrational number is irrational.

12. Mukta can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

OR

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

13. Find all zeroes of the polynomial $x^3 + 3x^2 - 4x - 12$, if one of its zeroes is –3.

14. What is the frequency of the class 20–40 in the following distribution?

Age (years)	Number of Persons
more than or equal to 0	83
more than or equal to 20	55
more than or equal to 40	32
more than or equal to 60	19
more than or equal to 80	8

15. Find the unknown entries a, b, c, d, e in the following distribution of heights of students in a class:

Heights	150-155	155-160	160-165	165-170	170-175	175-180	Total
f	12	b	10	d	e	2	50
C.f.	a	25	c	43	48	f	

16. Find the value of x for which $DE || BC$ in the following Fig. 3.

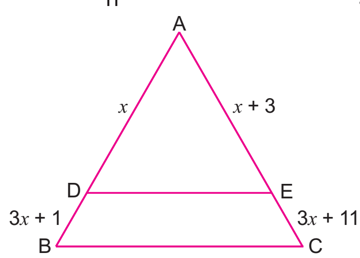


Fig. 3

17. In the given Fig. 4, in ΔABC, $B = C$ and $BD = CE$. Prove that $DE || BC$.

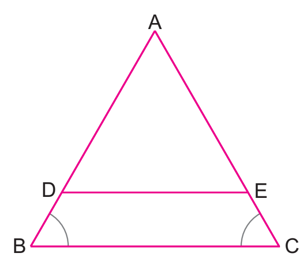


Fig. 4

18. Prove that $(\operatorname{cosec}\theta - \cot\theta) = \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$ is an identity.

Section – C

Question numbers 19 to 28 carry 3 marks each.

19. Show that any positive odd integer is of the form $6q+1, 6q+3$ *or* $6q+5$ where q is any positive integer.

20. Check whether 8^n can end with the digit 0 (zero) for any natural number 'n'.

OR

Show that $3\sqrt{5}$ is irrational number.

21. Solve the following system of linear equations graphically.

$$x+y=3, \quad 3x-2y=4$$

State whether the equations are consistent or not.

22. If the polynomial $6x^4+8x^3+17x^2+21x+7$ is divided by another polynomial $3x^2+4x+1$, the remainder comes out to be $ax+b$, find a and b.

OR

If a polynomial $f(x)=x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out be $(x+a)$, find k and a.

23. If A and B are acute angles, such that $\tan A=\frac{1}{2}, \tan B=\frac{1}{3}$ and $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A.\tan B}$, find $A+B$.

OR

If $\sin(A+B)=1$ and $\cos(A-B)=\frac{\sqrt{3}}{2}$, $0°< A+B \le =90°, A>B$, then find A and B.

24. Find the unknown entries *a, b, c, d, e, f* in the following distribution of heights of students in a class:

Height (in cm)	150–155	155–160	160–165	165–170	170–175	175–180
Frequency	12	b	10	d	e	2
Cummutative Frequency	a	25	c	43	48	f

25. The perpendicular from A on side BC of a ΔABC intersects BC at D such that $DB = 3CD$. Prove that $2AB^2=2AC^2+BC^2$.

26. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

27. Evaluate: $\frac{\cos 58°}{\sin 32°}+\frac{\sin 22°}{\cos 68°}-\frac{\cos 38° \operatorname{cosec} 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$

28. Find the mean marks and modal marks of students for the following distribution:

Marks	**Number of Students**
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43

60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

Section – D

Question numbers 29 to 34 carry 4 marks each.

29. Prove the following identity.

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\cos A+\sin A$$

OR

Prove that:

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\tan A+\cot A=1+\sec A.\text{cosec } A$$

30. Prove that:

$$\frac{1}{\cos\theta+\sin\theta-1}+\frac{1}{\cos\theta+\sin\theta+1}=\text{cosec }\theta+\sec\theta$$

31. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

OR

Prove that, if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, the other two sides are divided in the same ratio.

32. A man travels 600 km partly by train and the rest by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

33. Solve:

$$ax+by=a-b$$
$$bx-ay=a+b$$

34. Draw an ogive and the cumulative frequency polygon for the following frequency distribution by less than method.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	7	10	23	51	6	3

Answers

Chapter–1: Real Numbers

Summative Assessment

Multiple Choice Questions

1. (*b*)	2. (*c*)	3. (*b*)	4. (*d*)	5. (*c*)	6. (*c*)	7. (*c*)
8. (*b*)	9. (*b*)	10. (*d*)	11. (*d*)	12. (*c*)	13. (*d*)	14. (*a*)
15. (*b*)	16. (*c*)	17. (*c*)	18. (*c*)			

Exercise

A. Multiple Choice Questions

1. (*b*)	2. (*b*)	3. (*c*)	4. (*c*)	5. (*c*)	6. (*d*)	7. (*a*)
8. (*d*)	9. (*b*)	10. (*b*)				

B. Short Answer Questions Type–I

1. No, because an integer can be written in the form $4q$, $4q+1$, $4q+2$, $4q+3$.
2. No. $(3q+1)^2 = 9q^2+6q+1 = 3(3q^2+2q) + 1 = 3m+1$.
3. No, because $6^n = (2\times3)^n = 2^n\times3^n$, so the only primes in the factorisation of 6^n are 2 and 3, and not 5.
4. HCF = 75, as HCF is the highest common factor.
5. q has the factors of the form $2^n\times5^m$ for whole numbers n and m.
6. Since $0.1\overline{34}$ has non-terminating repeating decimal expansion, its denominator has factors other than 2 or 5.

C. Short Answer Questions Type–II

10. 4	11. $y=19$	12. 63	13. 1260	14. 75 cm
15. 2520 cm	16. $2^3\times5^4$, 0.0514			

Formative Assessment

Activity

1. Algorithm	2. Irrational	3. Lemma	4. Arithmetic	5. Terminating
6. Decimal	7. Real	8. Prime	9. Euclid	10. Product
11. Rational				

Rapid Fire Quiz

1. T	2. F	3. T	4. F	5. T	6. T	7. F
8. F	9. T	10. T	11. F			

Match the Columns

(*i*) (*b*) (*ii*) (*a*) (*iii*) (*e*) (*iv*) (*d*) (*v*) (*g*) (*vi*) (*f*) (*vii*) (*c*)

Oral Questions

3. No 4. No

10. The factors of q should be of the form 2^m5^n for some non-negative integers m and n.

11. No, it is irrational 12. Yes, rational 13. Yes, 2 14. 1

15. Rational and irrational 16. No, as it is non-terminating non-repeating 17. Two

18. $2+\sqrt{3}$ and $2-\sqrt{3}$ 19. 1 20. Even

Multiple Choice Questions

1. (*c*)	2. (*d*)	3. (*d*)	4. (*c*)	5. (*c*)	6. (*c*)	7. (*c*)
8. (*b*)	9. (*c*)	10. (*d*)				

Class Worksheet

1. (*i*) Terminate after 3 decimal, places (*ii*) Not terminate (*iii*) Not terminate
(*iv*) Terminate after 3 decimal, places (*v*) Terminate after 7 decimal places
2. (*i*) *c* (*ii*) *d* (*iii*) *d* (*iv*) *c* (*v*) *d* (*vi*) *b*
3. (*i*) True (*ii*) False
4. (*ii*) HCF of 847 and 2160 is 1, Therefore the numbers are co-prime 5. HCF = 6, LCM = 3024

Paper Pen Test

1. (*i*) *b* (*ii*) *c* (*iii*) *b* (*iv*) *a* (*v*) *d* (*vi*) *d*
2. (*i*) True (*ii*) True 3. (*i*) 3 4. (*ii*) HCF 24, LCM 360

Chapter–2: Polynomials

Summative Assessment

Multiple Choice Questions

	Ans.	Solution
1.	(*b*)	Sum of the roots = (–3)+2 = –1, Product of the roots = (–3)(2) = – 6 Required polynomial = $x^2 + x - 6$
2.	(*b*)	
3.	(*b*)	Let α, β, γ be the roots and $\alpha = 0$ Then $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ $\quad \beta\gamma = \frac{c}{a}$
4.	(*c*)	Sum of the roots = (–3) + 4 = 1, Product of the roots = (–3)(4) = –12 Required polynomial = $(x^2 - x - 12)$ or $\frac{x^2}{2} - \frac{x}{2} - 6$
5.	(*a*)	$\because$ (–3) is a zero $\quad (k-1)(-3)^2 + k(-3) + 1 = 0$ $9k - 9 - 3k + 1 = 0 \quad 6k = 8$ or $k = \frac{4}{3}$
6.	(*a*)	Let α, β, γ be the roots and $\alpha\beta = 3$ Then $\alpha\beta\gamma = -\frac{9}{2} \quad 3 \times \gamma = \frac{-9}{2}$ or $\gamma = \frac{-3}{2}$
7.	(*c*)	
8.	(*c*)	Let the roots be α and $\frac{1}{\alpha}$. Then $\alpha\left(\frac{1}{\alpha}\right) = \frac{m}{5}$ or $m = 5$ or m.
9.	(*a*)	
10.	(*b*)	$\frac{1}{\alpha} + \frac{1}{\alpha\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-1}{-1} = 1$
11.	(*b*)	
12.	(*d*)	
13.	(*d*)	2 and –3 are the roots $(2)^2 + (a+1)2 + b = 0$ and $(-3)^2 + (a+1)(-3) + b = 0$ $2a + b + 6 = 0$ and $-3a + b + 6 = 0$ on solving, we get $a = 0, b = -6$

Exercise

A. Multiple Choice Questions

1. (*c*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*c*) 6. (*b*) 7. (*b*)

B. Short Answer Questions Type–I

1. True 2. False 3. True 4. False 5. No 6. $\deg g(x) \le \deg p(x)$
7. $\deg p(x) < \deg g(x)$

C. Short Answer Questions Type–II

1. (*i*) $-2, \frac{2}{3}$ (*ii*) $\frac{-1}{7}, \frac{2}{3}$ (*iii*) $\frac{-3\sqrt{2}}{2}, \frac{\sqrt{2}}{4}$ (*iv*) $\sqrt{30}, -\sqrt{30}$ (*v*) $\frac{2}{\sqrt{3}}, 3\sqrt{3}$
(*vi*) $a, \frac{1}{a}$ (*vii*) $\frac{-2}{3}, \frac{1}{2}$ (*viii*) $\frac{1}{4}, \frac{1}{4}$

3. (*i*) $\frac{1}{3}(3x^2 - 2x - 1); \frac{-1}{3}, 1$ (*ii*) $x^2 - 4\sqrt{3};\ 2(3)^{\frac{1}{4}}, -2(3)^{\frac{1}{4}}$
(*iii*) $\frac{1}{2\sqrt{5}}(2\sqrt{5}x^2 + 3x - \sqrt{5}); \frac{-\sqrt{5}}{2}, \frac{1}{\sqrt{5}}$ (*iv*) $\frac{1}{16}(16x^2 - 42x + 5); \frac{1}{8}, \frac{5}{2}$
4. $x^3 + 3x^2 - 8x - 2$ 5. (*i*) No (*ii*) No (*iii*) Yes
6. $a = -2, b = -8$ 7. (*i*) $-5, \frac{3}{2}$ (*ii*) $-\frac{1}{2}$
8. (*i*) $2x^2 - 3 = 2(x^2 + 1) - 5$ (*ii*) $x^3 + 1 = 0.(x^4) + (x^3 + 1)$ (*iii*) $x^2 + 1 = 1(x^2 - 1) + 2$
9. (*i*) $\frac{37}{9}$ (*ii*) $\frac{215}{27}$ (*iii*) $\frac{-215}{18}$ 10. $k = \frac{-2}{3}$

D. Long Answer Questions

1. (*i*) $\frac{1}{9}(9x^2 - 85x + 36)$ (*ii*) $\frac{1}{3}(3x^2 - 35x + 92)$ 2. $-\sqrt{3}, -1$
3. $x^2 - 2x + 3$ 4. –5, 7 5. $19x + 1$ 6. $x - 2$ 7. (*i*) $\frac{-13}{216}$ (*ii*) $\frac{-2}{3}$
8. –2, 1, 4 9. (*i*) $\frac{10001}{16}$ (*ii*) $\frac{-36}{5}$ 10. $\frac{1}{16}(16x^2 - 65x + 4)$

Formative Assessment

Activity

1. Remainder 2. Polynomial 3. Dividend 4. Variable 5. Factor 6. Constant
7. Real 8. Cubic 9. Zero 10. Root 11. Identity 12. Degree 13. Linear

Think Discuss and Write

1. Yes, $x^7 + x - 1$ 2. False, $x^3 + 1$ is a binomial of degree 3
3. False, $4x^2$ is a monomial of degree 2 4. Yes, $4x^3 + 3x^2 + 2x + 1$ is a cubic polynomial

Oral Questions

1. T 2. F 3. No 4. $\deg g(x) \le \deg p(x)$
5. Degree of Quotient = 1, Degree of Remainder = 1 6. Yes 7. T 8. No
9. Same sign 10. F 11. F, because it is equal to 3.

Multiple Choice Questions

1. (*a*) 2. (*b*) 3. (*d*) 4. (*a*) 5. (*b*) 6. (*c*) 7. (*b*)
8. (*a*) 9. (*b*) 10. (*c*) 11. (*c*) 12. (*d*) 13. (*a*) 14. (*c*) 15. (*d*)

Match the Columns

(*i*) (*d*) (*ii*) (*a*) (*iii*) (*c*) (*iv*) (*d*) (*v*) (*f*) (*vi*) (*e*)

Class Worksheet

Rapid Fire Quiz

1. (*i*) F (*ii*) F (*iii*) T (*iv*) F (*v*) T (*vi*) F (*vii*) T
   (*viii*) F (*ix*) F (*x*) F (*xi*) T
2. (*i*) *a* (*ii*) *c* (*iii*) *a* (*iv*) *b* (*v*) *b* (*vi*) *c*
3. (*i*) False (*ii*) True, deg divisor ≤ deg dividend if remainder is zero
4. (*i*) Zeroes are $\frac{4}{3}$ and $\frac{-3}{2}$ (*ii*) Quotient $= -4x^2 - 5$, Remainder $= 3x + 13$
5. (*i*) $x^2 + 2\sqrt{3}x - 9$, Zeroes are $-3\sqrt{3}$, $\sqrt{3}$ (*ii*) The zeroes are $\sqrt{2}, \frac{-2\sqrt{2}}{3}, \frac{-\sqrt{2}}{2}$
6. (*i*) $6x - 2x \neq -4x$ (*ii*) $6x - 2x \neq -4x$
7. Step I: 10, 2, 2, 2, *x*, 2; Step 2: $5x + 2 = 0$, $x - 2 = 0$; Zeroes are $-\frac{2}{5}, 2$

Project Work

1. Quadratic 2. Atmost two 3. (*a*) two (*b*) one (*c*) zero

Paper Pen Test

1. (*i*) *b* (*ii*) *d* (*iii*) *d* (*iv*) *a* (*v*) *b* (*vi*) *a*
2. (*i*) False (*ii*) True 3. (*i*) Zeroes are $-\sqrt{2}, -\frac{1}{2}$ (*ii*) $g(x) = -4x^2 - 3x + 6$
4. (*i*) $k = 3$, Quotient $= x^2 + 3$, 2 is the zero of $x^3 - 2x^2 + 3x - 6$
   (*ii*) When $a = 5, b = -3$ and when $a = -1, b = 3$, zeroes are -1, 2, 5

Chapter–3: Pair of Linear Equations in Two Variables

Summative Assessment

Multiple Choice Questions

	Ans.	Solution
1.	(*a*)	$\frac{a_1}{a_2} = \frac{6}{3} = 2, \frac{b_1}{b_2} = \frac{-7}{-4} = \frac{7}{4}, \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ unique solution
2.	(*d*)	$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{5}{15} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-10}{-30} = \frac{1}{3}$ $\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Infinitely many solutions
3.	(*c*)	The system will be inconsistent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ *i.e.* $\frac{1}{2} = \frac{3}{k} \neq \frac{-4}{-7}$ or $k = 6, k \neq \frac{21}{4}$
4.	(*d*)	The system will have unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ *i.e.* $\frac{k}{6} \neq \frac{-1}{-2}$ or $k \neq 3$

5.	(c)	The given system has infinitely many solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ i.e. $\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$ or $a + b = 6$ ...(i) $2a - b = 9$ ...(ii) On solving (i) and (ii), we get $a = 5$, $b = 1$
6.	(a)	$am \neq bl$ $\frac{a}{l} \neq \frac{b}{m}$ i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ It has a unique solution
7.	(c)	Since the system represents coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ i.e. $\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{7}{4a+b}$ or $8a+2b = 7a+7b$ or $a - 5b = 0$
8.	(d)	
9.	(c)	Since the lines are parallel $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e. $\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$ or $k = \frac{15}{4}$, $k \neq -5$
10.	(b)	2 (3) + 5 (–2) × 4 = 6 –10 + 4 = 0 4 (3) + 10 (–2) + 8 = 12 – 20 + 8 = 0
11.	(a)	Let number of Re 1 coins be x and ₹ 2 coins be y Then, $x + y = 50$ ...(i) $x + 2y = 75$ ...(ii) On solving (i) and (ii), we get $x = 25$, $y = 25$
12.	(b)	Let the units digit be x and the tens digit be y Then number $= 10y + x$ Reversed number $= 10x + y$ $10y + x - 18 = 10x + y$ $9x - 9y + 18 = 0$ $x - y + 2 = 0$ ...(i) Also $x + y = 12$ ...(ii) On solving (i) and (ii), we get $x = 5$, $y = 7$ The number = 75

Exercise

A. Multiple Choice Questions

1. (a) 2. (c) 3. (a) 4. (d) 5. (c) 6. (d) 7. (b)
8. (b) 9. (c) 10. (b) 11. (c) 12. (b)

B. Short Answer Questions Type–I

1. False, it should be –1
2. Yes, since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
3. No, because $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so the equations represent parallel lines.
4. Yes, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

5. 0   6. Infinite   7. No, since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so it has a unique solution

C. Short Answer Questions Type–II

1. (*i*) $4x + y = 1$ (*ii*) $6x - 4y + 3 = 0$ (*iii*) $6x - 4y + 14 = 0$ 2. (*i*) consistent   (*ii*) inconsistent

3. $k = -6$ 4. (*i*) $a = 4, b = 8$ (*ii*) $a = 5, b = 1$ (*iii*) $a = -1, b = \frac{5}{2}$

5. $x + y + 1 = 0, x - y = 5$, Infinitely many   6. 14, $\frac{-5}{2}$

7. $x = 340, y = -165, \lambda = \frac{-1}{2}$ 8. (0, –2), $\left(0, \frac{1}{5}\right)$ (2, –1); $\frac{11}{5}$ sq. units

9. $a = 5, b = 2$ 10. $x = 85°, y = 55°$ 11. $x = 33, y = 50°$, $A = 70°$, $B = 53°$, $C = 110°$, $D = 127°$

12. (*i*) $x = 6, y = 8$ (*ii*) $x = 2, y = 3$ (*iii*) $x = a^2, y = b^2$
(*iv*) $x = 2, y = -3$ (*v*) $x = 4, y = 9$ (*vi*) $x = \frac{1}{2}, y = \frac{-3}{2}$
(*vii*) $x = 8, y = 3$ (*viii*) $x = \frac{1}{3}, y = -1$ (*ix*) $x = 5, y = 1$
(*x*) $x = 3, y = 4$ (*xi*) $x = \frac{-1}{2}, y = \frac{1}{4}$ (*xii*) $x = 1, y = 3$
(*xiii*) $u = 2, v = 1$ (*xiv*) $x = 4, y = 5$

13. (*i*) Inconsistent   (*ii*) consistent $x = 2, y = -3$ (*iii*) consistent $x = -1, y = -1$

14. 40 years   15. 100 students in hall A, 80 students in hall B

16. length = 20 m, Breadth = 16 m   17. 40°, 140°

18. (*i*) $x = 3, y = 2$, (0, 3.5), (0, –4)   (*ii*) $x = 2, y = 3$, (0, 6) and (0, –2)

19. (*i*) $x = 1, y = 2$ (5, 0) (– 2 , 0)   (*ii*) $x = 1, y = 2$, (4, 0), (–3, 0)

20. (i) $x = 1, y = -1$ (*ii*) $x = \frac{-1}{2}, y = 2$ (*iii*) $x = m + n, y = m - n$ (*iv*) $x = 11, y = 8$

21. $10x + y = 3 + 4(x + y)$ and $10x + y + 18 = 10y + x$

D. Long Answer Questions

1. (0, 0), (4, 4) (6, 2)   2. ₹ 10, ₹ 15   3. 6 square units   4. 10 km/h, 40 km/h

5. 69 or 96   6. 2.5 km/h   7. (*i*) $x = 2, y = -1$ (*ii*) $x = 1, y = 4$

8. $x = 2, y = 4$, 12 sq. units   9. Scheme A → ₹ 12000,   Scheme B → ₹ 10,000   10. 36

11. $\frac{4}{7}$ 12. Father's age = 42 years,   Son's age = 10 years

13. Speed from point A = 40 km/ h, from point B = 30 km/h

14. 100 km/h, 80 km/h   15. 60 km/h, 40 km/h   16. ₹ 215

17. ₹ 600, ₹ 40   18. One man in 36 days, One woman in 18 days

19. 25   20. 36

Formative Assessment

Activity:1

1. Consistent   2. Infinite   3. One   4. Unique   5. Line   6. Elimination   7. Parallel

Oral Questions

1. A pair of linear equations which has either unique or infinitely many solutions.

2. A straight line.
3. When it has no solution.
4. Yes.
5. Yes.
6. It has infinitely many solutions.
7. Two coincident lines

Activity: 2 Hands on Activity

1. $\frac{l}{t} \neq \frac{m}{n}$
3. Unique solution, Intersecting lines
4. $k \neq -6$, *i.e.*, all values except – 6
5. consistent
6. All values except 10

Activity: 3 (Analyses of graph)

1. (2, 0), (4, 0)
2. (0, –2), (0, 4)
3. Unique solution: (3, 1)
4. 1 sq. unit
5. 9 sq. units

Multiple Choice Questions

1. (*d*) 2. (*b*) 3. (*a*) 4. (*d*) 5. (*b*) 6. (*c*) 7. (*a*)
8. (*d*) 9. (*b*) 10. (*b*) 11. (*b*) 12. (*b*) 13. (*b*) 14. (*b*) 15. (*a*)

Rapid Fire Quiz

1. T 2. F 3. T 4. F 5. T 6. F 7. T 8. T

Match the Columns

(*i*) (*d*) (*ii*) (*e*) (*iii*) (*a*) (*iv*) (*f*) (*v*) (*b*) (*vi*) (*c*)

Class Worksheet

1. (*i*) *b* (*ii*) *c* (*iii*) *a* (*iv*) *a* (*v*) *b*
2. (*i*) True (*ii*) False
3. (*i*) $x = 7$ and $y = 9$, values –1 and $\frac{30}{7}$ (*ii*) $x = 20$, $y = 30$
$A = 130°$, $B = 100°$, $C = 50°$, $D = 80°$
4. (*i*) Area of trapezium = 8 sq. units
(*ii*) Speed of the boat in still water = 10 km/h; Speed of the stream = 4 km/h

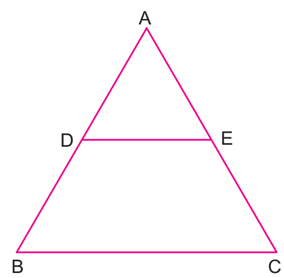
Paper Pen Test

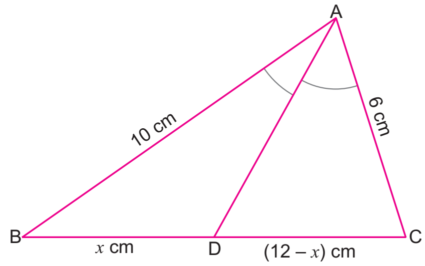
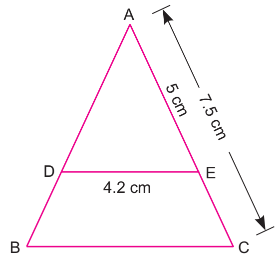
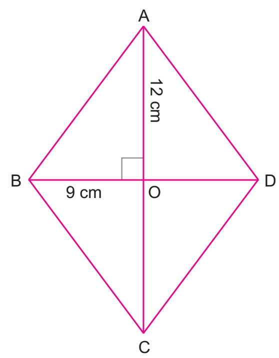
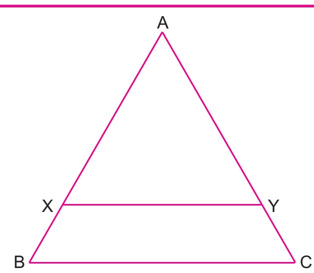
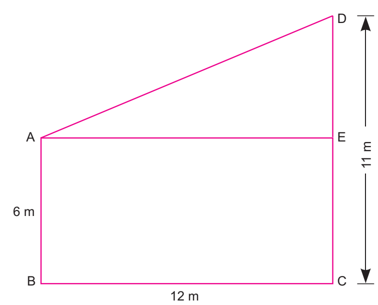
1. (*i*) *c* (*ii*) *c* (*iii*) *c* (*iv*) *c* (*v*) *c*
2. (*i*) True (*ii*) True
3. (*i*) $a = 3$, $b = 1$ (*ii*) $x = a^2$, $y = b^2$
4. (*i*) $x = 1$, $y = 4$, Areas = 8 sq. units, 2 square units; ratio = 4 : 1
(*ii*) Speed of the bus is 60 km/h; Speed of the train is 90 km/h

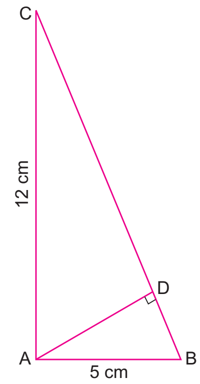
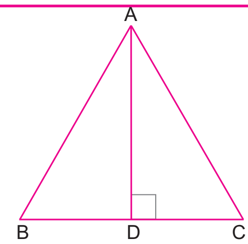
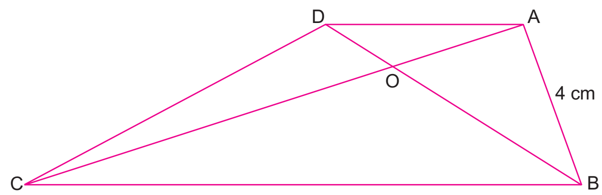
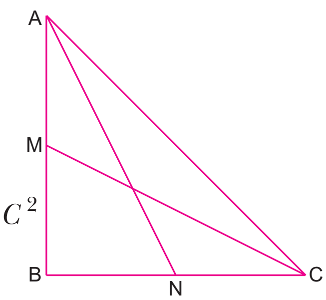
Chapter–4: Triangles

Summative Assessment

Multiple Choice Questions

	Ans.	Solution
1.	(*b*)	Since $DE \parallel BC$ $\frac{AD}{DB} = \frac{AE}{EC} = \frac{2}{3}$ $\frac{6}{EC} = \frac{2}{3}$ or $EC = \frac{3 \times 6}{2} = 9$ cm $AC = AE + EC = 6 + 9 = 15$ cm 

2.	(*c*)	As AD is the bisector of A. $\frac{AB}{AC} = \frac{BD}{DC}$ $\quad \frac{10}{6} = \frac{x}{12-x}$ $120 - 10x = 6x \quad x = 7.5$ $BD = 7.5$ cm 
3.	(*d*)	Since $DE \parallel BC$ By AA corollary, $\Delta ADE \sim \Delta ABC$ $\frac{AE}{AC} = \frac{DE}{BC}$ or $\frac{5}{7.5} = \frac{4.2}{BC}$ Or $\quad BC = \frac{7.5 \times 4.2}{5} = 6.3$ cm 
4.	(*a*)	Ratio of areas of two similar triangles is the square of the ratio of their corresponding sides
5.	(*a*)	$\frac{AB}{DE} = \frac{1}{2} = \frac{BC}{EF}$ $\quad$ or $\frac{8}{EF} = \frac{1}{2}$ *i.e.* $EF = 16$ cm
6.	(*b*)	$AO = OC = 12$ cm, $BO = OD = 9$ cm In right ΔAOB $AB^2 = AO^2 + OB^2 = 12^2 + 9^2$ $= 144 + 81 = 225$ $AB = 15$ cm 
7.	(*c*)	$\frac{AB}{BX} = 4 = \frac{AC}{YC}$ $\quad (\because XY \parallel BC)$ $4 = \frac{AC}{2}$ or $\quad AC = 8$ cm $AY = AC - YC$ $= 8 - 2 = 6$ cm 
8.	(*b*)	*If* AB and CD are the poles, then $\quad AB = CE = 6$ cm, $BC = AE = 12$ m $DE = 11 - 6 = 5$m In right ΔADE, $AD^2 = AE^2 + ED^2$ $= 12^2 + 5^2 = 144 + 25 = 169$ $AD = 13$ cm 

9.	(*b*)	In right ΔABC $BC^2 = AB^2 + AC^2$ $= 12^2 + 5^2 = 169$ $BC = 13$ cm $AD \times BC = AB \times AC$ $AD = \frac{5 \times 12}{13} = \frac{60}{13}$ cm 
10.	(*c*)	In right ΔACD $AD^2 = AC^2 - CD^2$ $= BC^2 - CD^2 \qquad (\because AC = BC)$ $= (2CD)^2 - CD^2$ $= 4CD^2 - CD^2 = 3CD^2$ 
11.	(*b*)	Since $\frac{AO}{OC} = \frac{DO}{OB}$ and $\angle AOB = \angle DOC$ $\Delta AOB \sim \Delta COD$ $\frac{AO}{OC} = \frac{DO}{OB} = \frac{AB}{CD} = \frac{1}{2}$ or $\frac{4}{CD} = \frac{1}{2}$ or $CD = 8$ cm 
12.	(*b*)	In right ΔABC $AC^2 = AB^2 + BC^2$ $= (AN^2 - BN^2) + (CM^2 - BM^2)$ $= AN^2 - \left(\frac{1}{2}BC\right)^2 + CM^2 - \left(\frac{1}{2}AB\right)^2$ $= AN^2 + CM^2 - \frac{1}{4}(BC^2 + AB^2) = AN^2 + CM^2 - \frac{1}{4}AC^2$ or $AC^2\left(1 + \frac{1}{4}\right) = AN^2 + CM^2$ or $5AC^2 = 4(AN^2 + CM^2)$ 

Exercise

A. Multiple Choice Questions

1. (*c*) 2. (*d*) 3. (*b*) 4. (*b*) 5. (*d*) 6. (*a*) 7. (*a*)
8. (*b*) 9. (*d*) 10. (*c*) 11. (*c*) 12. (*a*) 13. (*c*) 14. (*b*)
15. (*c*) 16. (*a*) 17. (*c*)

B. Short Answer Questions Type–I

1. Yes, $26^2 = 24^2 + 10^2$
2. False, a rectangle and a square has each angle equal to 90° but the two figures are not similar.

3. Yes, by AAA criterion   4. No, it will be $\frac{4}{25}$

5. No, because the angles should be the included angle between the two proportional sides.

6. No, $B = \quad Y$ 7. Yes, because $\frac{DL}{LE} = \frac{DM}{MF} = 3$ 8. 6 cm   9. 25 cm   10. 1 : 9

C. Short Answer Questions Type–II

2. 4.8 cm   3. $x = 1$ 4. 17 cm   5. DB = 3.6 cm, CE = 4.8 cm

6. No   7. Yes   9. 18 cm   10. 9 m   13. 60°   14. 1 : 4   15. 10 m

16. (i) 13 cm   25. 11 or 8

D. Long Answer Questions

1. $2\sqrt{5}$ cm, 5 cm, $3\sqrt{5}$ cm   2. 8 cm, 12 cm, 16 cm   9. $\frac{25}{81}$ 10. $\frac{2-\sqrt{2}}{2}$

Formative Assessment

Activity:1

1. Similar   2. Equiangular   3. Line   4. Right angled   5. Parallel

6. Congruent   7. Thales   8. Pythagoras   9. Square

Oral Questions

1. Two polygons of the same number of sides are similar, if their corresponding angles are equal and their corresponding sides are in the same ratio or proportion.
2. If two polygons are similar, then the same ratio of the corresponding sides is referred to as the scale factor.
3. In world maps, blueprints for the construction of a building, etc.
4. Any two circles, two squares, two photographs of same persons but different size, etc.
5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
6. If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then the two triangles are similar.
7. If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
8. False, corresponding angles should be equal.
9. No, corresponding sides are proportional. They may not be equal.
10. True
11. False

Multiple Choice Questions

1. (*d*)   2. (*c*)   3. (*a*)   4. (*c*)   5. (*b*)   6. (*a*)   7. (*a*)

8. (*c*)   9. (*b*)   10. (*c*)   11. (*b*)   12. (*a*)   13. (*c*)   14. (*c*)

15. (*d*)   16. (*c*)

Match the Columns

(*i*) (*d*)   (*ii*) (*a*)   (*iii*) (*c*)   (*iv*) (*b*)

Rapid Fire Quiz

1. F   2. F   3. T   4. F   5. T   6. F   7. F

8. Same   9. Square   10. Right   11. Right

Word Box

1. congruent 2. similar 3. congruent 4. scale factor 5. equiangular
6. Basic proportionality 7. Pythagoras 8. parallel 9. corresponding sides
10. similar 11. congruent 12. equal, proportional

Class Worksheet

1. *(i)* *c* *(ii)* *c* *(iii)* *d* *(iv)* *c* 2. *(i)* True *(ii)* False
3. 125 cm^2 4. 4.750 km
5. *(i)* 2.4 cm *(ii)* Yes, $\Delta ABC \sim \Delta QRP$ by SSS similarly criterion
(iii) Yes, $25^2 = 24^2 + 7^2$ 6. *(i)* Similar *(ii)* EF, BC, FD *(iii)* Congruent

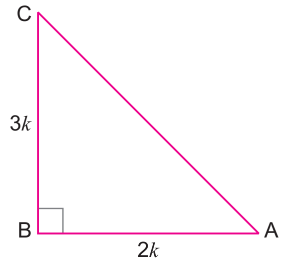
Paper Pen Test

1. *(i)* *c* *(ii)* *d* *(iii)* *c* *(iv)* *a* *(v)* *c* 2. *(i)* False *(ii)* False
3. *(ii)* 9 : 1 4. $AB = 9$ units; $BC = 12$ units; $CA = 15$ units; $DE = 18$ units; $DF = 30$ units; $EF = 24$ units

Chapter–5: Introduction to Trigonometry

Summative Assessment

Multiple Choice Questions

	Ans.	Solution
1.	*(b)*	Given $\tan A = \frac{3}{2}$ Let $AB = 2k, BC = 3k$ Then, $AC^2 = (3k)^2 + (2k)^2 = 13k^2$ $AC = \sqrt{13}\,k$ *i.e.*, $\cos A = \frac{AB}{AC} = \frac{2k}{\sqrt{13}k} = \frac{2}{\sqrt{13}}$ 
2.	*(d)*	$\sin(\alpha + \beta) = 1$ $\alpha + \beta = 90°$ $\cos(\alpha - \beta) = \cos(90° - \beta - \beta) = \cos(90° - 2\beta) = \sin 2\beta$
3.	*(d)*	$\sin\alpha = \frac{1}{\sqrt{2}}$ $\alpha = 45°, \cos\beta = \frac{1}{\sqrt{2}},$ $\beta = 45°$ $\tan(\alpha + \beta) = \tan(45° + 45°) = \tan 90°$ = Not defined
4.	*(a)*	$\because$ ΔABC is right-angled at C $A + B = 180 - C = 90°$ $\cos(A + B) = \cos 90° = 0$
5.	*(c)*	$\cos 9\alpha = \sin\alpha$ $\cos 9\alpha = \cos(90° - \alpha)$ $9\alpha = 90° - \alpha$ or $\alpha = \frac{90}{10} = 9$ $\tan 5\alpha = \tan 45° = 1$
6.	*(c)*	$\frac{\sin 60°}{\cos 30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = 1$

7.	(*b*)	Given expression $= [\text{cosec}\,(75°+\theta) - \sec\{90° - (75+\theta)\} - \tan(55°+\theta) + \cot(90° - (55+\theta)]$ $= \text{cosec}\,(75°+\theta) - \text{cosec}\,(75°+\theta) - \tan(55°+\theta) + \tan(55°+\theta)$ $= 0$
8.	(*b*)	Given expression $= \dfrac{\sin^2 22° + \sin^2(90-22°)}{\cos^2 22° + \cos^2(90-22°)} + \sin^2 63° + \cos 63° \sin(90-63)$ $= \dfrac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \cos 63°$ $= 1 + 1 = 2$
9.	(*c*)	$\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \dfrac{4\tan\theta - 1}{4\tan\theta + 1}$ (Dividing numerator and denominator by $\cos\theta$) $= \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$
10.	(*a*)	$\sin(2\times 0) = \sin 0° = 0$ and $2\sin 0° = 2\times 0 = 0$
11.	(*c*)	$\dfrac{2\tan 30°}{1-\tan^2 30°} = \dfrac{\frac{2}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \sqrt{3} = \tan 60°$
12.	(*b*)	$9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A) = 9\times 1 = 9$
13.	(*c*)	$(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\,\theta)$ $= \left(1+\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)\left(1+\dfrac{\cos\theta}{\sin\theta}-\dfrac{1}{\sin\theta}\right)$ $= \dfrac{(\cos\theta+\sin\theta+1)(\sin\theta+\cos\theta-1)}{\sin\theta\cos\theta}$ $= \dfrac{(\cos\theta+\sin\theta)^2-(1)^2}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} = \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$
14.	(*c*)	$\sec\theta+\tan\theta = x$ $\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta = x^2$ $1+\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta = x^2$ $1+2\tan\theta(\tan\theta+\sec\theta) = x^2$ $1+2x\tan\theta = x^2$ or $\tan\theta = \dfrac{x^2-1}{2x}$
15.	(*b*)	$\cos^4 A - \sin^4 A = (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$ $= 1[\cos^2 A - (1-\cos^2 A)] = 2\cos^2 A - 1$

Exercise

A. Multiple Choice Questions

1. (*a*)	2. (*b*)	3. (*b*)	4. (*c*)	5. (*c*)	6. (*a*)	7. (*a*)
8. (*a*)	9. (*d*)	10. (*c*)	11. (*d*)	12. (*d*)	13. (*b*)	14. (*c*)
15. (*c*)	16. (*c*)	17. (*d*)	18. (*d*)	19. (*b*)	20. (*a*)	21. (*c*)

22. (*b*) 23. (*b*)

B. Short Answer Questions Type–I

1. True 2. False 3. True 4. True 5. False 6. $\frac{1}{9}$ 7. 1

8. 1 9. 1 10. $\alpha = 1$ 11. 2

C. Short Answer Questions Type–II

1. $\sin A = \frac{3}{5}$, $\tan A = \frac{3}{4}$, $\cot A = \frac{4}{3}$
2. $\cos A = \frac{7}{25}$, $\tan A = \frac{24}{7}$, $\operatorname{cosec} B = \frac{25}{7}$
3. $\sin A = \frac{5}{13}$, $\cot A = \frac{12}{5}$
4. $\sin\theta = \frac{3}{4}$, $\cos\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = \frac{3}{\sqrt{7}}$, $\sec\theta = \frac{4}{\sqrt{7}}$, $\cot\theta = \frac{\sqrt{7}}{3}$
5. (*i*) 1 (*ii*) 0
7. $\sin Q = \frac{7}{25}$, $\cos Q = \frac{24}{25}$
8. $\sin\beta = \frac{\tan\beta}{\sqrt{1+\tan^2\beta}}$, $\cos\beta = \frac{1}{\sqrt{1+\tan^2\beta}}$, $\operatorname{cosec}\beta = \frac{\sqrt{1+\tan^2\beta}}{\tan\beta}$, $\sec\beta = \sqrt{1+\tan^2\beta}$, $\cot\beta = \frac{1}{\tan\beta}$
9. $\sin\theta = \frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$, $\operatorname{cosec}\theta = \sqrt{10}$, $\sec\theta = \frac{\sqrt{10}}{3}$, $\cot\theta = 3$
10. 0
11. $\frac{3}{5}$ 12. 2 13. $\frac{-13}{3}$ 14. 9 15. 1 16. $BC = 3\sqrt{3}$ cm, $AC = 6$ cm
17. $A = B = 45°$ 18. $\sin(A+B) = \frac{\sqrt{3}}{2}$, $\cos(A-B) = 1$ 25. 1
29. $\frac{12}{7}$ 30. $x = 30°$ 31. $x = 45°$ 36. 2 37. 1 38. 2 39. 0
40. $-\frac{1}{7}$ 41. $A = 44°$ 44. $\frac{225}{64}$ 46. $\frac{1}{3}$ 47. 3 48. $\frac{2a^2}{a^2+b^2}$ 50. 8

Formative Assessment

Activity

1. Cotangent 2. Identity 3. Cosecant 4. Square 5. Tangent
6. Zero 7. Right triangle 8. Secant 9. One 10. Trigonometry
11. Cosine

Multiple Choice Questions

1. (*d*) 2. (*b*) 3. (*d*) 4. (*a*) 5. (*b*) 6. (*c*) 7. (*a*)
8. (*c*) 9. (*c*) 10. (*b*) 11. (*d*) 12. (*b*) 13. (*a*) 14. (*b*)
15. (*c*)

Match the Columns

(*i*) (*d*) (*ii*) (*c*) (*iii*) (*e*) (*iv*) (*b*) (*v*) (*a*)

Rapid Fire Quiz

1. F 2. T 3. T 4. F 5. F 6. F 7. T
8. F 9. T 10. F 11. F 12. T 13. T 14. T
15. T 16. F 17. T 18. F 19. T 20. T

Oral Questions

1. $\cos A$ 2. Yes 3. 0 4. $\sec^2\theta$ 5. 1 6. AB

7. An equation which holds true for all values of the variable.

8. 1 9. Yes 10. cot A 11. Hypotenuse 12. False

13. Increase because as we increase θ, the side opposite to right angle will increase and the ratio of $\tan\theta$ will also increase.

14. It will increase 15. $\cot\theta = \frac{\cos\theta}{\sin\theta}$ 16. $1+\tan^2\theta = \sec^2\theta$ 17. False 18. No

Class Worksheet

1. (*i*) *b* (*ii*) *c* (*iii*) *b* (*iv*) *a* (*v*) (*b*) (*vi*) (*d*) (*vii*) (*c*) (*viii*) (*c*)
2. (*i*) False (*ii*) True
3. (*i*) F (*ii*) F (*iii*) T (*iv*) F (*v*) F (*vi*) T
4. (*i*) 6 (*ii*) 0 (*iii*) 0 (*iv*) 0° (*v*) 3 (*vi*) 1
5. (*i*) increases (*ii*) decreases (*iii*) 1 (*iv*) 0 (*v*) Tri, gon, metron
6. (*i*) T (*ii*) F (*iii*) F (*iv*) T (*v*) F (*vi*) T

Paper Pen Test

1. (*i*) *b* (*ii*) *c* (*iii*) *d* (*iv*) *a* (*v*) *d* (*vi*) *a*
2. (*i*) False (*ii*) True 3. (*ii*) $\theta = 90°$ 4. (*ii*) LHS = RHS = $\frac{-15}{113}$

Chapter–6: Statistics

Summative Assessment

Multiple Choice Questions

	Ans.	Solution
1.	(*c*)	
2.	(*a*)	Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{6+4p+30+24+20+12}{14+p}$ $6.4 = \frac{92+4p}{14+p}$ or $89.6 + 6.4p = 92 + 4p$ or $2.4p = 2.4$ or $p = 1$
3.	(*b*)	(see table below)
4.	(*c*)	(see table below) Modal class is 30 – 40

Solution 3:

Classes	Cumulative Frequency	Frequency
5,000 –10,000	150	18
10,000 –15,000	132	14
15,000 –20,000	118	33
20,000 –25,000	85	17
25,000 –30,000	68	26
30,000 –35,000	42	42

Solution 4:

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	3	9	15	30	18	5

Modal class is 30 – 40

5.	(c)	

Classes	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 205
Frequency	4	5	13	20	14	7	4
Cumulative Frequency	4	9	22	42	56	63	67

$\frac{n}{2} = \frac{67}{2} = 33.5$, Median class = 125 – 145

Modal class = 125 – 145

Required difference = 145 – 125 = 20

Exercise

A. Multiple Choice Questions

1. (c) 2. (a) 3. (a) 4. (d) 5. (b) 6. (a) 7. (c)
8. (c) 9. (d) 10. (c) 11. (b) 12. (c)

B. Short Answer Questions Type–I

1. Median 2. Mode = 3 Median – 2 Mean 3. 25, 30 4. 30–40
5. 300–350 6. 55–65 7. 12.5–16.5 8. 8 9. 82

10. False, because for calculating the median for a grouped data, we assume that the observations in the classes are uniformly distributed.

11. False, it depends on the data. 12. False, it depends on the data.

C. Short Answer Questions Type –II

1. $p = 20$ 2. $p = 1$ 3. $k = 6$ 4. $p = 20$ 5. 3.54 6. 31 years
7. 36.36 8. $f_1 = 8, f_2 = 12$ 9. $p = 7$ 10. ₹ 211 11. 109.92

12. 14.48 km/l, No

13.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90–100
Number of Students	10	40	80	140	170	130	100	70	40	20

14. (i) Less than Type

(i) Less than Type		(ii) More than Type	
Age (in years)	Number of students	Age (in years)	Number of students
		More than or equal to 10	300
Less than 20	60	More than or equal to 20	240
Less than 30	102	More than or equal to 30	198
Less than 40	157	More than or equal to 40	143
Less than 50	227	More than or equal to 50	73
Less than 60	280	More than or equal to 60	20
Less than 70	300		

15. $f_1 = 8, f_2 = 7$ 16. $f = 25$ 17. ₹ 5800 18. 201.7 kg 19. 65.63 hours

20. Mode = 36.8 years, Mean = 35.38 years

D. Long Answer Questions

1. Mean = 38.2 , Median = 35, Mode = 43.33
2. Mean = 37.2, Median = 39.09, Mode = 42.67
3. Mean = 169, Median = 170.83, Mode = 175
4. Mean = 145.20, Median = 138.57, Mode = 125
5. 58.33
6. Average = 170.3 sec, Median = 170 sec.
7.

(i) Distance (in m)	No. of Students	Cumulative frequency
0–20	6	6
20–40	11	17
40–60	17	34
60–80	12	46
80–100	4	50

(*iii*) 49.41 m

8. 21.25 cm
9. Median = 17.81 hectares, Mode = 17.78 hectares
10. Median = ₹ 17.5 lakh
11. 46.5 kg
12. ₹ 138

Formative Assessment

1. Mode 2. Ogive 3. Mean 4. Class mark 5. Assumed mean
6. Median 7. Frequency 8. Data 9. Interval 10. Class size
11. Empirical

Multiple Choice Questions

1. (*c*) 2. (*d*) 3. (*b*) 4. (*c*) 5. (*a*) 6. (*d*)
7. (*a*) 8. (*c*) 9. (*a*) 10. (*b*) 11. (*b*)

Rapid Fire Quiz

1. F 2. T 3. F 4. T 5. F 6. F 7. T
8. F 9. F

Match the Columns

(*i*) *d* (*ii*) *e* (*iii*) *b* (*iv*) *g* (*v*) *h* (*vi*) *a* (*vii*) *c*
(*viii*) *f*

Oral Questions

1. Mode = 3 Median – 2 Mean
2. Yes
3. Median
4. Ogive
5. Class size
6. Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ where l is the lower limit of the modal class, f_0, f_1, f_2 are the frequencies of the class preceding the modal class, the modal class and the class succeeding the median respectively and h is the class size.
7. Mean, $\bar{x} = a + \frac{\Sigma fidi}{\Sigma fi}$, where a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i.
8. Mean

9. No, because when we calculate the mean of a grouped data, we assume that the frequency of each class is centered at the mid-point of the class. Due to this the two values of mean, namely those from ungrouped data and grouped data are rarely the same.

10. The positional mid value when a list of data has been arranged in ascending or descending order.

Class Worksheet

1. (*i*) *a* (*ii*) *d* (*iii*) *b* (*iv*) *a* (*v*) *b*
2. (*i*) False (*ii*) False
3. 20
4. 50, 55, 52.5
5. $a + \frac{\sum d_i f_i}{\sum f_i}$
6. $a + \frac{\sum f_i u_i}{\sum f_i} \times h$
7.

Class Interval	*x*	*f*	*u*	*fu*
0 – 100	50	2	– 3	– 6
100 – 200	150	8	– 2	– 16
200 – 300	250	12	– 1	– 12
300 – 400	350	20	0	0
400 – 500	450	5	1	5
500 – 600	550	3	2	6
		50		**– 23**

$\bar{x} = 304$

8. (*i*) mode (*ii*) uniform (*iii*) modal (*iv*) 3, mean, mode (*v*) median (*vi*) cumulative frequency of the median class
9. frequency of the class succeeding the modal class

Paper Pen Test

1. (*i*) *d* (*ii*) *c* (*iii*) *a* (*iv*) *b* (*v*) *d* (*vi*) *d*
2. (*i*) False (*ii*) False
3. (*i*) $a = 12,\ b = 13,\ c = 35,\ d = 8,\ e = 5,\ f = 50$ (*ii*) Mode = ₹ 11,875
4. (*i*) Mean = 48.41; Median = 48.44 (*ii*) Median weight = 46.5 kg

Model Question Paper – 1

1. (*c*) 2. (*c*) 3. (*d*) 4. (*d*) 5. (*a*) 6. (*c*) 7. (*b*)
8. (*c*) 9. (*b*) 10. (*b*)

11. No. As prime factorisation of $6^n (6^n = 2^n \times 3^n)$ does not contain 5 as a factor.

12. $x^2 - x + 1$ *OR* $p = \frac{-5}{8}$ 13. $k = 2$ 15. 21 cm^2 16. $\frac{1}{10}$

17.

Marks obtained	Number of students
Less than 10	5
Less than 20	8
Less than 30	12
Less than 40	15
Less than 50	18

Less than 60	22
Less than 70	29
Less than 80	38
Less than 90	45
Less than 100	53

18. No, it is not always the case. The values of these three measures can be the same. It depends on the type of data.

19. 17 21. $\frac{2}{3}, \frac{-1}{7}$ *OR* $x^2 + 2x - 3$ 22. 40 km/h, 30 km/h *OR* 50 years, 20 years

25. $\frac{1}{\sqrt{3}}$ 27. $P = 11$ 28. ₹ 11875

29. $k = -3$, zeroes of $2x^4 + x^3 - 14x^2 + 5x + 6$ are 1, – 3, 2 and $-\frac{1}{2}$, zeroes of $x^2 + 2x - 3$ are 1, –3.

30. 6 sq. units 33. $\frac{2\sqrt{3}}{3}$ 34. $p = 5, q = 7$

Model Question Paper – 2

1. (*c*) 2. (*d*) 3. (*a*) 4. (*c*) 5. (*b*) 6. (*d*) 7. (*d*)

8. (*c*) 9. (*c*) 10. (*a*) 11. 13 12. 0 *OR* $\frac{13}{36}$ 13. $k = -6$ 14. 60 cm

15. yes, $\because AC^2 = AB^2 + BC^2$ and $B = 90°$ 17. 17.3 18. 0

20. 63 21. $k = -9$, quotient $= x^2 + 5x + 6$, zeroes are 3, –2, –3

22. $3x^2 + 8x + 4, -2, -\frac{2}{3}$ *OR* 100 in hall A and 80 in hall B. 25. $\frac{1}{\sqrt{2}}$ 27. 53 28. 25

29. $1, \frac{1}{2}$ 30. (0, 0)(6, 2)(4, 4) 33. $\frac{5}{2}$ 34. 201.81

Model Question Paper – 3

1. (*c*) 2. (*c*) 3. (*a*) 4. (*b*) 5. (*d*) 6. (*d*) 7. (*a*)

8. (*b*) 9. (*d*) 10. (*b*) 11. 435 12. NO

13. $x = 5, y = 3$ *OR* $x - y = -4, 2x + 3y = 7$; infinitely many pairs 17. 27.6 18. 30 – 40

20. HCF = 24, LCM = 685008

21. $x = 3, y = 2$; Lines intersect the y-axis at the points (0, –1) and (0, 11) 22. $\sqrt{3}, -5\sqrt{3}$ *OR* 6

25. $\frac{1}{\sqrt{3}}$ 27. $f = 8$ 28. 154 29. $1, -\frac{1}{2}, 2 + \sqrt{3}$ and $2 - \sqrt{3}$

30. 83 *OR* 100 km/h, 80 km/h 32. $1 + \frac{1}{\sqrt{3}}$ 34. 21.25

Model Question Paper – 4

1. (*b*) 2. (*c*) 3. (*c*) 4. (*b*) 5. (*c*) 6. (*d*) 7. (*d*)

8. (*c*) 9. (*b*) 10. (*b*) 11. NO 12. $a = 3$ *OR* $x^2 - 4x + 1$

13. $x = 3, y = -1$ 14. $x = 2$ 15. 7 cm 17. $x = 62$ 18. NO 20. 625

21. $\frac{\sqrt{2}}{4}, \frac{-3\sqrt{2}}{2}$ 22. Inconsistent *OR* $Q = x - 2$, $R = 3$ 25. $A = 45° B = 15°$

26. $\frac{1}{2}$ 27. 28 28. 106.1 29. $\sqrt{5}, \sqrt{5}+\sqrt{2}, \sqrt{5}-\sqrt{2}$

30. 12 and 4 *OR* 70 days and 140 days 33. $1+2\sqrt{3}$ 34. 138.6

Model Question Paper – 5

1. (*c*) 2. (*c*) 3. (*c*) 4. (*c*) 5. (*b*) 6. (*a*) 7. (*c*)

8. (*d*) 9. (*c*) 10. (*c*) 11. 180, 15 12. 108

13. $k = 10$ *OR* $p^2 + q^2 = 0$ and $r \neq 0$ 17. 7.4 18. 26 20. 999720 21. $-\frac{1}{2}$, 1

22. $x = 1, y = 2$, (5, 0) (–2, 0) *OR* 20 paise coins = 25 and 25 paise coins = 25 25. $2\sqrt{3}$

26. $A = 45°, B = 15°$ 27. 40.61 *OR* $p = 7$ 28. $f_1 = 8, f_2 = 12$

29. 10 km/h, 4 km/h 30. $2, -2, \sqrt{7}, -7$ 32. 2

34. Mean=26.4,Median=27.2, Mode=29.09

Model Question Paper – 6

1. (*b*) 2. (*c*) 3. (*b*) 4. (*a*) 5. (*d*) 6. (*b*) 7. (*c*)

8. (*b*) 9. (*c*) 10. (*d*) 11. 15 12. $x = 1$, $y = 2$ *OR* $x = 3$, $y = 2$

13. $k = 7$ 14. $\frac{49}{64}$ 15. Median Class: 145–150, Modal Class: 145–150

16.

More than 50	More than 55	More than 60	More than 65	More than 70	More than 75
50	48	42	34	20	5

17. 10 m 21. $21x^2 - 2x - 8$ 23. $BC = 5\sqrt{3}$, $AC = 10$ cm 24. $x = 12$ 25. 39

26. 18 *OR* ₹ 18000 and ₹ 14000 27. 2.7 cm 29. $x = 2$, $y = 4$; (–1, 0), (2, 4), (5, 0)

30. 58.75 32. $\sqrt{2}, -\sqrt{2}$, 1, 4

Model Question Paper – 7

1. (*d*) 2. (*b*) 3. (*c*) 4. (*a*) 5. (*c*) 6. (*b*) 7. (*d*)

8. (*c*) 9. (*c*) 10. (*a*) 11. $m = 300$, $n = 50$

12.

Less than 145	Less than 150	Less than 155	Less than 160	Less than 165	Less than 170
10	18	38	50	56	60

13. Median Class: 20–30, Modal Class: 20–30 15. 60° 17. $k = -7$ 18. 1

19. 39.71 20. 34.75 21. $\sqrt{5}, \frac{-3\sqrt{5}}{20}$ 26. ₹ 400, ₹ 30 *OR* 42 or 24

27. $25\sqrt{21}$ cm^2 29. $x = 2$, $y = 3$; (0, 6), (0, 1), (2, 3) 33. 17.5

34. $\sqrt{3}, -\sqrt{3}$, 2, –3

Model Question Paper – 8

1. (*a*) 2. (*c*) 3. (*b*) 4. (*a*) 5. (*d*) 6. (*b*) 7. (*b*)

8. (*a*) 9. (*a*) 10. (*c*) 11. $a = 23$, $b = 11$, $c = 7$ 12. consistent

13. $x = 2, y = -3$ 14. 170.2 15. 39.7

16. No, the correct correspondence $\frac{EF}{ST} = \frac{DE}{TU}$ 17. No, 18. *OR* $\frac{1}{2}$

21. 7, –5 *OR* ±18 22. 93 24. 24 cm 25. $\frac{6-\sqrt{3}}{2}$ *OR* 1 26. 0

27. 44.29 28. 62

29. $x = 2, y = 3$, Area of triangle with x-axis=7.5 sq. units, Area of triangle formed with y-axis= 5 sq. units. *OR* $(0, -7)$ $(0, 1)$

30. Speed of sailor = 10 km/h, Speed of current = 2 km/h 34. $f_1 = 9, f_2 = 16$

Model Question Paper – 9

1. (*b*) 2. (*b*) 3. (*d*) 4. (*b*) 5. (*c*) 6. (*c*) 7. (*c*)

8. (*b*) 9. (*d*) 10. (*c*) 11. 9 12. 6

13. No, since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so it has a unique solution 14. 53.18 17. 26.25 18. $\frac{16}{29}$

21. $k = 6$ 22. $pq = r$ *OR* 27 25. $x = 10$ *OR* 69.43%

26. 147.2 mm 30. 14x – 10 31. $x = 3, y = 3$; 1:1

34. Median = 53 *OR* Median = 220

Model Question Paper – 10

1. (*b*) 2. (*c*) 3. (*a*) 4. (*c*) 5. (*c*) 6. (*d*) 7. (*b*)

8. (*c*) 9. (*c*) 10. (*a*)

12. Speed of rowing = 6 km/h, Speed of current = 4 km/h *OR* $x = -2, y = 5$ and $m = -1$

13. –3, –2, 2 14. 23 15. $a = 12, b = 13, c = 35, d = 8, e = 5, f = 50$ 16. $x = 3$

21. $x = 2, y = 1$, consistent 22. $a = 1, b = 2$ *OR* $k = 5$ and $a = -5$

23. $A + B = 45°$ *OR* $A = 60°, B = 30°$ 24. $a = 12, b = 13, c = 35, d = 8, e = 5, f = 50$

27. $\frac{6-\sqrt{3}}{3}$ 28. Mean = 51.75, Mode = 55

32. Speed of train = 100 km, Speed of the car = 80 km/h 33. $x = 1, y = -1$